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INDECSA

The winner of the INDECS award for the best published article – INDECSA – for Volume 2 and year 2004, is Dr. Serghey A. Amelkin.

Dr. Amelkin is awarded for his article *Accumulation and consumption in microeconomic system*, INDECS 2(2), 119-125, 2004. That article received the largest number of votes during the process conducted by the Commission for choosing the best article in accordance with the propositions for INDECSA, as stated in the official web site of the Journal.

The official INDECSA presentation ceremony was held on May 26, 2005, in Zagreb – Croatia, during Decos 2005 – International Workshop *Describing Complex Systems*.

In the name of the INDECS Council and in our personal names let us congratulate Dr. Amelkin for winning the INDECSA and wish him fruitful future researches and success in professional life.

Zagreb, 1 July 2005 — Josip Stepanić and Josip Kasač

LIST OF REFEREES

The referees of articles published in the journal INDECS in year 2004, listed in alphabetic order, are:

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The contribution of the referees to the quality of the articles published is acknowledged.

Zagreb, 1 July 2005

Josip Stepanić

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THERMODYNAMIC MODEL OF CAPITAL EXTRACTION IN ECONOMIC SYSTEMS

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SUMMARY

In this paper the properties of the wealth function of an economic system are studied. An economic analog of the Gibbs-Duhem equation is derived. Equilibrium states and limiting profit extraction regimes in non-equilibrium economic systems are obtained for the Cobb-Douglas wealth function.

KEY WORDS

nonequilibrium and irreversible thermodynamics, economics, econophysics, financial markets, business and management

CLASSIFICATION

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INTRODUCTION

A *macro-system* is one that includes a large number of elements. It can only be controlled on a macro level by changing parameters averaged over the ensemble of its elements. Thermodynamic systems containing large numbers of molecules provide a classical, long-studied example of macro-systems. Economic systems containing large numbers of economic micro-agents provide a second important class of macro-systems. Further examples of macro-systems are given by migration systems, segregated systems, whose elements interact through a uniform single medium, etc.

An important feature of macro-systems is that direct contact between two macro-systems leads to a stochastic interaction between their elements on a micro-level. This occurs spontaneously and is irreversible, because it is necessary to supply the systems with external energy or capital to return them to their initial states.

Mathematical models of macro-systems can be divided into structural analytical models, which derive a system's behaviour from the behaviour and statistical properties of its micro-elements, and phenomenological models, which directly model macro behaviour. The macro-system (thermodynamic) approach to economics was developed by von Neumann, Samuelsen, Lihnerovich, Rozonoer, Martinás and others. A comprehensive list of references can be found in the reviews [1, 7] and the monographs [9, 10].

In this paper we will employ the following definitions:

1. The state of a macro- system is described by two types of variables – extensive and intensive. The former are proportional to the scale of the system, while the latter are independent of scale change. For example, in thermodynamics, volume, internal energy and mass are extensive while concentration is intensive. In economics, endowments of resources and capital are extensive, while resource prices are intensive. Extensive variables in an isolated macro-system obey balance equations. For systems with transformers such as chemical reactors or production companies, these balances govern the transformation of one type of extensive variables into another, for example.
2. We consider three types of sub-systems:

   Systems with infinite capacity and constant intensive variables (*reservoirs*). For example, heat reservoir in thermodynamics or market in economics where the trading flows are so large that the influence of an individual trader on prices is infinitely small and prices are constant(prices do not depend on the trading volume).

   Finite capacity systems, with intensive variables that depend on its extensive variables for fixed time scale. For example, the temperature of a thermodynamic system with finite heat capacity depends on its internal energy. For economic system with finite capacity resource's estimate depend on its endowment. We will also refer to a finite capacity economic system as economic system.

   Active systems with controllable intensive variables. For example, a working body of a heat engine or an economic intermediary that operates between economic systems.
3. Kinetics of exchange processes. The difference of intensive variables of two contacting macro-systems with finite capacities leads to an emergence of exchange flows. These flows in turn depend of the intensive variables of contacting systems and are directed in such a way that the values of intensive variables move closer. In equilibrium these values are the same and there are no flows.

Similarly to the approach adopted in finite-time thermodynamics ([9, 16, 17]) we assume that the economic system under consideration consists of subsystems in internal equilibrium and that all irreversibility is concentrated on the contact surfaces between these subsystems.

The maximal work problem of transforming a non-organized form of energy (heat, chemical energy) into an organized form (mechanical work, electric current, work of separation) plays the major role in thermodynamics. Its solution led to the introduction of exergy, the maximal amount of nonorganised energy that can be transformed into work. This measure does not take into account the rate of work (transformer's power). Accounting for this constraint led to the replacement of exergy with a more general notion of *work capacity* [12].

The problem of extracting the maximal capital from a macro-system, with subsystems that have different initial endowments of liquid capital and illiquid capital in the form of various resources, represents microeconomic analogy of thermodynamics' maximal work problem. In thermodynamics the work can be fully transformed into other forms of energy. Similarly all capital (money, basic resource) in microeconomics can be fully transformed into any other resources. Other resources can only be transformed into money if there is demand for it from the other economic system. We shall call the limiting amount of money that can be extracted from economic system subject to some conditions the *profitability* of this system. Production of work or extraction of capital is not possible unless the system includes active subsystems. In thermodynamics they are heat engines and other transformers, in economics they are economic intermediaries or production companies.

The special variable that gives a quantitative measure of irreversibility of system's processes plays a central role in macro-system's theory. When macrosystem approaches the equilibrium the value of this variable increases. In equilibrium it attains maximum. In thermodynamics this measure is called entropy. It has been proven that entropy is a function of extensive variables and is an extensive variable itself. Therefore the entropy is a homogeneous function of the degree one. In microeconomics equilibrium economic system is described by the wealth function S that depends on the stock of resources N and stock of capital N_0. Note that wealth function of a system that consists of a number of non-uniform subsystems is not additive. Furthermore, in the general case subsystems' wealth functions can have different dimensionality. The principle difference between thermodynamic and economic macro-systems is that in thermodynamics an exchange of only one type of material or energy is possible (heat transferred from a hot to a cold body). During this exchange the entropy of one of contacting subsystems can decrease but the entropy of the other will increase in such a way that the total system's entropy tends to maximum. In economics all exchanges are voluntary. Therefore the wealth function of each participant does not decrease. In most cases that is only possible for a multi-resources' exchange.

PROPERTIES OF WEALTH FUNCTION AND ANALOGY OF GIBBS-DUHEM EQUATION

The state of an economic system can be described by the vector of $N = (N_1; \ldots; N_n)$ resources and capital N_0. These are extensive variables. Economic system is prepared to sell resource N_i at a price that is not less than pi, and to buy it at a price that is not higher than p_i. We shall call p_i the equilibrium price estimate of the i-th resource by economic system. These estimates themselves are economic system's intensive variables. For a finite-capacity economic system p depends on N and N_0.

Suppose that active subsystem interacts with economic system by buying and selling its resources in such a way that the state of economic system changes cyclically and the exchange is executed at the equilibrium prices p. Then the increase of economic system's capital is

$$\mathrm{d}N_0 = -\oint p\mathrm{d}N . \qquad (1)$$

If $\mathrm{d}N_0$ was not equal zero than active subsystem would be able to extract arbitrary large profit by exchanging with one economic system without changing the state of the environment (if $\mathrm{d}N_0 < 0$ then active subsystem extracts resource in direct cycle, if $\mathrm{d}N_0 > 0$ then it does it in inverse cycle). This is not feasible in economics (Ville Axiom [14]) and therefore the integral (1) must be equal zero and a function $M(N)$ exists such that

$$p_M \mathrm{d}M = \sum_{i=1}^{n} p_i \mathrm{d}N_i \qquad (2)$$

Let us construct function S, such that

$$\mathrm{d}S = p_0(\mathrm{d}N_0 + p_M \mathrm{d}M) = p_0\left(\mathrm{d}N_0 + \sum_{i=1}^{n} p_i \mathrm{d}N_i\right). \qquad (3)$$

From Pfaffian forms theory it is known that for two variables N_0 and M there exists integrating multiplier p_0, such that $\mathrm{d}S$ is total differential.

The formal proof of the existence of the wealth function $S(N_0; N)$ in an economic system is similar to this sketch [2, 4, 10, 15]. The multiplier $p_0 = \mathrm{d}S/\mathrm{d}N_0$ is the estimate of the basic resource (capital) and the estimate of the i-th resource is

$$p_i = \frac{1}{p_0}\frac{\partial S}{\partial N_i}, \quad i = 1, \ldots, n. \qquad (4)$$

Expressing $\mathrm{d}N_0$ from (3) we get

$$\mathrm{d}N_0 = \frac{\mathrm{d}S}{p_0} - \sum_{i=1}^{n} p_i \mathrm{d}N_i . \qquad (5)$$

The wealth function and all its arguments are proportional to the scale of the system. Therefore, it is a homogeneous function of the degree one. From Euler theorem it follows that it can be written in the following form

$$S(N_0, N) = p_0\left(\mathrm{d}N_0 + \sum_{i=1}^{n} p_i \mathrm{d}N_i\right) = \sum_{i=0}^{n} \frac{\partial S}{\partial N_i}\mathrm{d}N_i \qquad (6)$$

The estimates of resources and capital $p_0(N)$; $p_i(N)$ here are homogeneous functions of the degree zero.

From (6) follows that

$$N_0 = \frac{S}{p_0} - \sum_{i=1}^{n} p_i N_i , \qquad (7)$$

$$\mathrm{d}N_0 = \frac{\mathrm{d}S}{p_0} + S\cdot\mathrm{d}\left(\frac{1}{p_0}\right) - \sum_{i=1}^{n} (p_i \mathrm{d}N_i + N_i \mathrm{d}p_i) . \qquad (8)$$

Comparison of equations (8) and (5) yields the following equation that links capital's estimate and resources' estimates

$$S\cdot\mathrm{d}\left(\frac{1}{p_0}\right) - \sum_{i=1}^{n} N_i \mathrm{d}p_i = 0 . \qquad (9)$$

Similarly comparing the differential S from (6) with the expression (3), we get

$$N_0 \mathrm{d}p_0 + \sum_{i=1}^{n} N_i \mathrm{d}(p_0 p_i) = 0 . \qquad (10)$$

The conditions (9) and (10) follow from the existence of function S and its homogeneity. They are economics analogies of Gibbs-Duhem equations. The following conditions also follow from the existence of function S:

$$\frac{\partial(p_0 p_i)}{\partial N_j} = \frac{\partial(p_0 p_j)}{\partial N_i} = \frac{\partial^2 S}{\partial N_i \partial N_j}, \tag{11}$$

$$\frac{\partial p_0}{\partial N_j} = \frac{\partial(p_0 p_j)}{\partial N_0} = \frac{\partial^2 S}{\partial N_0 \partial N_j}. \tag{12}$$

It is easy to see that from conditions (11), (12), it follows that

$$\frac{\partial p_i}{\partial N_j} + p_i \frac{\partial p_j}{\partial N_0} = \frac{\partial p_j}{\partial N_i} + p_j \frac{\partial p_i}{\partial N_0}, \quad i, j = 1, \ldots, n. \tag{13}$$

Conditions (11) and (12) are economic analogies of Maxwell equations.

One of the forms of wealth function that obeys the conditions (3 – 10), is the Cobb-Douglas function

$$S_k = A \prod_{i=0}^{n} N_i^{\gamma_i}, \tag{14}$$

with $A > 0$ a constant, $\gamma \geq 0$ and $\sum_{i=0}^{n} \gamma_i = 1$. An alternative form of S proposed by Martinás is

$$S_M = \sum_{i=1}^{n} g_i N_i \ln\left(\frac{N_0}{k_i N_i}\right), \tag{15}$$

where g_i and k_i are some constants.

For economic reservoir

$$S^0 = p^0\left(N_0^0 + \sum_{i=1}^{n} p_i^0 N_i^0\right), \tag{16}$$

where p_{00} and p_{i0} are constants. The dimensionality of function S is in units of currency of the corresponding economic system. The dimensionality of the estimates p_i is the amount of capital per unit of i-th resource.

Demand and supply functions are defined as dependencies of the amounts of resource sold (bought) on the price. If we consider its time-dependent version then demand and supply functions will describe the dependence of the flow of the traded resource on its price. This flow is equal zero if the price C_i is equal to the estimate p_i. The equations (9 – 13) show that estimates can not be arbitrary functions of resources' endowments. They must be homogeneous functions of zero degree that obey these equations.

This makes possible to model empirical data for nearly equilibrium economic systems. In [13] it was demonstrated on historic data for Sweden that the conditions (11) and (12) held during the periods when it was nearly equilibrium and broke down during economic crisis of 1930-th. Fulfilment of the conditions (11) and (12) guarantees the existence of a function S.

EQUILIBRIUM IN ECONOMIC SYSTEMS

We consider an economic system that has a wealth function and which includes m subsystems with given initial endowments of resources $N_\nu(0)$, $\nu = 1, \ldots, m$.

SYSTEM WITH ECONOMIC RESERVOIR

Economic reservoir corresponds to the perfect competition market with constant prices. They are determined by exogenous factors or by the conditions of non-accumulation of resource on the market.

The conditions of equilibrium in such systems is reduced to the equality of the resources' estimates in all subsystems to the market's prices

$$p_{i\nu}(\overline{N_\nu}) = p_i^0, \quad i = 1, \ldots, n, \nu = 1, ..., m. \qquad (17)$$

Here $\overline{N_\nu} = (\overline{N_0}, \overline{N_1}, ...)$ is vector of stocks of resources in equilibrium for ν-th subsystem. The balances of capital in each subsystem

$$\sum_{i=1}^{n}\left[\overline{N_{i\nu}} - N_{i\nu}(0)\right]p_i^0 = N_{0\nu}(0) - \overline{N_{0\nu}}, \quad \nu = 1, ..., m, \qquad (18)$$

are to be added to conditions (17). The system (17) and (18) determines $(n+1)m$ variables $\overline{N_{i\nu}}$, $i = 0, \ldots, n$, $\nu = 1, ..., m$.

