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ACCUMULATION AND CONSUMPTION IN MICROECONOMIC SYSTEM

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SUMMARY

Two main processes are common for an economic system. They are consumption and accumulation. The first one is described by utility function, either cardinal or ordinal one. The mathematical model for accumulation process can be constructed using wealth function introduced within the frame of irreversible microeconomics. Characteristics of utility and wealth functions are compared and a problem of extreme performance of resources exchange process is solved for a case when both the consumption and accumulation exist.

KEY WORDS

irreversible microeconomics, accumulation, consumption, wealth function, maximum income determination problem

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INTRODUCTION

Investigation of resources exchange processes using irreversible microeconomics methods is based on introduction of wealth function is an extensive parameter of an economic agent[1] [1]. Arguments of the wealth function are resources stocks in the agent at issue. So, it is postulated that welfare of the economic agent is monotonous dependence on the stocks. The higher level of the wealth function the agent can reach the more successful the agent is from the economic point of view. But there is consumption process besides stockpiling in any economic process. And utility theory emphasises the consumption process influence on prosperity of the agent. A. Smith wrote [2]:

> *The revenue of an individual may be spent either in things which are consumed immediately, and in which one day's expense can neither alleviate nor support that of another, or it may be spent in things more durable, which can therefore be accumulated, and in which every day's expense may, as he chooses, either alleviate or support and heighten the effect of that of the following day.*

So, because there are two main processes influencing the well-being of the systems, we should consider both of the processes simultaneously.

UTILITY FUNCTION

Let us consider features of wealth and utility functions. These functions describe welfare effects of accumulation and consumption processes correspondingly. Let us start from the utility function. Existence of the utility function is the basic principle of neoclassical microeconomic theory [3]. Utility function associates each point in space of the resources consumption intensities with a number. It is a definition of the utility function. The more value of this number the more welfare of the system is. The important feature of the consumption process is that any voluntary process in an economic system should increment the utility function. It means that *all consumption processes are not reversible ones*.

Another feature of utility function conception is that the system started from an arbitrary point will reach the equilibrium instantly.

It happens due to the model of time using in neoclassical theory. The crux of this model is following. The axis of time is divided onto several intervals so that intersection of any two intervals is equal to empty set and conjunction of all these intervals is equal to whole time axis. The point at each interval is selected and all processes in the system at issue concentrate to this point. That is why it is hard to understand whether arguments of the utility functions are either fluxes of consuming resources or stocks of the resources to be consumed in this time interval. That is why in the most of textbooks one can find "a basket" instead "a flux" or "a stock" and dimensions of arguments of the utility function as "kg", "items", etc. An example of this model is Edgeworth box - a model of resource exchange - where exchange of the resources is assumed to pass until a no trade equilibrium point at a contract curve would be reached. Then consumption occurs (this process is not considered in the model) and new turn of the process begins from the same initial point.

There exist three main concepts of utility function. They are cardinal utility, ordinal utility and a theory of revealed preferences. Cardinal utility function is postulated to be measured, the second partial derivatives are determined for it, Hessian is negatively defined, utility of sum of the subsystems is equal to sum of the subsystems utilities. Ordinal utility and concept of revealed preferences, which is close to ordinal utility, are very useful for description of resource exchange. It is connected with impossibility to measure either utility or increment of utility. Both of these concepts are used in economic theory. Cardinal features of the utility function are sufficient in the theory of risks, where some assumptions are made about the second derivatives of the utility function. Concept of revealed preferences is the basement of the microeconomic theory of consumption.

As the cardinal utility can be measured the unit of this function should be introduced. This unit is called "util" [4]. Why economists did not use units of money to measure the utility of economic agents? A possible reason is following one. The only optimal basket of goods corresponds to each value of the agent's income. Then it is possible to consider the agent's income as a argument of the utility function. This transformation is made in the theory of risks to determine propensity of the economic agent to run risks. It means that the same value of money income can correspond to different values of the utility functions for different agents. That is why another unit, common for all economic agets, is necessary for cardinal utility function.