Suppose that the wealth functions for each subsystem have Cobb-Douglas form (14)

$$S_\nu = A_\nu \prod_{i=0}^{n} N_{i\nu}^{\gamma_{i\nu}}, \quad \nu = 1, ..., m. \qquad (19)$$

The estimates then are

$$p_{0\nu} = S_\nu \frac{\gamma_{0\nu}}{N_{0\nu}}, \quad p_{i\nu} = \frac{\gamma_{i\nu} N_{0\nu}}{\gamma_{0\nu} N_{i\nu}} = S_\nu \frac{\gamma_{i\nu}}{p_{0\nu} N_{i\nu}}. \qquad (20)$$

Let us introduce the variable

$$V_\nu = N_{0\nu} + \sum_{i=1}^{n} p_i^0 N_{i\nu}, \quad \nu = 1, ..., m, \qquad (21)$$

called *capitalization* of the ν-th subsystem in terms of market prices. The condition (18) states that capitalization is constant during equilibrium exchange

$$\overline{V_\nu} = V_\nu(0) = V_\nu. \qquad (22)$$

The conditions (17) take the form

$$\frac{\gamma_{i\nu} \overline{N_{0\nu}}}{\gamma_{0\nu} \overline{N_{i\nu}}} = p_i^0, \quad \nu = 1, ..., m, i = 1, \ldots, n. \qquad (23)$$

The solution of the system (22) and (23) has the form

$$\overline{N_{i\nu}} = V_\nu \frac{\gamma_{i\nu}}{p_i^0}, \quad \overline{N_{0\nu}} = V_\nu \gamma_{0\nu}, \quad i = 1, \ldots, n, \nu = 1, ..., m. \qquad (24)$$

The corresponding equilibrium value of the wealth function is

$$\overline{S_\nu} = V_\nu \gamma_{0\nu}^{\gamma_{0\nu}} \prod_{i=1}^{n} \left(\frac{\gamma_{i\nu}}{p_i^0}\right)^{\gamma_{i\nu}}, \quad \nu = 1, ..., m. \qquad (25)$$

If the vector of market's prices p_0 is not determined by the external factors but is set at such level that all resources offered at the *auction* are sold then in addition to the conditions of equilibrium (17) the capital balance (18) and conditions of non-accumulation of resources on the market are needed

$$\sum_{\nu=1}^{n}\left[N_{i\nu}(0) - \overline{N_{i\nu}}\right] = 0, \quad i = 1, \ldots, n. \qquad (26)$$

These conditions determine n variables p_i^0.

If S_ν has the form (19) then the equations (26) take the form

$$\frac{1}{p_i^0} \sum_{\nu=1}^{m} V_\nu \gamma_{i\nu} = \sum_{\nu=1}^{m} N_{i\nu}(0), \quad i = 1, \ldots, n. \qquad (27)$$

We denote

$$\overline{N_{i\Sigma}} = \sum_{\nu=1}^{m} N_{i\nu}(0)\,, \qquad \overline{N_{ij}} = \sum_{\nu=1}^{m} \gamma_{i\nu} N_{j\nu}(0)\,,$$

and after taking into account (21) the conditions (27) can be rewritten as a linear system

$$\overline{N_{i0}} + \sum_{j=1}^{n} p_j^0 \overline{N_{ij}} - p_i^0 \overline{N_{i\Sigma}} = 0\,, \quad i = 1, \ldots, n, \tag{28}$$

that determines price vector p^0.

SYSTEMS WITHOUT ECONOMIC RESERVOIR

Since economic exchange is a voluntarily action by an agent, it is possible to exchange a resource if and only if this resource estimates for contacting systems have opposite signs. For example, production waste has negative estimate for one subsystem and positive for another, which have a capability to process it. If these estimates have the same sign in both contacting systems then only an exchange where at least two resources are traded can take place (flow of a resource and counter flow of capital and counter flow of another resource (barter)). It turns out that a state where vectors of estimates p for all subsystems are identical and any exchange that increases the wealth function of ν-th function

$$S_\nu = p_{0\nu}\left(N_{0\nu} + \sum_{i=1}^{n} p_i N_{i\nu}\right) = p_{0\nu} V_\nu\,, \tag{29}$$

reduces the wealth function of at least one other contacting subsystems, is an equilibrium state. That is, in economics (unlike thermodynamics) all Pareto-optimal states turned out to be equilibrium states. Some of these equilibrium states correspond to an exchange via an auction when the prices are determined by the conditions of non-accumulation (26). In this case capitalization V_ν of each subsystem in equilibrium is equal to the initial capitalization, which determines the equilibrium distribution of the basic resource.

If functions S_ν have all the same dimensionality (which is not always the case) then it is possible to single out the state in the Pareto set for which the value of the wealth function is maximal. This means that transfer into another equilibrium state would not give wealth function gains to some subsystem sufficient high to offset losses to wealth function of other subsystems.

It is clear that this maximal wealth function state corresponds to the equality of capital estimates

$$p_{0\nu} = p_0\,, \quad \nu = 1, \ldots, m. \tag{30}$$

which determines, jointly with conditions of equilibrium and conditions of non-accumulation, the distribution of all resources.

EXTRACTION OF CAPITAL

Consider a system with an active subsystem whose goal is to extract capital. For simplicity we assume that this subsystem is an intermediary, which resells resources without processing it.

UNCONSTRAINT EXCHANGE TIME

Extraction of capital from a system is only possible if the its initial state is non-equilibrium, that is, if vectors of estimates $p_\nu(0)$ for its subsystems are different. The process terminates in equilibrium when

$$p_{i\nu}(\overline{N_{0o}}, \overline{N_\nu}) = p_i^0\,, \quad i = 1, \ldots, n, \nu = 1, ..., m. \tag{31}$$

Maximum of the extracted capital corresponds to the minimum of the following expression

$$\sum_{\nu=1}^{m} \overline{N_{0\nu}} \to \min\,. \tag{32}$$

Intermediary buys resources at lowest prices (from subsystems with estimates of the i-th resource lower than p_i^0) and sells it at the highest prices (to subsystems with estimates higher than p_i^0). Both buying and selling are reversible with zero Increments of wealth function. The state of equilibrium is determined by $m\times n$ conditions (31), m reversibility conditions

$$S_\nu(\overline{N_{0o}},\overline{N_\nu}) = S_\nu(N_{0\nu}(0), N_\nu(0)) = S_\nu, \quad \nu = 1, ..., m, \tag{33}$$

and condition of non-accumulation of resources by the intermediary

$$\sum_{\nu=1}^{n}\left[N_{i\nu}(0) - \overline{N_{i\nu}}\right] = 0, \quad i = 1, \ldots, n. \tag{34}$$

The system (31), (33) and (34) determines $(m+1)n$ subsystems' state variables and n equilibrium estimates p_i. Naturally in equilibrium in a system with an intermediary N_0 and N are different from equilibrium during a direct exchange. Maximum of the extracted capital is

$$M = \sum_{\nu=1}^{m}\left[N_{0\nu}(0) - \overline{N_{0\nu}}\right]. \tag{35}$$

For the Cobb-Douglas wealth function (19) the conditions of equilibrium take the form (23), where instead of condition of constant capitalisation (22) one needs to use the condition of constancy of S_ν

$$A_\nu \prod_{i=0}^{n} \overline{N_{i\nu}}^{\gamma_{i\nu}} = S_\nu(0), \quad \nu = 1, ..., m, \tag{36}$$

jointly with equation (34).

DISSIPATION AND CAPITAL EXTRACTION IN A FINITE TIME IN A CLOSED ECONOMIC SYSTEM

If the duration of the process is finite and constraint then the increment of the wealth function and the amount of extracted capital depend on the demand and supply functions (that is, the dependencies of the flow rates of resources on the price differentials). When an intermediary buys resource from economic system in a finite time is has to increase the offered price above the equilibrium price. As a result it spends more capital. Similarly during a sale in a finite time an intermediary has to give a discount on the equilibrium price. This reduces its capital. The product of the flow between two EA on the difference between the prices of buying and selling describes the current losses of capital due to the factor of irreversibility capital dissipation)

$$\sigma(p,c) = g(p,c)(p-c). \tag{37}$$

Capital dissipation measures irreversibility of the processes in the system.

Reciprocity conditions for flows that linearly depend on price differences

The causes of resource-exchange flows (their "driving forces") is the differential between resources' estimate by the economic systems and the price offered by an intermediary. Suppose that deviations from the equilibrium are small and the flows can be considered as linear functions of the price estimate differences.

The driving force for the i–th resource is $\Delta_i = p_i - c_i$. We denote the flow directed to economic system as positive. We get

$$g_i = \sum_{\nu=1}^{m} a_{\nu i}\Delta_\nu = \sum_{\nu=1}^{m} a_{\nu i}(p_\nu - c_\nu), \quad i = 1, \ldots, n. \tag{38}$$

We shall call matrix A with elements $a_{i\nu}$ the matrix of kinetic coefficients of economic system. It determines kinetics of its exchange with environment. The flow of resource causes the counter-flow of capital such that

$$\frac{dN_0}{dt} = -\sum_{i=1}^{n} c_i g_i \ . \qquad (39)$$

The change in the value of the wealth function here is

$$\frac{dS}{dt} = \frac{\partial S}{\partial N_0}\frac{dN_0}{dt} + \sum_{i=1}^{n}\frac{\partial S}{\partial N_i} g_i = -p_0\sum_{i=1}^{n} c_i g_i + p_0\sum_{i=1}^{n} p_i g_i =$$

$$= p_0\sum_{i=1}^{n}(p_i - c_i)g_i = p_0\Delta^T A\Delta \ . \qquad (40)$$

Δ is the vector of driving forces.

Because capital's estimate $p_0 > 0$, resource exchange can be executed with buyer's and seller's consent and wealth function does not decrease, the matrix is positive. Let us show that it is also symmetrical. Indeed, if driving forces are expressed in terms of flows using equation (38) then for any infinitesimally short time period the expression (40) will take the form

$$\frac{dS}{dp_0} = dN^T B dN \ , \qquad (41)$$

where dN is the vector-column of increases of resources' stocks, $B = A^{-1}$. The elements $b_{i\nu}$ of this matrix are

$$b_{i\nu} = \frac{\partial^2}{\partial N_i \partial N_\nu}\left(\frac{S}{p_0}\right) = b_{\nu i} \ , \quad i = 1, \ldots, n, \ \nu = 1, \ldots, m. \qquad (42)$$

Thus, the matrix B is positive and symmetrical. Therefore its inverse demand/supply matrix is also symmetrical and positive near equilibrium. The following reciprocity relations hold: *the effect of the difference between price and estimate of the ν-the resource on the flow of i-th resource is the same as the effect of the difference between the price and estimate of the i-th resource on the flow of the ν-th.*

The optimal buying (selling) of resource for linear resource exchange

System with one finite-capacity economic subsystem. Consider a system with one finite capacity subsystem (economic system) and an active subsystem (intermediary). Suppose the initial and finite states of economic system are given $N(0) = (N_0(0), N_1(0), \ldots, N_m(0))$, $\overline{N} = (N_0(\tau), N_1(\tau), \ldots, N_m(\tau))$. The intermediary sets such vector of prices $c(t) = (c_1(t), c_2(t), \ldots, c_m(t))$, that the final capital of economic system $N_0(\tau)$ is minimal. First we assume that the flow depends linearly on the driving forces

$$g = A\Delta = A(c - p) \ . \qquad (43)$$

Matrix A with $m\times m$ elements a_{ij} is positive and symmetrical, $(c - p)$ is the vector with elements

$$\Delta_i = c_i - p_i(N). \qquad (44)$$

We denote $\delta_i = N_m(\tau) - Ni(0)$ and rewrite the problem as follows

$$\overline{N_0} = N_0(\tau) = N_0(0) + \int_0^\tau \sum_{i=1}^{m} c_i(t) \sum_{j=1}^{m} a_{ij}(c_j - p_j)dt \to \min_{c(t)} \ , \qquad (45)$$

subject to constraints

$$\int_0^\tau g_i(t)dt = -\delta_i \ , \quad i = 1, \ldots, m. \qquad (46)$$

The problem (44 – 46) corresponds to maximum of the extracted capital as $M = N_0(0) - N_0(\tau)$.

Let us express c in terms of g using (43)

$$c(g) = p + A^{-1}g = p + Bg \ . \qquad (47)$$

The problem

$$\overline{N_0} = N_0(0) + \int_0^{\tau} g^T (p + Bg)dt \to \min_g , \tag{48}$$

subject to constraints (46) gives lower bound on the optimal solution of the problem (44 – 46), because the condition (45) has been deleted. If this solution is realisable, that is, if it obeys the condition (45), then the solution of the problem (48) and (46) is also a solution of (44 – 46).

Because matrix B is symmetrical and positive the problem (48) and (46) is a convex averaged problem of non-linear programming. Its optimal solution is constant and equal to

$$g_i^* = -\frac{\delta_i}{\tau} = \frac{N_i(0) - N_i(\tau)}{\tau}, \quad i = 1, \ldots, m. \tag{49}$$

The corresponding solution of the equations (45) is realisable ($N_i^*(t) \geq 0$)

$$N_i^*(t) = N_i(0) - \frac{N_i(0) - N_i(\tau)}{\tau} t, \quad i = 1, \ldots, m. \tag{50}$$

Substitution of this dependence into $p_i(N)$ will determine $p_i^*(t)$ and the equation (47) yields the optimal price $c^*(t)$.

System with a number of economic subsystems. Consider a system with n economic systems and an intermediary. Intermediary buys resource from some subsystems and sells it to others. The maximum of the extracted capital corresponds to its minimum in all economic systems at time τ. That is, the solution of the problem

$$\sum_{\nu=1}^{m} N_{\nu 0}(\tau) = \sum_{\nu=1}^{m} \left(N_{\nu 0} - \int_0^{\tau} \sum_{i=1}^{n} c_{i\nu}(t) g_{i\nu}(t,c) \mathrm{d}t \right) \to \min_c , \tag{51}$$

where the conditions (45) hold for each economic system, and condition (46) is replaced with the condition of non-accumulation of resources by the intermediary

$$\sum_{\nu=1}^{m} \int_0^{\tau} g_{i\nu}(t)dt = 0, \quad i = 1, \ldots, n. \tag{52}$$

The values of $\overline{N_{i\nu}}$ in this problem are free.

Because both buying and selling should proceed optimally, the flows of resource should be constant and must obey the conditions (49)

$$g_{i\nu}^* = \frac{N_{i\nu} - \overline{N_{i\nu}}}{\tau}, \quad i = 1, \ldots, n, \nu = 1, ..., m. \tag{53}$$

From (53) it follows that the criterion (51) is determined by the subsystems' final states

$$\overline{N_0} = \sum_{\nu=1}^{m} \overline{N_{\nu 0}}(\overline{N_\nu}) \to \min_{\overline{N_\nu}} . \tag{54}$$

$\overline{N_\nu}$ must be chosen in such a way that $\overline{N_0}$ is minimal subject to constraint (52), which takes the form

$$\sum_{\nu=1}^{m} \overline{N_{\nu i}} = \sum_{\nu=1}^{m} N_{\nu i}(0) = \overline{N}(0), \quad i = 1, \ldots, n. \tag{55}$$

The conditions of optimality (54) and (55) on $\overline{N_{\nu i}}$ for this problem take the form

$$\frac{\partial \overline{N_{\nu 0}}}{\partial \overline{N_{\nu i}}} = -\lambda_i, \quad i = 1, \ldots, n, \nu = 1, ..., m. \tag{56}$$

Since $\partial \overline{N_{\nu 0}} / \partial \overline{N_{\nu i}} = -c_{\nu i}(\tau)$, (56) is reduced to the condition that at time τ the buying and selling prices must be the same for all economic systems for each kind of resources $c_{\nu i}(\tau)$ =

λ_i, $\forall\ \nu$. The conditions (55) and (56) consist of $n(1 + m)$ equations with respect to unknowns λ_i, $i = 1, \ldots, n$ and $\overline{N_{\nu i}}$, $i = 1, \ldots, n$, $\nu = 1, ..., m$. The dependencies $\overline{N_{\nu 0}}$ on $\overline{N_{\nu i}}$, in turn are determined by $g^*_{i\nu}(\overline{N_{\nu i}})$, $p_\nu(N)$ and by the matrix B via the equation (47).

Their substitution into (54) allows us to find the minimum of the residual capital and therefore, the maximum of the extracted capital.

Conditions of optimal trading for non-linear dependence of flows on price differences

Consider the problem of optimal buying (selling) of resource for non-linear resource-exchange law. This problem for scalar resource was considered in [11], [12]. We denote the amount of resource as ΔN and the duration of exchange as τ. The problem of optimal buying takes the form

$$\overline{N_0} = N_0(\tau) \to \min_{c(t)}, \tag{57}$$

subject to constraints

$$\int_0^\tau g(c, p(N_0, N))\mathrm{d}t = \Delta N, \tag{58}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -g(c, p(N_0, N)),\ N(0) = N^0, \tag{59}$$

$$\frac{\mathrm{d}N_0}{\mathrm{d}t} = cg(c, p(N_0, N)),\ N_0(0) = N_0^0, \tag{60}$$

In this problem $c(t)$ is the price set by the intermediary, $p(N_0, N)$ is resource estimate by the subsystem, $g(c, p(N_0, N))$ is the flow of resource that depend on c and p in such a way that

$$\begin{aligned} &\mathrm{Sign}(g) = \mathrm{Sign}(c - p) \\ &g(c, p) = 0 \qquad c = p. \end{aligned} \tag{61}$$

The conditions of optimality for the problem (57-60) have the form ([11, 12])

$$\frac{\mathrm{d}}{\mathrm{d}N}\frac{\partial g/\partial c}{g^2(p,c)} = \frac{(\partial g/\partial p)(\partial p/\partial N_0)}{g^2(p,c)}. \tag{62}$$

In [11] and [12] it is also shown that criterion (57) is equivalent to criterion of minimal dissipation

$$\sigma = \int_0^\tau g(c, p)(c - p)\mathrm{d}t \to \min. \tag{63}$$

We consider the optimal buying problem for vector flows, where the flow of each i-th resource g_i ($i = 1, \ldots, n$) depends on the vector of prices $c = (c_1, \ldots, c_n)$ and estimates $p = (p_1, \ldots, p_n)$. Here the minimum of spent capital corresponds to the problem with criterion (57) subject to constraints

$$\sigma = \int_0^\tau g(c, p)(c - p)\mathrm{d}t \to \min. \tag{63}$$

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$$\int_0^{\tau} g_i(c, p(N_0, N))\mathrm{d}t = \Delta N_i\ , \tag{64}$$

$$\frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}t} = -g_i(c, p(N_0, N)), \quad N_i(0) = N_i^0 \quad i = 1,\ \ldots, n\ , \tag{65}$$

$$\frac{\mathrm{d}N_0}{\mathrm{d}t} = \sum_{i=1}^{n} g_i(c, p(N_0, N)), \quad N_0(0) = N_0^0\ . \tag{66}$$

The maximal amount of residual capital decreases monotonically when τ increases, tending to reversible limit we already found. Indeed, if this dependence was not monotonic then the intermediary could stop exchange at $\tau_\mathrm{i} < \tau$ when this capital was minimum. This means that during an optimal process the r.h.s. of the equation (66) has one sign. This allows us to choose the new independent variable

$$\mathrm{d}t = \frac{\mathrm{d}N_0}{\sum_{i=1}^{n} c_i g_i(c, p)}\ , \tag{67}$$

and replace minimisation of the residual capital with minimisation of the duration of the process for given $\overline{N}_0$

$$\tau = \int_{N_0^0}^{\overline{N}_0} \frac{\mathrm{d}N_0}{\sum_{i=1}^{n} c_i g_i(c, p)} \to \min_{c(t)}\ , \tag{68}$$

subject to constraints

$$\int_{N_0^0}^{\overline{N}_0} \frac{\mathrm{d}N_i}{\mathrm{d}N_0}\mathrm{d}N_0 = -\int_{N_0^0}^{\overline{N}_0} \frac{g_i(c, p(N_0, N))\mathrm{d}N_0}{\sum_{i=1}^{n} c_i g_i(c, p)} = \overline{N}_i - N_i^0 = \delta_i\ , \tag{69}$$

$$\frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}N_0} = -\frac{g_i(c, p(N_0, N))}{\sum_{i=1}^{n} c_i g_i(c, p)}, \quad N_i(N_0^0) = N_i^0 \quad i = 1,\ \ldots, n\ . \tag{70}$$

We assume that the solution of the problem (68 – 70) is not degenerate ($\psi_0 = -1$) and denote the scalar product as follows

$$\sum_j c_j g_j = (c, g), \quad \sum_j \psi_j g_j = (\psi, g)\ .$$

The Hamiltonian function of this problem is

$$H = -\frac{1 + (\psi, g)}{(c, g)}\ , \tag{71}$$

$$\frac{\mathrm{d}N_\mathrm{i}}{\mathrm{d}N_0} = -\frac{g_i}{(c, g)}, \quad N_i(N_0^0) = N_i^0, \quad N_i(\overline{N}_0) = \overline{N}_i, \quad j, i = 1,\ \ldots, n\ . \tag{72}$$

The weak conditions of optimality here are

$$\frac{\partial H}{\partial c_j} = 0 \Rightarrow H(c, g, \psi) = \frac{\sum_i \psi_i (\partial g_i / \partial c_j)}{g_j + \sum_i c_i (\partial g_i / \partial c_j)}\ , \tag{73}$$

$$\frac{\partial \psi_i}{\partial N_0} = -\frac{\partial H}{\partial N_i} \Rightarrow \frac{\partial \psi_i}{\partial N_0} = -\frac{1}{(c, g)} \sum_j \left\{ \left[\psi_j - c_j H(c, g, \psi)\right] \frac{\partial g_j}{\partial N_i} \right\}, \quad i = 1,\ \ldots, n\ . \tag{74}$$

Conditions (74) show that for an optimal process the expression in the r.h.s. of this equation has the same value for all *j*. The boundary conditions for adjoint variables are determined by the boundary conditions $N(0) = N_0^0$ and $N(\tau) = \overline{N}$. $\overline{N}_0$ can be viewed as a parameter. This parameter can be determined from the condition that in the optimal process the integral (68) equals τ. Conditions (74) are a system of linear equations with respect to vector of adjoint variables ψ. After elimination of ψ from (73) the optimality conditions can be reduced to a form similar to (62).

MAXIMAL RATE OF PROFIT EXTRACTION IN OPEN ECONOMIC SYSTEM

STATIONARY STATE OF AN OPEN ECONOMIC SYSTEM WITH LINEAR RESOURCE-EXCHANGE LAWS, PRINCIPLE OF MINIMAL CAPITAL DISSIPATION

Consider an open microeconomic system shown in Fig. 1.

Suppose the system is in a stationary state; each of its *n* subsystems ($i = 1, \ldots, n$) exchanges *m* types of resources g_{ij}^{ν} ($i, j = 1, \ldots, n$; $\nu = 1, \ldots, m$) with other subsystems; the flows of resources depend linearly on the differences of estimates $\Delta_{ij}^{\nu} = \overline{p_j^{\nu} - p_i^{\nu}}$. For each subsystem and each resource these flows are constrained by the conditions of the balance

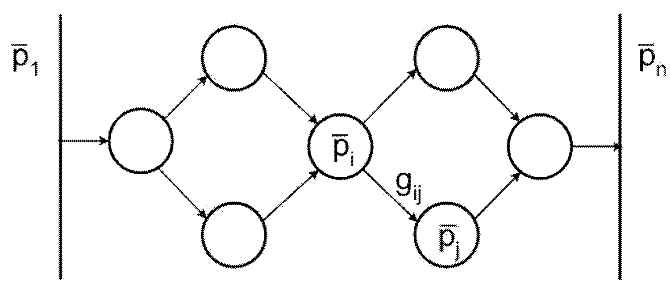

Figure 1. The structure of an open microeconomic system.

$$\sum_{j=1}^{n} g_{ij}^{\nu}(\Delta_{ij}) = 0, \quad i = 2, \ldots, n-1, \quad \nu = 1, \ldots, m. \tag{75}$$

Here Δ_{ij} is the vector of driving forces with components Δ_{ij}^{μ}.

If these flows depend linearly on the estimates' difference then (see (38))

$$g_{ij}^{\nu} = \sum_{\mu=1}^{m} a_{ij}^{\mu\nu} \Delta_{ij}^{\mu}, \quad i, j = 1, \ldots, n, \quad \mu, \nu = 1, \ldots, m. \tag{76}$$

For i-th subsystem ($i = 2, \ldots, n$-1) the vector of resources' estimates p_i depends on its endowments of resources. The market prices $\overline{p}_1$ and $\overline{p}_n$ for corresponding markets where these resources are bought and sold are fixed. The capital dissipation can be written as

$$\sigma = \frac{1}{2} \sum_{i,j=1}^{n} \sum_{\nu=1}^{m} g_{ij}^{\nu} \Delta_{ij}^{\nu}. \tag{77}$$

The multiplier 1/2 appears here because each flow enters (77) twice. The function σ characterises the irreversible losses necessary for maintaining resources' flows (trading costs). After taking into account (76) the capital dissipation can be rewritten as the following quadratic form

$$\sigma = \frac{1}{2}\sum_{i,j=1}^{n}\sum_{\mu,\nu=1}^{m} a_{ij}^{\mu\nu}\Delta_{ij}^{\mu}\Delta_{ij}^{\nu} . \tag{78}$$

If all matrices A_{ij} with elements $a_{ij}^{\mu\nu}$ are positive then the matrix of this quadratic form is positive and $\sigma \geq 0$. After taking into account the reciprocity relations the condition of minimum of σ on $\overline{p}_{i\nu}$ $(i = 2, ..., n-1)$ is

$$a_{ij}^{\mu\nu} = a_{ij}^{\mu\nu}, \quad i, j = 1, ..., n; \quad \mu,\nu = 1, ..., m . \tag{79}$$

This condition and equality $\Delta_{ij}^{\nu} = p_j^{\nu} - p_i^{\nu}$ lead to the equalities

$$\sum_{j=1}^{n}\sum_{\mu=1}^{m} a_{ij}^{\mu\nu}\Delta_{ij}^{\mu} = 0, \quad i = 2, ..., n-1; \quad \nu = 1, ..., m . \tag{80}$$

which coincide with balance equations (75) if the flows have the form (76). Therefore the following statement holds: *stationary regime in an open microeconomic system that consists of internally equilibrium subsystems with flows that depend linearly on the estimates' differentials corresponds to such a distribution of resources between subsystems that capital dissipation is minimal*

This is the economic analogy of Prigogine minimal dissipation principle in irreversible thermodynamics.

CAPITAL EXTRACTION IN OPEN MICROECONOMIC SYSTEM WITH AN INTERMEDIARY

Consider economic system with an intermediary, two markets (economic reservoirs) and subsystems (Fig. 2).