WEALTH FUNCTION

Let us consider an economic agent characterised by stocks of resources X. Let X_0 be base resource namely money. Then any resource exchange process can be considered as superposition of process of exchange resource to money. For example, barter exchange is represented as two processes proceeding simultaneously. They are buying of the first resource such that amount of money gained from the selling process should be spent for buying. A value of each resource v including the base one is introduced [1]. It allows one to consider the wealth function as a state function of the system. This function was introduced in [1, 5, 6]. State function means that when the system changes its state the increment ΔS depends on change of the stock ΔX only and for any cyclic process in a space of resources stocks increment of the wealth function is equal to zero, because initial and final states in such a process coincide.

Let us formulate an additional restriction for the wealth function $S(X)$. This function for the system consisting of several agents should be equal to sum of the wealth functions of all agents. It other words the wealth function should be extensive variable of the system. It means that function $S(X)$ should be homogeneous of the first order [1].

Let us discuss the units of the wealth function. In a case when a value v_ν does not depend on X we can measure the wealth function in units of the ν-th resource. Then we accept $v_\nu = 1$.

In the opposite case when all values depend on X then the measuring unit of S has to be selected, it is similar problem to the selection of the measing unit for physical entropy. There is a special case, at least as theoretical possibility, the introduction of "ideal money". This resource does not take part in resource exchange processes. Ideal money has to fulfill the following conditions:

- value of this resource $v_{im} = 1$. It does not depend on the resources stocks,
- values of other resources does not depend on ideal money stock,
- the change of stock of ideal money is equal zero for all subsystems,

and measure the wealth function in units of ideal money. It means that one can use units of some currency in the only case when no exchange process including this currency flux is in the system at issue. In the case when all stocks are used in exchange processes one should introduce ideal money or some kind of "utils" (non-monetary units as it was mentioned in [8] to measure the wealth function.

One could see that features of the wealth function are very close to features of cardinal utility function. But there are some significant differences [7]. They are

- the wealth function is a state function of an economic agent,
- the arguments of the wealth function are stocks and the values of resources determines the flows of the resources. It allows to use a model of continuous time of processes in a macrosystem consisting of different agents,
- the wealth function determines a value of the base resource. It allows to describe either voluntary or compulsory processes.

More detailed description of the wealth function features can be found in [8].

In a consumption process the wealth function is decreasing. If q_i is the consumed quantity, then $\Delta S = -\Sigma_i\, v_i \cdot q_i < 0$. Incorporating the utility concept gives a possibility to consider the consumption as a no loss process too. The wealth decrease is compensated by the utility increase, that is $S(X - q_i) - S(X) + U(q_i) \geq 0$.

In an "equilibrium process" the equality can be used, then $-S(X - q_i) + S(X) = U(q_i)$, the utility of a consumption bundle is the wealth decrease caused by the consumption. In non-equilibrium situation the wealth decrease must be smaller than the utility produced by the consumption.

DESCRIPTION OF THE SYSTEM

Let us consider a system consisting of two subsystems namely a seller and a buyer. The seller is an active subsystem [9]. The aim of this subsystem is to maximise its stock of the base resource (money). This subsystem can fix the price p for resource exchange process. The buyer is characterised by existence of processes of accumulation and consumption in the subsystem. It has resources stocks $X = (X_0, ..., X_n)$, where X_0 is the base resource stock. Resources stocks increase because of exchange processes with environment and decrease because of consumption process $q(t)$:

$$\dot{X} = g(p, v) - q(t), \tag{1}$$

where $g(p, v)$ is intensity of resources exchange depending on difference between the value v of resource by the buyer and price p fixed by the seller, $q(t)$ is intensity of consumption.

RESOURCE EXCHANGE PROCESS

To determine the behavior of a subsystem with consumption let us consider a problem on minimal capital dissipation for the system [10]. The problem at issue is following:

To determine price dependency with respect to time corresponding to the maximal income of the seller while it sells the given amount G of the resource during the given time period $[0, \tau]$.