The markets are described by the resources price vectors p_+ and p_-, linear resource exchange kinetics (linear dependence of flows on differentials of resources' prices (estimates))

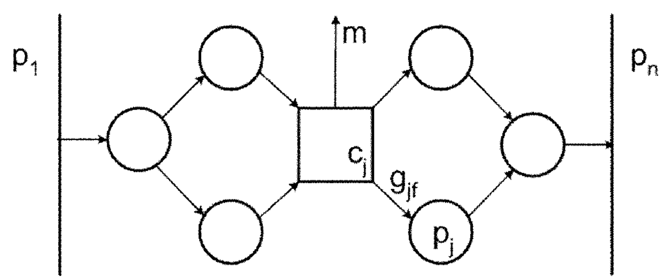

Figure 2. The structure of an open microeconomic system with an intermediary.

$$g_{ij\nu}(\Delta_{j\nu}) = \sum_{k=1}^{m} a_{ikj\nu}\Delta_{kj\nu}, \quad j,\nu = 1, ..., n . \tag{81}$$

Here $g_{ij\nu}$ is the flow of i-th resource between j and ν subsystems, $\Delta_{kj\nu} = p_{k\nu} - p_{kj}$. If one of the contacting subsystems is an intermediary which sets the price c_{kj} for buying k-th resource from i-th subsystem then $\Delta_{kjf} = c_{kf} - c_{kj}$. We denote the matrix of exchange coefficients between j-th and ν-th subsystems as $A_{j\nu}$ and between j-th subsystem and economic intermediary as A_{jf}. The flow of resource-exchange then is

$$g_{j\nu} = A_{j\nu}\Delta_{j\nu}, \quad g_{if} = A_{if}\Delta_{if} . \tag{82}$$

The flow of capital extracted from the system is

$$m = \sum_{j=1}^{n} c_j^T A_{jf} \Delta_{jf} \to \max_{c_j}, \qquad (83)$$

where c_j is the price vector with components c_{kj}. The flow of capital m attains maximum on c_j subject to the condition of non-accumulation of resources by the intermediary

$$\sum_{j=1}^{n} A_{jf} \Delta_{jf} = 0. \qquad (84)$$

The condition (84) is the system of linear equations $k = 1, ..., m$ that links the prices to the resource estimates p_{kj} for each of the passive subsystems. The problem (83) and (84) is convex and has a unique solution.

The resource estimates p_j, in turn, depend on the endowments resources N_j and capital N_{0j} as well as on the wealth function $S_j(N_j, N_{0j})$ of each economic system. They can be found from the condition that in a stationary state for any price vector $c = (c_1, ..., c_j, ..., c_n)$, the values of p_{kj} (estimates of the k-th resource in j-th subsystem) minimise the capital dissipation

$$\sigma = \frac{1}{2} \sum_{j,\nu=1}^{n} \Delta_{j\nu}^T A_{j\nu} \Delta_{j\nu} + \sum_{j=1}^{n} \Delta_{jf}^T A_{jf} \Delta_{jf} \to \min_{p}. \qquad (85)$$

Solution of the problems (85) and (83), (84) allows us to find the maximal flow of profit m, the corresponding resources' estimates p_j, $j = 1, ..., n$ and, if the wealth function is known, the distribution of resources between subsystems. The problem (85) should be solved subject to the condition of non-negativity of stocks N_j and N_{0j} in all subsystems that constraint the feasible set of estimates p.

CONCLUSION

In this paper we considered economic analog of the classical macro- system problem of extraction of an organized resource from a macro-system. In particular, we were concerned with the problem of extracting maximal capital from an economic system in infinite and finite times and with the problem of determining the maximal rate of capital extraction.

Conditions for the extraction of maximal capital from an open and a closed system with multi-component linear resource-exchange kinetics were obtained. The conditions that must hold for a stationary state in economic macrosystem with and without an intermediary were obtained.

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TERMODINAMIČKI MODEL IZDVAJANJA KAPITALA IZ EKONOMSKOG SUSTAVA

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SAŽETAK

U radu su proučena svojstva funkcije bogatstva ekonomskog sustava. Izveden je ekonomski analogon jednadžbe Gibbs-Duhem. Ravnotežna stanja i granični režimi izdvajanja dobiti iz neravnotežnog ekonomskog sustava su dobiveni za Cobb-Douglas funkciju bogatstva.

KLJUČNE RIJEČI

neravnotežna i ireverzibilna termodinamika, ekonomija, ekonofizika, financijska tržišta, poslovanje i menadžment

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ENTROPY – OUR BEST FRIEND

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We are the mirror as well as the face in it.
We are tasting the taste this minute
of eternity. We are pain
and what cures pain, both. We are
the sweet cold water and the jar that pours.
Rumi

SUMMARY

The paper tries to tackle the question of connection between entropy and the living. Definitions of life as the phenomenon that defies entropy are overviewed and the conclusion is reached that life is in a way dependant on entropy – it couldn't exist without it. Entropy is a sort of medium, a fertile soil, that gives life possibility to blossom. Paper ends with presenting some consequences for the field of artificial intelligence.

KEY WORDS

entropy, autopoiesis, life, living systems

CLASSIFICATION

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WHAT IS LIFE?

The aim of the present paper is search for new understanding of the role of entropy in connection to life and showing some consequences that arise from this. If I want to reach this goal, I have to start by listing some of the most common answers to question "What is life?"

In searching for the principal, determining characteristic of life we normally tend to slip into the enumeration of its vital functions like metabolism, reproduction, growth etc. Karl von Frisch's book "Du und das Leben" from the year 1949 is an example of such an approach. The deficiencies, or at least borderline cases (crystals, viruses, the planet Earth…), of such definitions are not hard to find. Looking for the characteristic functions of living organisms is important for medical and some biological purposes, but it does not tell us enough about the phenomenon of life itself.

Maturana and Varela [1] characterise the prevailing attitude of contemporary biology to the question of life as a combination of the physical-chemical and evolutionary approach. The first one explains biological processes from the point of view of chemical reactions going on inside living organisms. It focuses on processes such as cellular respiration and metabolism, the synthesis of proteins and also the genetic code, which is supposed to contain all information necessary for the synthesis of proteins and for life and the development of the organism in general. The second approach explains the emergence of biological processes as the result of random variations of the genetic code and natural selection of the phenotypes in which the genetic information gets realised. The first line of thought considers its basic biological unit to be the gene, for the second one this is the species[1].

Maturana and Varela [1, 2] do not question the physical-chemical foundation of living systems nor their gradual development through continuous interactions with the environment. They only doubt that the units of research selected this way (genes, species) could present us with a basis for our understanding of what is life in its essence. They claim that the question: What do all living systems have in common that makes us classify them as living beings? remains unanswered and always tacitly present somewhere in the background, even if most biologists tend to avoid it [1; p.74].

It is interesting that one of the most influential works on the question of life had not been written by a biologist but by a physicist. In his book "What is Life?" Erwin Schrödinger [3] presents a view of life starting from an utterly different perspective from contemporary biology. He takes into account the uniform nature of living beings, by which he manages to avoid reduction. Schrödinger suggests the following answer to the question: When do we consider something to be alive?;

"When it ´feeds´ on negative entropy." [3; ch.7].

The theory that living beings create negative entropy (the so-called syntropy or negentropy) has been picked up and developed in the last decades by the chemist Ilya Prigogine in his concept of *dissipative structures* (see e.g. [4]). A similar conception of the living can also be found in the work of one of the forefathers of cybernetics – Heinz von Förster, who compares living beings to the Maxwell demon in order to present the idea that living beings are actually entropy-retarders.

It is important to notice that in all the variants of the described theory the basic units of research are living beings in their entirety and not just one selected function or process (e.g. reproduction or metabolism). If the entropic definition of life is to appear plausible, we cannot consider living beings to be closed systems, as in such systems entropy can only grow or remain unchanged.

Living beings therefore must be open systems. But despite the fact that they are open, they are nevertheless also clearly separated from the environment in some way. This separation is, ontologically speaking, much stronger than for example the separation of the dewar (which can be considered to be an approximation of a closed system) from its environment. Thus, living systems are not closed in terms of the exchange of energy and matter, but they are "closed" in terms of preserving their identity. To emphasise these distinctions, Maturana and Varela distinguish between structurally and organisationally open or closed systems. Living organisms are thus structurally open and organisationally closed systems.

Schrödinger gave an expanded entropy equation for this kind of systems: $dS = d_eS + d_iS$, where dS stands for the entire change of entropy of a living system, d_eS stands for the flow of entropy through the system and d_iS stands for the production of entropy inside the system due to irreversible changes occurring in it. While the diS member is always positive, the d_eS member can also be negative and in its absolute value bigger than d_iS, meaning that the entire change of entropy in an open system can be less than zero. Thus, an open system can change in the direction of increased orderliness. Of course, this ordering in open systems feeds on the order of the (closed) wider system, which contains these open systems – namely, the environment. This containing system still change in the direction of lesser order according to the second law of thermodynamics. The increase of entropy represents the flow of entropy that has negative value from the point of view of the contained open systems and enables them to increase their inner order.

Under certain circumstances open systems can continuously perform work. For a system to be able to do that, it must not be in the state of stable equilibrium, rather, it has to "search" for such equilibrium [5]. Let us consider Bertalanffy's example of the water reservoir with high potential energy: one might open the reservoir and the water would start flowing from it in the direction of lower potential energy until it would reach a state in which its potential energy could go no lower (the state of stable or at least local equilibrium). In the meantime, it could perform some work, for example, it could make a turbine go. But it is obvious that this performing could only last for a limited period of time.

IN WHAT STATE SHOULD A SYSTEM BE IN ORDER TO PERFORM WORK CONTINUOUSLY?

That is the key question. Let me explain why: In the second part of his autobiography Karl Popper included a chapter entitled "Conversations with Schrödinger" [6]. In this chapter Popper challenges Schrödinger's definition of life (that which "feeds" on negative entropy) with the claim that also a common oil stove or a self-winding watch can do that. Therefore, that cannot be the defining characteristic of life. Schrödinger answers that living beings remain in the stationary state of relatively high orderliness (i.e. low entropy) by continuously extracting order from the environment (and are thus continuously capable of performing work).

The realisation that living beings are structurally open systems that can lower their inner entropy on the credit of the environment is important, but it is even more important to find out how they (we) manage to do that *continuously*.

The answer is: Structurally open systems have to search for the state of stable equilibrium, but in order to do that they must constantly remain outside that state. The systems succeeding in this are far from the so-called thermal equilibrium. The imperfective aspect of the verb "to search" implies a major change in our line of thought. We are dealing with systems whose goal is not to reach a given state, but to continue *searching for such a state*. The most appropriate way of describing them is perhaps the Zen saying: *the path is the goal*. In the rest of the paper I will reflect upon such searching systems.

NEGOTIATING

The state in which we find structurally open systems capable of continuously performing work is called the stationary state. The term "stationary" is perhaps not the most appropriate, since the described systems are actually constantly in motion. Stagnation would mark the change of their organisation and thus their identity. Naturally, their mobility cannot be of just any kind – it has to be *regulatory*. The characteristic variable (variables) of the system has to be maintained inside of given delimitations. Like a rope-walker, who has to keep balancing herself. The rope-walker "functions" far from the state of balance – the finding of a balanced position for her would mean certain death (or at least unspectacular messing around in the safety net).

Exactly the same goes for living beings. They need to balance fixation and flexibility. Just as adaptation to a given life-space is essential, so is the permanent fluidity. If the system became completely fixed, it would not only lose its "stationary" organisation, but it would also become unable to preserve its dynamic stability with the environment. The importance of this conclusion cannot be overemphasised. I believe that here we touch upon the principal pattern, characteristic of living systems. As we can see, the decrease of entropy or the preservation of available energy inside the system is not a finite task (one that has a reachable goal). It is vital that the system *persists in doing this*.

The linear way of thinking that we are used to cannot handle the imperfective aspect of the described process very well. If we take it, for example, that "searching" is the essence of the described systems, one would imagine a system searching for something and once it finds that thing, its task is done. Searching (like all other processes which can be described reasonably well) is a linear thing. It is a process with a beginning and an end and a transitory function: from *x* to *y*, from the problem to the solution. This roughly depicts the view on life (and cognition) held by prevailing approaches in artificial intelligence: living beings are supposed to be a type of so-called autonomous agents (entities that can solve problems in changing environment).

But the "searching"[2] as described here is intrinsically imperfective. We do not search to find, but *to keep searching*. Once we find it, we are lost. Obviously, the process in question is recursive and circular. We seem to have problems with describing it. (Zen koans appear to be the most appropriate way of doing this).

"A MISTAKE IS NOT AN ERROR" [7]

Before I go on, let me indicate one of the consequences of the described pattern. Pask characterises it with the sentence: "A mistake is not an error." [7]. Continuous, self-centred "searching" (or "testing") which enables us (living systems) to survive is also reflected in our experiential world. In this perspective, the "imperfections" found in any performing or concept (if we only delve deep enough analytically) prove to be necessary. The "fuzziness" of borders, indeterminacy and similar "bugs" are not just signs of a momentary incapability or lack of precision of the observer (or deficiencies of the theory), but a reflection of the amazing property of living (in this case also conscious) beings, which allows them to continuously adapt to a changing (entropic!) environment.

This ability of constant manoeuvring, this continuous search for equilibrium that is an aim in itself (I hope that the recursion here is obvious) is called *negotiating* by Varela, Thompson and Rosch [8]. The authors believe that this ability of living beings *to negotiate their way through a world*, which is "not pre-given and determined, but *constantly formed by a series of actions, which we engage in*, is a necessary condition for a richly interlaced and independent experiential world" [8].

LIFE IS A SYSTEM-PROCESS (SYSTEM-PHENOMENON)

An aeroplane is an aeroplane whether it happens to be flying at the moment of observation or not. Similarly, a computer remains a computer also when turned off. The organisation of its components, that is, the network of its internal relationships due to which we classify such a system as a computer (the identity of the system) is independent of whether the entire system is operational (performs certain operations) in a given moment or not. In living systems, the situation is completely different[3]. They must function constantly in order to exist. Their identity is therefore determined by their functioning or dynamics and not their structure. We can thus conclude that *existence is the only really important product of the functioning of living beings*.

In order to understand the nature of living systems, we must establish *a double view* [9], which allows us to see the pattern and the components through which it is embodied at the same time. Besides, this living system cannot be observed non-temporally – a frozen picture (i.e. only by exploring its structure) cannot tell us whether the system in question is indeed alive or not.

At this point we could ask ourselves: Is life a phenomenon? Is it perhaps a characteristic of a system? Or is it a particular kind of dynamics, which can "happen" to a particular type of systems? The answer is hard to find, since (in scientific language) there exist no appropriate categories for describing such phenomena. Life is a combination of dynamics and entity (structure) that changes. It is neither structure nor process, it is structure-process. Consciousness is neither body nor mind, it is body-mind. We are dealing here with indivisible wholes, composed of two levels: dynamics and structure that gets realised through it. More accurately: if we divide them, their essence is lost.

The scientific language endeavouring to follow as much as possible the ideal of the mathematical-logical language, does not include any appropriate structure for dealing with that. In logic there exist entities (logical variables), their properties and relations between them. In mathematics we find mathematical structures, their properties, relations between these structures and an active part in the form of functions. But there is no possibility of describing a structure-process. Because of that, the scientific language must necessarily objectify living beings (and from here the next step of trivialization is just around the corner). Now we can understand why Maturana and Varela felt a need to create a new language [1], which would be capable of dealing with circular phenomena (or better structure-phenomena) of the living world.

Exploring the properties of the living is not the only area affected by the deficiencies of the existing scientific language and concepts. A widely known example is also the problem of quantum physics undecided about how to classify quantum entities. These manifest a "double" nature: they have some properties of "proper", "normal" particles, but on the other hand they also show wave properties (a wave is *a pattern* of dynamics, a non-material thing). The problem was solved by de Brogli through coining the term *wave-particle* (and an appropriate mathematical formalisation – the de Brogli wave equation). Thus physics was forced to accept a completely new type of system – *a system which remains what it is only so far as it keeps doing what it is doing*[4]. A similar problem was encountered by computer experts who solved it in a pragmatic way, by introducing the so-called object-oriented languages in which the definition of some of the variables includes also their properties and functions.

Thus, life is a wave-particle, or rather, a structure-process. A living system remains alive as long as it has this double nature – "double" only from the point of view of the linear analytical-reductionist thinking. From a cybernetic point of view, the structure-process is a basic unit. And the conclusion from chapter two applies to it: if we "cut" it, the creative circle is broken and with that the domain it used to create is lost.

The above insight also suggests that life is a continuous process that cannot be interrupted. Thus, “turned off” living systems are the impossibility.

AUTOPOIESIS

It is quite obvious that the standard (scientific) vocabulary does not include an appropriate term for describing the imperfective process of searching for equilibrium or negotiating. There is also no model available to account for such processes. Let us take a look at what can actually be stated about them:

Since we are dealing with a repetitive process “curved on itself” it is safe to assume that there is recursion involved at some point. In the language of cybernetics, one could say that we are dealing with a self-regulatory system with a feedback mechanism. The big question is, of course, what are the elements of this mechanism and mostly – what is the essential variable preserved by the system. In the case of living beings, this feedback mechanism cannot be considered to have been “programmed” from the outside in order to regulate a certain variable (as in the case of the thermostat – heater system), but a loop functioning autonomously. This means it is also independent in “selecting” its manner of functioning.

This is exactly what Maturana and Varela had in mind when coining the term autopoiesis to describe the essential characteristic of the living. Autopoietic systems are those (auto-regulatory) systems, which preserve their own auto-regulatory nature. Thus, they do not preserve some externally prescribed parameter (e.g. the temperature in a room), but their very own manner of functioning. Notions such as “nature” or “manner of functioning” are mere approximations and do not describe the essence of autopoietic systems too well. That is why Maturana and Varela used the notion of organisation. Autopoietic systems are those, whose organisation has the property of preserving itself (*auto-poiesis*: self-production).

According to Maturana and Varela, the organisation of the system determines its class identity and must remain unchanged if this identity is to be preserved. From the point of view of organisation the concrete realisation of the components of the system is of no importance. What matters are only the abstract characteristics of the system and relations between them, which make this system belongs to a particular class. We can see that for most systems organisation is determined from the “outside”, i.e., by the observer. But there is an exception, autopoietic systems determining their identity (organisation) by themselves. And this self-defining is the source of their autonomy, according to Maturana and Varela.

BACK TO THE CONNECTION BETWEEN ENTROPY AND THE LIVING

The autopoietic theory is consistent with Schrödinger‘s thesis of life as that which defies entropy. Both theories discuss the living system as a closed system separated from the environment by its activity. This activity is self-preservation. While Maturana and Varela determine the meaning of this term, they do not explain how the emergence of such a system comes about nor what are the elements of the feedback mechanism that keeps living beings going in a state far from thermal equilibrium (i.e. maintains their flexibility – openness – while at the same time fixes them). I am afraid that also the present article will not solve these problems. But it can at least give it a try.

When talking about the autonomy of living systems that does not mean that a living (autopoietic) system could function without the environment. An autopoietic system is *structurally open and organisationally closed*. The above connection between the Schrödinger‘s definition and the autopoietic one renders another essential bond between the system and environment: the (entropic) dynamics of the environment allows for the autopoietic dynamics of the living system. If we "turn off" entropy, we also turn off the conditions for life.

The state of continuous searching (negotiating) can be reached only through the co-operation of two opposing forces. At this point we can think of the numerous examples of "self-propelled" feedback systems offered by cyberneticists.

ARTIFICIAL LIVING SYSTEMS?

Černigoj [10] defines autopoietic systems as

"any form of auto-regulation based on auto-regulatory systems, the recursive parameter of which is the organisation of the system inside which this auto-regulation takes place".

This definition emphasises the ability of the systems in question to dynamically change their own structure and with it also the auto-regulatory *mechanisms* (which are considered to be something static, what in the language of cognitive science would be designated as hard-wired). According to Černigoj, auto-regulatory processes allow for the self-organisation of the system and with this also for its non-trivial adaptation to the environment. This implies that the system is capable of *creating novelties* through which it can adapt even to unpredicted changes in the environment (which is non-trivial itself!). An auto-regulatory system that could balance the states of its recursive parameters exclusively by auto-regulatory *mechanisms* would be, by definition, incapable of such adaptation, since the essence of auto-regulatory mechanisms lies in the fact that they are based on pre-existing arrangements. Auto-regulatory mechanisms can compensate only for the disturbances the compensating of which they were intended for (when speaking of man-made auto-regulatory mechanisms) or those to which they are adapted (when speaking of naturally evolved auto-regulatory mechanisms). But we should keep in mind that from the point of view of an individual organism these adaptations exist as predetermined biological facts, whose philogenetic development is possible only due to the auto-regulatory processes working in the background.

In other words, the autopoietic organisation enables the system to change the strategies of its functioning. This is possible because the only constant in an autopoietic system is its organisation (i.e. its capability for preserving its capability of preserving its capability of preserving…). As long as this goes on, the system can change its structure and manners of functioning. An autopoietic system has no other "task" but to preserve its organisation. Its behaviour is of a negotiating character; one could say that given segments (bodily or temporal) of its being perform certain tasks, but if we look at it as a whole, we see that their only "task" is to endure. There is no way of telling if autopoietic systems have any other task (or if they are saving particular "problems", as can often be heard in the field of artificial intelligence).

This fact has far-reaching implications. Understood this way, the notion of autopoiesis implies that it is impossible to artificially create a system embodying this type of organisation! How are we supposed to design a robot or a computer programme whose only duty is the preservation of its preservation of its preservation…? (The task of a robot is always to fulfil the orders of its programmer.) It is a big mistake on the part of the researchers of artificial intelligence and artificial life *to set their systems inside a trivial environment*[5], *an entropy-free environment* (usually, that is some sort of a virtual world in a computer). As we have seen, entropy is not an enemy to life but rather a stimulus allowing for its constant battle, constant search for new forms of auto-regulation and adaptation (the task of which is again searching for new ways…). Entropy would represent an "enemy" or a problem to be "solved" only in case if life were a perfectly linear thing (in which case the solution would be quite simple. We could, for example, make a simple vacuum container, put a stone inside it and we would thus have made a system capable of defying entropy changes for a very long time). Fortunately, it is not so. Entropy or the imminent danger of decomposition, of death, of the termination of the autopoietic organisation is, so to speak, a necessary condition for the preservation of life.

It would appear that entropy in some way exists on a similar level as life itself – on the level of gestalt. Let us consider the following: (just like life) entropy as such cannot be modelled. We can model, for example, gas diffusion, we can model random increase of disorder etc. But the very essence of entropy is impossible to grasp. From this point of view, it would appear that life and entropy go hand in hand at some invisible level, only the reflections of which are visible to the scientific eye.

Thus, artificial systems are not located in an entropic environment. The first thing that comes to mind is, of course: then let us build an entropic environment and set our "living" software inside it. But as mentioned above, it is not as simple as that. It seems that building an artificial entropic system is just as complicated as building an artificial living system. As I mentioned before: we are able to model some of the effects of entropy, just as we are able to model certain processes of life. Thus, we are able to counterfeit *the appearance*, but not the autopoietic organisation itself. This mistaking of the appearance (of the structure) for its organisation (identity) is called the "PacMan sindrome" by Riegler [11]. Programmers, who want to create conditions as "natural" as possible for their artificial agents, tend to build *a priori* determined concepts into them. Artificial agents thus enter into interactions with man-determined entities such as "food" and "enemy", who make sense only to the programmer of the system. Doing so, they ignore questions like: How did the organisms get the idea that a given type of entity represents food? How can they "know" that some other being is a dangerous opponent? Beasts do not come equipped with little signs saying "I am your enemy". And even if they did – how could cognitive beings manage to learn to understand the meanings of such signs? Autopoietic systems create system-independent inner states or "meanings". But artificial systems are unable to create their meanings.

Another possible solution would be to expose our programme (or robot) to real/natural entropy. And here is where we notice the disadvantages of Schrödinger's definition and the advantages of the one formulated by Maturana and Varela. Maybe we could create an autonomous agent capable of changing its parts once they malfunctioned. But this would still not mean that we are dealing with a living system. It would still be a robot given a particular task (set by the programmer!). By its functioning it would not constitute itself as an ontologically separate unit. Living systems are autopoietic, meaning that they keep constituting themselves – not only in the physical sense but also in the sense of constituting themselves as ontological categories which get separated from the rest of the world exactly because of this self-constitution. Living systems preserve their organisation, but organisation cannot exist without a structure to be embodied in (this is what I was trying to point out when mentioning the "double view"). Entropy presents a constant threat to this structure and thus provides an opportunity for the autopoietic organisation to get realised.

CONCLUSION

Let me try to give a conclusion based on what was said previously:

Living systems must function in a non-trivial (entropic) environment. Non-triviality of the environment is a necessary condition for preserving the negotiating nature of living/autopoietic systems.

Entropy allows for a constant dynamics of living systems, the existence of which is obvious just as long as they remain active. A constant threat of decomposition, chaos and destruction is what keeps us going. We fight against them all our lives, but nevertheless they are our best friends. They are our allies making our existence possible, for this never-ending battle is just what life actually is.

REMARKS

[1]One of exceptions is perhaps Dawkins who tries to combine both aspects in his theory of the selfish gene and severe criticism of non-evolutionary conceptions.

[2]An alternative term would also be „striving".

[3]Let us consider the possibility that what was said in the above paragraph is wrong and we could make an artificial living being. Can you imagine switching such a creature off (just for the night, not to spend too much energy)?

[4]Whenever they can, physicists still talk about the "double" nature of "particles" and they are still irritated by the fact that they are unable to determine exactly both the position (a quantity defining the physical component) and the speed (defining the "kinetic" component) of such particles.

[5]A mistake also made by Varela who claimed that his cellular automata programmes, that simulated the organisational closure, were autopoietic systems.

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ENTROPIJA – NAŠ NAJBOLJI PRIJATELJ

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SAŽETAK

U radu je naznačeno pitanje povezanosti entropije i života. Definicije života kao pojave koja proturiječi entropiji su analizirane. Zaključeno je da je život na određeni način ovisan o entropiji – jer bez nje ne bi mogao postaojati. Entropija poprima značenje vrste medija, plodnog tla, koje omogućuje razvoj života. Članak završava prezentiranjem dijela posljedica za područje umjetne inteligencije.

KLJUČNE RIJEČI

entropija, autopoiesis, život, živi sustavi

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ABOUT MODELLING OF COMPLEX NETWORKS WITH APPLICATIONS TO TERRORIST GROUP MODELLING

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SUMMARY

Based on available data on various organisations and networks, the article explores some key properties of the structure of a terrorist organisation. Analytical focus is on lower levels of organisational hierarchy, where network structure with exponential distribution of the number of links among network nodes is clearly visible. Such networks tend to grow organically, are very efficient in information diffusion, and are robust regarding stochastic failures and targeted attacks. These network features are illustrated by recent example based on network data about September 11, 2001 attacks on New York and Washington.