Let us formalise this problem for linear dependence of intensity of resource exchange process and scalar resource:

$$g(p, v) = \alpha(v - p), \tag{2}$$

where v is value of the resource determined by the buyer, p – price fixed by the seller. The objective functional is the total income of the seller

$$\int_0^\tau p\alpha(v-p)\mathrm{d}t \to \max_p . \tag{3}$$

Restrictions for the set of possible solutions are total amount of transferred resource

$$\int_0^\tau \alpha(v-p)\mathrm{d}t = G . \tag{4}$$

and balance equation for the resource stock of the buyer

$$\dot{X} = \alpha(v-p) - q(t), \quad X(0) = X_0 , \tag{5}$$

where $q(t)$ is intensity of consumption.

Hamilton function for Problem (3 – 5) can be written as following

$$H = \alpha(v-p)[p - \lambda + \psi(t)] - \psi(t)q(t) . \tag{6}$$

Here λ and $\psi(t)$ are undetermined multipliers; λ is a constant and $\psi(t)$ is a function of time such that $\psi(\tau) = 0$. Note that v is a function of stocks of the buyer. That is why necessary conditions of optimality $\partial H/\partial p = 0$, $\dot{\psi} = -\partial H/\partial X$ have the following form:

$$p = \frac{v + \lambda - \psi(t)}{2} , \tag{7}$$

$$\dot{p} = -\alpha \frac{\partial v}{\partial X} \frac{v - \lambda + \psi(t)}{2} . \tag{8}$$

Comparison of (9) with (6) gives

$$\dot{p} = \frac{\partial v}{\partial X}[\dot{X} + q(t)] . \tag{9}$$

If v is linear function of X: $v = A\text{-}c\cdot X$, then, taking into account (9) solution can be found from the linear differential equation

$$\dot{X} = -\alpha c X + \xi(t) , \tag{10}$$

where

$$\xi(t) = -\frac{\alpha c}{2}\int_0^t q(\theta)\mathrm{d}\theta + \frac{\alpha}{2}[A - \lambda] - q(t) + K ,$$

with λ determined from (4) and K from the condition $\lambda(\tau) = 0$. Solution of (10) is

$$X(t) = \mathrm{e}^{-\alpha c t}\left[\int_0^t \xi(\theta)\mathrm{e}^{\alpha c \theta}\mathrm{d}\theta + X(0)\right] . \tag{11}$$

For constant value of q function $\xi(t)$ is linear

$$\xi(t) = at + b ,$$

where a and b are constants dependent on λ.

Equation (11) can be rewritten as

$$X(t) = X(0)\mathrm{e}^{-\alpha c t} + \frac{\alpha}{c}\left[t - \frac{1}{c} + \frac{b}{a}\right] . \tag{12}$$

CONCLUSION

Process of consumption significantly influences the resource exchange. Pure exchange of resources is not a real process. There should be a reason why an economic agent needs the resource. One of the reasons is accumulation of the resource. Another one is consumption. Intensity of the consumption is determined by utility function of the agent. But the determination of this intensity is a secondary problem. In a general case the economic agent should solve a multicriteria problem where it takes into account increments of welfare due to accumulation, consumption and all other processes motivating exchange of the resource. But the driving force of the resources exchange process is the difference between vectors of values determined by the wealth function of corresponding subsystem and prices independently of the reasons motivating the economic activity.

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REMARKS

[1]That wealth function, introduced by K. Martinás is not the welfare function of traditional microeconomics.

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AKUMULACIJA I POTROŠNJA U MIKROEKONOMSKIM SUSTAVIMA

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SAŽETAK

Dva glavna procesa su uobičajena za ekonomski sustav: potrošnja i akumulacija. Potrošnja je opisana funkcijom korisnosti, kardinalnom ili ordinalnom. Matematički model za akumulaciju konstruira se pomoću funkcije bogatstva koju se uvodi u okviru ireverzibilne mikroekonomije. Značajke korisnosti i funkcije bogatstva su uspoređene. Problem krajnje mogućnosti procesa izmjene resursa riješen je za slučaj kad postoje i potrošnja i akumulacija.

KLJUČNE RIJEČI

ireverzibilna mikroekonomija, akumulacija, potrošnja, funkcija bogatstva, problem određivanja najveće dobiti