KEY WORDS

complex networks, network structure, network properties

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INTRODUCTION

After catastrophic terrorist attacks by kidnapped airlines on New York and Washington in September 2001 the interest for al-Qaeda terrorist organisation in public and media rose immediately. Experts and analysts all over the world started to offer various explanations of al-Qaeda's origins, membership recruitment, modes of operation, as well as of possible ways of its disruption. Journalists in search of hot topics took over and publicized most of the publicly available materials, often revising them further and making them even more intriguing and attractive for wide audiences.

One could thus read or hear that al-Qaeda is "a net that contains independent intelligence", that it "functions as a swarm", that it "gathers from nowhere and disappears after action", that it is "an *ad hoc* network", "an atypical organisation", extremely hard to destroy, especially by traditional anti-terrorist methods. In a similar tone, the day after July 7, 2005 simultaneous explosions in London subway British Home Secretary referred to the attack as coming "out of the blue in a way that there was no knowledge of beforehand in any respect whatsoever" [1]. This statement is all the more surprising when we know that many state intelligence and security services have been exerting serious efforts for more than four years in attempting to disrupt al-Qaeda network. Descriptions like the ones above sound extraordinary, sometimes almost fantastic, and provoke questions such as which master mind, if any, created such powerful organisation and what efforts are needed to put it into operation. Fortunately, the amount of concrete data and facts on al-Qaeda and similar organisations is constantly growing and our understanding of their ways of functioning is improving.

The main purpose of this article is to shed some light on the structure of contemporary, relatively decentralised terrorist organisations of which al-Qaeda is a primary example. The intent is also to show how and why these networked organisations possess particular properties. As we shall see, many of these regularities are common for a whole spectrum of organisational forms, of which terrorist organisations are only a small part.

HIERARCHICAL ELEMENTS OF ORGANISATIONAL STRUCTURE

Military, paramilitary and other similar organisations have usually been associated with hierarchical structure. Execution of orders without complaints, minimal deviation from plans, close coordination resulting from strict division of labour, and precise common guidelines are all deemed necessary for the efficiency and control of such organisations.

Numerous terrorist organisations, especially those preoccupied with ideological or nationalist goals, are structured according to similar principles. Although this may sound paradoxical, these organisations attempt to strictly control the amount of violence. Violence is usually subordinate to the achievement of other, often political, goals such as acquiring sympathies in public, attracting attention, exerting pressure needed for political negotiations, exhausting the opposing side, and lowering its morale. For example, it is probably more than a pure coincidence that the IRA refrained from using violence some ten days after the London subway attacks [2]. The desire to distinguish itself from perpetrators of massive, non-discriminating violent acts was likely one of the factors influencing IRA's decision. To control the amount of violence it is necessary that organisation's "political arms" control terrorist activities. Lower-ranking members need to be under strict control of higher headquarters in order to prevent undesirable individual transgressions. This is why the organisations like the Red Brigades, ETA, or IRA are strictly hierarchically structured [3] (see Fig. 1 also).

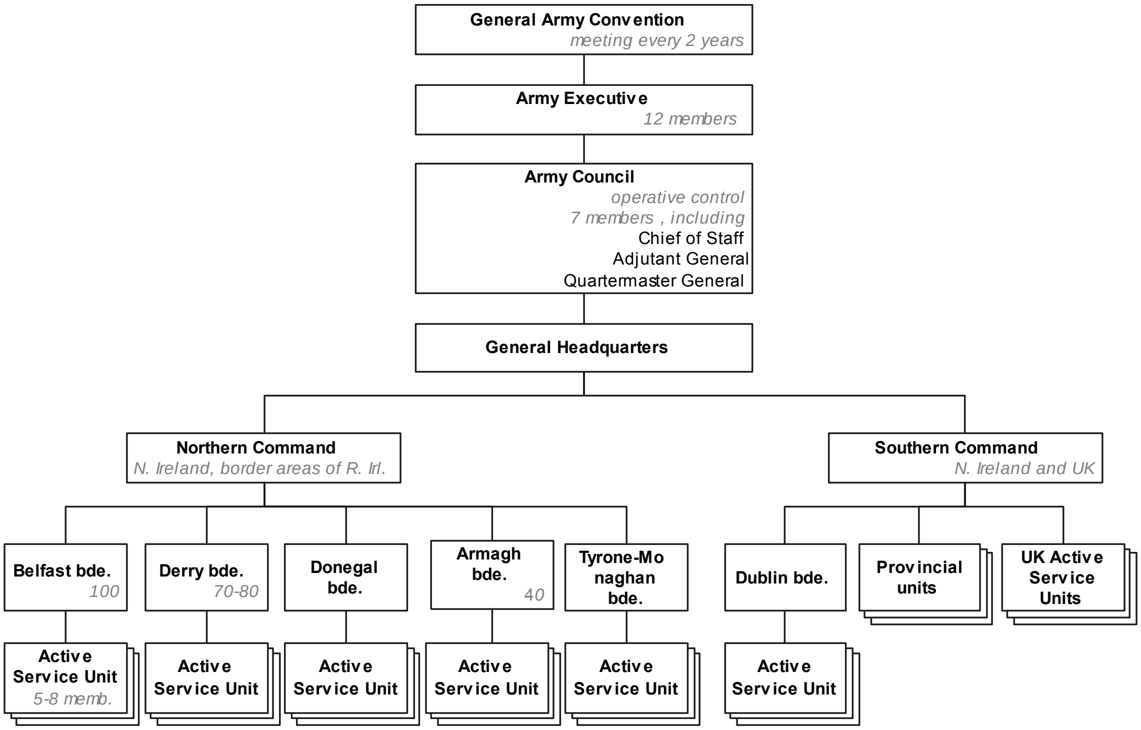

Figure 1. Organisational structure of the IRA in the second half of the nineties: hierarchical organisation with territorially organised divisions (based on [4]).

relatively recently are oriented towards maximisation of casualties and terror. For them, terror is more the aim in itself, than it is the means of political fight. Such groups have much less interest in maintaining strict hierarchical structure, which would ensure operative control over terrorist activities. One of such groups is al-Qaeda, especially when viewed as a wider structure encompassing various groups and individuals inspired by the spiritual leadership of Osama bin Laden.

Al-Qaeda in a narrower sense, as an organisation formed and led by bin Laden, also possesses hierarchical structure [5 – 7], but hierarchy is only one form of its complex structuring. Figure 2 presents the top of al-Qaeda organisation in the way the organisation of corporations or public institutions is usually depicted. The organisation is headed by Emir – the leader, which is the indisputable position of Osama bin Laden. The Emir is supported by Shura or the High Council, the activities of which are not much known of. In 2001 Shura supposedly had twenty to thirty members, many of which were not living in Afghanistan. Members of Shura have high authority and high degree of freedom, but are at the same time absolutely loyal to bin Laden and the organisation. The frequency of their meetings, their ways of communication, and the content of Shura's decisions are not publicly known.

In 2001 al-Qaeda had four organisational units, called committees, responsible for political and religious, financial, military, including terrorist, activities, and for propaganda and relations with media (Fig. 2). Military committee consisted of three departments. One of them was responsible for external military activities conducted during nineties in Chechnya, Bosnia, Kashmir, and perhaps some other countries affected by wars. Another department supervised terrorist operations all over the world. The third one was responsible for internal operations in Afghanistan, where al-Qaeda had entire military units. Special training camps were managed by this department and used for recruitment and training of new members from all over the world.

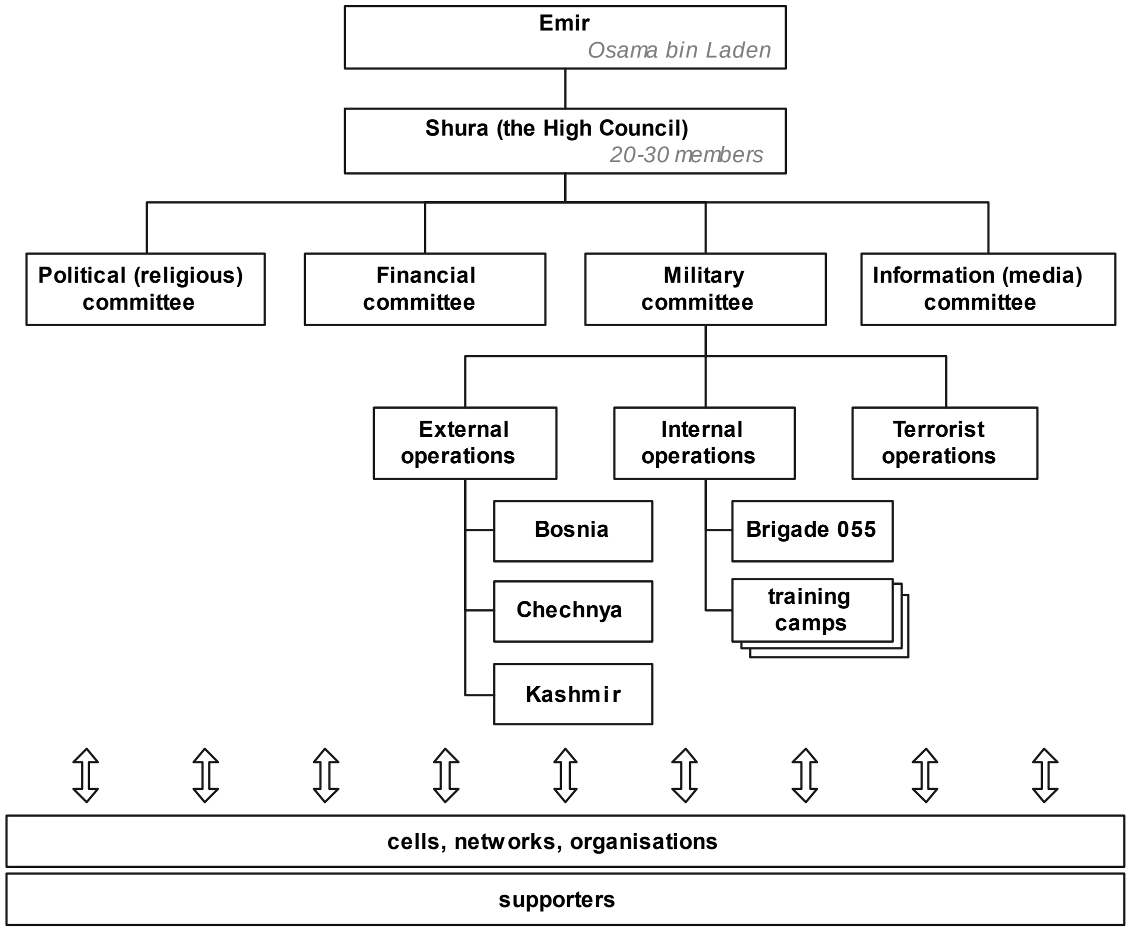

Figure 2. Hierarchical structure of al-Qaeda's core in 2001.

However, presented structural analysis, describing al-Qaeda as a hybrid organisation with its highest levels structured primarily according to functional principles, and with divisional structure at the lower levels, does not contain all the elements needed for understanding its functioning.

First, it must be noted that the formal structure of Figure 2 does not perfectly match all the relations and connections existing within the organisation, which are in reality much more complex. Additional relations including personal ties and multiple roles, which certain individuals attain, often exist aside from the specified lines of control. All of them constitute a network organisation, the lower hierarchical levels of which will be examined more closely in the forthcoming chapters. Moreover, connections between individuals and organisational units are often multiple, so that in reality multiple networks and hierarchies often exist. Thus we may speak about a whole meta-network of networks consisting of the operative network, the financial network, the administrative network, the network of trust, the knowledge network, the information network, and so on. The same node[1] may have different roles in these different networks. The complexity of the meta-network makes the structure of a terrorist network organisation extremely difficult to uncover [8].

Second, it is questionable whether a fundamentalist terrorist organisation can be analysed in the same ways and with the same tools as those used in structural analysis of modern business corporations. Membership in al-Qaeda undoubtedly has important spiritual dimension, and the organisation is at least as similar to a secret religious society as it is to a modern corporation. This opens another view on the structure of the organisation as a series of layers [9], which are often depicted as concentric circles. Members of the organisation believe that each subsequently inner circle is situated closer to a comprehensive revelation of the absolute truth in the centre (Fig. 3). The inner circles draw their indisputable authority in operative and tactical decisions from the assumed divine superiority. Advancement in such organisation is not so much related to the merits or deeds of an individual, as to his or her inner transformation in accordance with the organisation's spiritual teachings. Initiation rituals at certain organisational levels symbolise this spiritual development. The increased willingness to act in accordance with the organisation's goals is an important consequence of the increasingly resolute religious conviction.

The al-Qaeda's leadership does not exercise direct control over all activities of the organisation, not even over most of the activities carried out in the name of the organisation. This is not only because there is no need to control the amount of violence for attaining political goals. Control of violence is also not possible for technical and organisational reasons. What is then the role of leadership in such an organisation?

The leadership provides motivation for actions of the followers – a kind of a common vision or a narrative. It also provides organisational structure, doctrine and methods of operation, and, particularly, the means of personally connecting members of the organisation and the means of communication between them [10; p.324]. Using military terminology, for a network to achieve "self-synchronisation of dispersed forces", needed for its functioning, the leadership must ensure the unity of effort, define commander's intent and determine the rules of engagement [11; p.7].

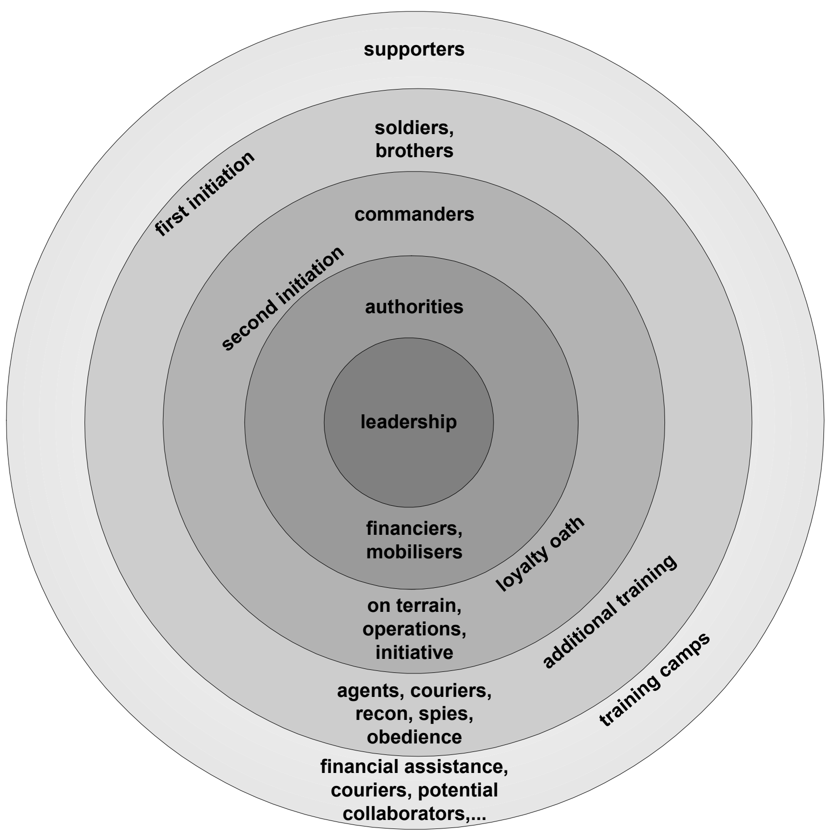

Figure 3. Concentric structure of al-Qaeda as a secret religious society.

It must be admitted that the al-Qaeda's leadership was exceptionally successful in fulfilling its duties. The commander's intent, i.e. the vision to fight for, was provided to membership through *fatwas* – religious decrees requesting death of Americans, their allies, and other infidels. The unity of effort was ensured through common fundamentalist Sunni religion, and reinforced by personal ties and mutual trust developed among the members in training camps in Afghanistan and in religious schools in Pakistan. The rules of engagement, including organisation, doctrine, techniques and methods of operation, were also developed through training, and they were even coded in a kind of training manual [12].

Once developed, "self-synchronised network of dispersed forces" may function without any direct interference or control of the leadership. The role of bin Laden, and probably of al-Zarqawi today in Iraq, becomes similar to the role of a manager of a business incubator [13]. While the manager provides entrepreneurs, or terrorists in this particular case, with start-up conditions, teaches them methods of operation, connects them with one another, monitors their more promising projects, and provides some inspiration and guidance, the initiatives and actions of newly-emerging leaders are autonomous in all other respects.

NETWORK ELEMENTS OF ORGANISATIONAL STRUCTURE

Most expert analyses and media reports on al-Qaeda's structure agree that all of al-Qaeda's organisational levels are structured according to hybrid network-hierarchical principles. Although complete data on the organisation are inaccessible, important studies of certain parts of al-Qaeda's network have been done. Valdis E. Krebs published one of particularly informative articles, reconstructing in considerable detail the portion of the network that prepared and executed September 11, 2001 attacks in the U.S. [14]. Much of the rest of our work is based on Krebs' findings.

Figure 4 depicts the network of airplane kidnappers – perpetrators of the attacks on New York and Washington. Four groups of kidnappers, each of which kidnapped one airplane, are marked with four different symbols for network nodes. Three groups consisted of five members, and one group of only four. One group did not complete its terrorist mission because passengers distracted terrorists enough to miss the planned target, but unfortunately not enough to escape crashing the plane. According to Christopher Allen's analysis [15], based on studying size of functional groups successfully collaborating over the Internet, successful small teams consist of 5-9 members, and the optimal size is 7-8. It is interesting to note that the only terrorist group that did not complete its mission had only four members, which is one below Allen's lower bound of five.

Let us now focus on the number of connections between team members and on the topology of the network. Bold lines in Figure 4 denote "old ties" between terrorists that had existed even before preparation of the attacks began. These ties of trust are typically formed through common schooling or common lodging. We can see that the kidnappers were only weakly connected, as each of them personally knew only three other network members on average. The average length of shortest path[2] between two nodes of the network is as high as 4,75, meaning that members were socially quite distant from one another. Low connectedness certainly favours secrecy, but it may impede network's operability.

During preparation of the attacks, kidnappers connected themselves more closely, increasing thus network's operability. These newly formed ties are denoted in Figure 4 with thin grey lines. It remains unclear, however, why had not all members of the same team known each other prior to boarding the planes. In teams marked with circles and pentagons, for example, each member knew maximally two other members of the same team.

Figure 5 depicts the network of airplane kidnappers, augmented with nodes representing their supporting assistants. According to Krebs' analysis, this wider network had 62 members in total, of which 19 were kidnappers, and 43 assistants: organisers, couriers, financiers, scouts, counterfeiters etc. Allen found that successfully functioning large networks typically comprise 25-80 members, with optimal size between 45 and 50. Again, a close match exists between the results of Allen's analysis of collaborating networked groups and this particular example of a terrorist group.

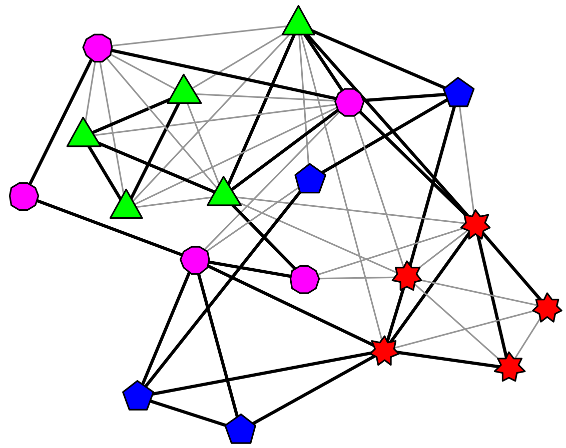

Figure 4. The network of airplane kidnappers participating in September 11, 2001 attacks in the U.S.

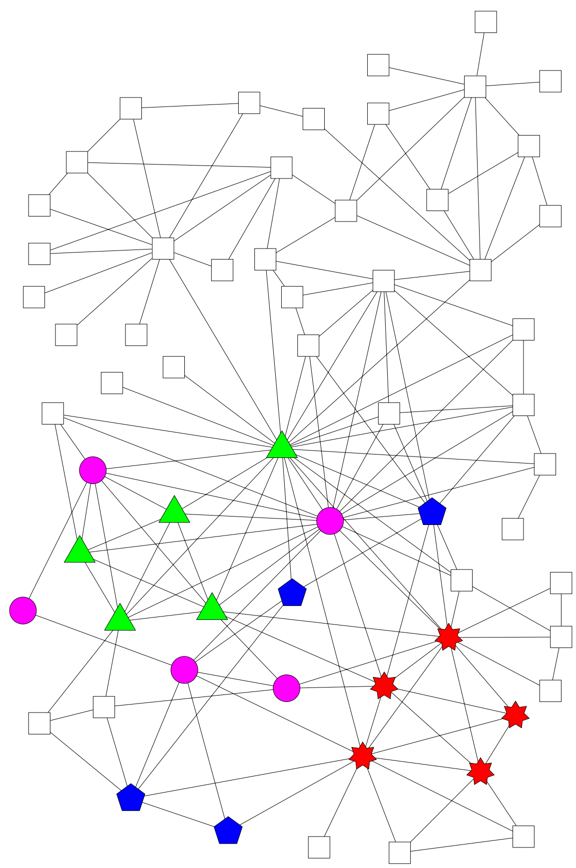

Figure 5. The network of airplane kidnappers and their supporters preparing September 11, 2001 attacks in the U.S.

Inspection of this network by standard measures of network structure [16 – 18] reveals firstly its low connectedness. A member of this network holds only 4,9 connections with other members on average[3], which means that average members are rather isolated from the rest of the network. (Try to imagine a group of 62 people of which you know only five – you would probably hardly feel a sense of belonging to such a group, or hardly expect any coordinated action of such a group.) Connectedness measure[4] of this network is only 0,08, meaning that only 8 % of all possible connections in the network really exist.

In spite of the low connectedness, however, nodes of this network are relatively close. The average length of shortest path between two nodes is 2,9, and the average closeness[5] of nodes is 0,35. Betweenness[6] is another important measure in social network analysis and it indicates a node's importance for communication among other nodes. The average betweenness of this network is 0,032, indicating relatively high average redundancy. However, betweenness of forty nodes is in fact less than 1 %, and only six nodes have betweenness higher than 10 %. These six nodes are obviously critical for information flow, especially the one with betweenness of almost 60 %, meaning that almost 60 % of communication paths among other nodes pass through this central node. This node represents Mohamed Atta, the leading organiser of the attack whose central position in the network is confirmed by other centrality indicators as well.

The values of centralisation measures[7] equal 23 % for degrees, 48 % for closeness, and even 56 % for betweenness of nodes. These results imply that links are relatively evenly distributed among nodes, but that some nodes' placements in network's topology are more significant than others'. Such nodes are in favourable positions regarding information diffusion and distribution of power, and are often referred to as central.

Distribution of degrees of nodes is particularly interesting. Degrees of nodes are exponentially distributed: the degree of most nodes is small, while only few nodes have high degree (Fig. 6). This property characterises the so-called scale-free networks [19, 20; pp.104-111][8], commonly found in diverse areas of science, technology, and society. The same property possess, for example, many traffic networks, networks of social contacts and social influence, networks of Internet servers, and many others, including also brain's neural network. Scale-free networks form spontaneously, without needing a particular plan or interventions of a central authority. Nodes that are members of the network for a longer time, that are better connected with other nodes, and that are more significant for network's functioning, are also more visible to new members, so that the new members spontaneously connect more readily to such nodes than to other, relatively marginal ones.

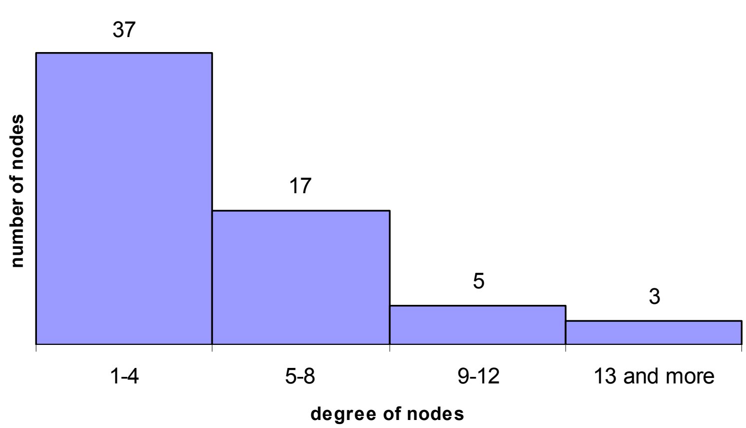

Figure 6. Distribution of degrees of nodes in the network of kidnappers and their supporters.

The al-Qaeda network was also organically formed, under influences of external conditions, operative needs and initiatives of group members. Most likely, the only important network's property resulting from intentional design is low connectedness, which is dictated by the need for secrecy and security of operation. Al-Qaeda's Training Manual states: "Cell or cluster methods should be adopted by the Organization. It should be composed of many cells whose members do not know one another, so that if a cell member is caught, the other cells would not be affected, and work would proceed normally." [12; Third Lesson].

Several other properties of scale-free networks have significant influence on network's functioning, so that the network may often appear as possessing almost supernatural properties. This may explain quotations describing al-Qaeda in the introductory chapter, whose authors were probably under impressions of a similar kind. The next two chapters will describe two of the potentially perplexing properties: speed of information diffusion through the network, and the network's resilience to loss of nodes.

INFORMATION DIFFUSION THROUGH THE NETWORK

When designing transport networks and telecommunication systems, including Internet networks, one of the most critical requirements is on the network's capacity of transferring certain amounts of commodities or information over certain distances in certain time. Methods of graph theory are often used for estimating network's capacity, including identification of directions, capacities, and perhaps transfer costs for each of the links.

Unfortunately, in the case of al-Qaeda network such methods are not particularly helpful because of their exceedingly demanding data requirements. Therefore we shall attempt to illustrate al-Qaeda network's information diffusion capabilities by means of an illustrative example. The example will use three networks of different structural types, but otherwise similar general properties. All three networks consist of 62 nodes. Attempts were also made at keeping the average degree of nodes close to 5 for all three networks. Schematic representations of the three networks are given in Figure 7.

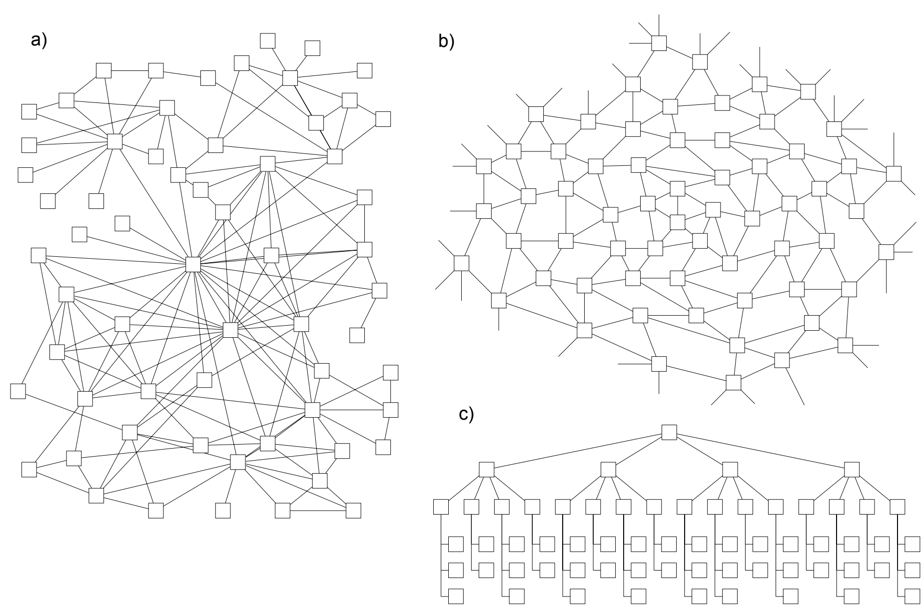

Figure 7. Three networks of different structural types with similar general properties: a) network with exponentially distributed degrees of nodes, b) network with uniformly distributed degrees of nodes, c) strictly hierarchical network.

First of the three networks (Fig. 7a) is Krebs' reconstruction of the network of airplane kidnappers and their supporters that we have already encountered in Figure 5. As we have also already noticed, this network's distribution of degrees of nodes is exponential. Degrees of nodes belonging to the second of the three networks (Fig. 7b) are uniformly distributed and each node is connected to exactly five other nodes. We can easily imagine this network as spreading over a sphere so that the outward-stretching links at borders of Figure 7b in fact connect nodes at the opposite borders. Such network does not possess any central or marginal nodes. As one can hardly find an example of organisation that would be structured in this way, this case is of primarily theoretical significance. Finally, the third network (Fig. 7c) represents standard hierarchical organisation with the structure of a tree. Note that the number of nodes in such network is always one more than the number of links, so that it is not possible to form the hierarchical network in which average degree of nodes would be five. As closest approximation we chose hierarchical tree consisting of 62 nodes, with degrees of all nodes at the second highest hierarchical level equalling five. This requirement automatically generates the rest of this network's structure.

In the first experiment, we were interested in speed at which information originating from the central node diffuses through the rest of each of the three networks. In the case of exponential network central node is located close to the middle of a graph. As we have already noted, this node possesses highest values of all three centrality indicators: degree, closeness, and betweenness. In the case of uniform network no node is distinguished by its centrality, so that the choice of the central information source is completely arbitrary. We have simply chosen the node in the middle of the graphical representation (Fig. 8b). Finally, in the case of hierarchical network central node is the one at the top of the hierarchy. It does not possess highest degree of all the nodes in the network, but is outstanding by its closeness and betweenness values.

We assume that, in each of the networks, information is released by the central node and that it diffuses through the rest of network in discrete steps. In the first step central node dispatches information to all of its neighbouring nodes, and in each next step each of the nodes that received information in the previous step dispatches it further to all of its neighbours. It is assumed that the time to traverse each of the links equals exactly one step, that there are no information losses, and that all links are of sufficient capacity to diffuse information further without any distortion. Time dynamics of information diffusion is schematically represented in Figure 8. Identically shaded areas comprise all the nodes that received information in the same time-step. Darker shading corresponds to areas that received information earlier. For each network, Table 1 contains percentages of nodes that received information in specified time-steps.

As can be seen in Figure 8 and Table 1, information diffusion from centre to periphery was slowest in the uniform network. The hierarchical and the exponential network both transferred information much faster, in only three time-steps. However, the transfer process was much more efficient in the exponential than in the hierarchical network. After only one step, information reached almost 40 % of nodes in the exponential, and only 8 % of nodes in the hierarchical network. After two steps, information reached over 90 % of nodes in the exponential network, and only about one third of the total number of nodes in the hierarchical network.

In the second experiment we investigated the speed of information diffusion in the opposite direction: from periphery to centre, i.e. the source of information is now a peripheral node. In the exponential network our choice for the starting peripheral node was one of the nodes located furthest from the central one, and with the value of betweenness equalling zero (Fig. 9a). In the uniform network the choice of both centre and periphery is completely arbitrary, so we simply repeated the same experiment as in the previous case. In the hierarchical network

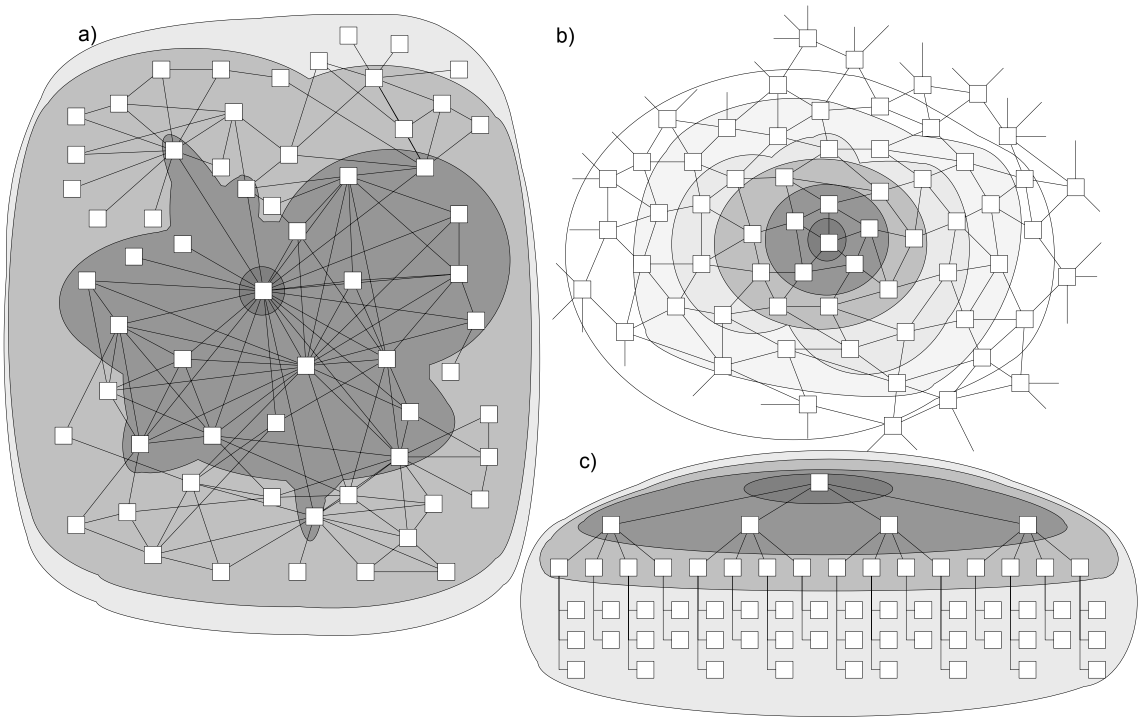

Figure 8. Information diffusion from centre to periphery: identically shaded areas comprise all the nodes receiving information in the same time-step.

Table 1. Percentage of nodes that received information for each time-step and each network – the case of information diffusion from centre to periphery.

Network	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Exp.	2 %	39 %	94 %	100 %			
Unif.	2 %	10 %	23 %	39 %	61 %	86 %	100 %
Hier.	2 %	8 %	34 %	100 %			

one of the nodes from the bottom of hierarchy was chosen as the starting peripheral node. Time dynamics for the case of information diffusion from periphery to centre is schematically represented in Figure 9. Darker shading again corresponds to areas that received information earlier. Table 2 summarizes percentages of nodes that received information in each of the time-steps.

Information diffusion was in this case slower in both the exponential and the hierarchical network. Diffusion through the exponential network completed fastest. What may surprise us is that the uniform network was in this case more efficient than the hierarchical network: in each intermediary time-step larger percentage of nodes received information in the former than in the latter network. This finding is in accordance with numerous empirical observations of serious inefficiencies in processing data and initiatives issuing from bottoms of hierarchies towards their upper levels. Most importantly, this particular case of an alarm notice, coming from a peripheral node and disseminating towards upper hierarchical levels, is typical in data gathering for intelligence purposes. It is therefore not surprising that restructuring of the existing hierarchical organisational structures became one of hot topics in the U.S. intelligence community after September 11, 2001 attacks.

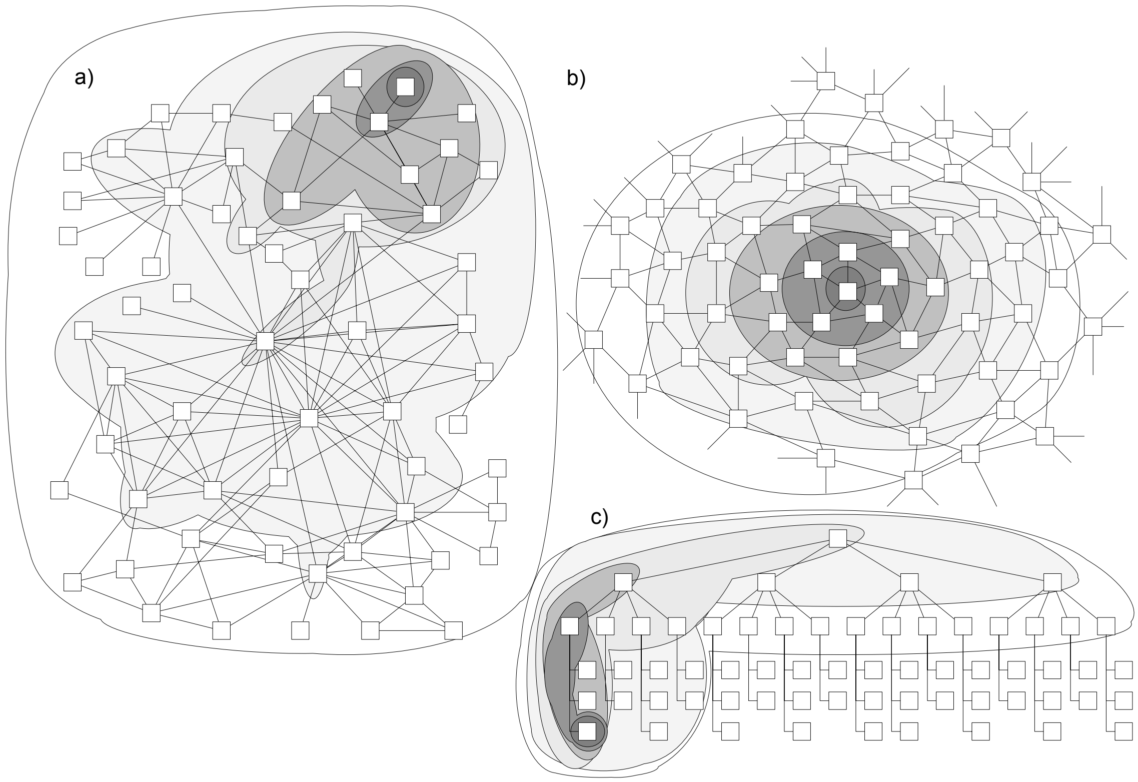

Figure 9. Information diffusion from periphery to centre: identically shaded areas comprise all the nodes receiving information in the same time-step.

Table 2. Percentage of nodes that received information for each time-step and each network – the case of information diffusion from periphery to centre.

Network	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Exp.	2 %	3 %	13 %	21 %	56 %	100 %	
Unif.	2 %	10 %	23 %	39 %	61 %	86 %	100 %
Hier.	2 %	3 %	8 %	15 %	34 %	53 %	100 %

The ease of information diffusion through networks with exponentially distributed degrees of nodes is often referred to as a "small-world property" [20, Ch. 3], meaning that it does not take many steps to get from one node to another. Together with secrecy of operation, this property significantly improves operational tempo of terrorist network's activities. In other words, terrorist operations can today be quickly prepared with very moderate requirements for personnel and assets, and without giving many clues at what is going on to external observers. In the right moment network may activate instantly, so that an observer is really left with the impression of terrorists "gathering from nowhere and disappearing after action". The metaphor of terrorists gathering as a swarm, and then quickly dispersing after action is also in place [21]. Another intriguing and analytically useful image is that of a critical mass of passive supporters to terrorists all over the world, who can rapidly self-organise and, through the process of "filtering", enable execution of deadly terrorist attacks anywhere in the world [22].

NETWORK'S RESILIENCE TO ATTACKS AND LOSS OF NODES

Resilience of a terrorist network in cases of arrestment, death, or any other loss of their members is of utmost importance for the network's survival. Conversely, network's vulnerability in such cases is critically important for successfully destabilising the network. There are at least three indicators of network's destabilisation [23]: first, the information flow through the network is seriously reduced, possibly to zero; second, the network, as a decision body, can no longer reach consensus; and third, network, as an organisation, loses the ability to effectively perform its tasks.

Removal of a central node in a hierarchical network has drastic consequences for the information flow, decision making, and task execution. Therefore very detailed rules regulating inheritance of commanding duties exist in, e.g., military organisations for protection against the loss of a commander.

Investigations of networks with exponentially distributed degrees of nodes show, however, extraordinary resilience of such networks to loss of nodes. This is the consequence of their redundant design, which is particularly important, thoroughly investigated, and intentionally applied in traffic, telecommunication and other technical networks. Robustness of terrorist networks is further enhanced by their multi-layered structure. As we have already noted, multiple connections exist among the members so that the whole network may be viewed as one huge meta-network of various functional sub-networks. Obviously, removal of one node does not affect each of the sub-networks uniformly, so that many sub-networks will often not experience serious disturbances.

Finally, network's flexibility is an element contributing perhaps most to its robustness. As there are no strict hierarchical rules, individuals easily change their roles depending on external circumstances. Therefore, when a node is lost, surrounding nodes quickly establish new connections and share responsibilities of the lost member. The network changes its structure and adapts to new circumstances without prolonged loss of functionality. The process of recovery is completely different than in hierarchical organisations, where the organisational structure does not change when a node is lost, but another member occupies the empty position and takes over all the responsibilities of the predecessor.

Robustness analysis of scale-free networks was first motivated by safety requirements for technical networks, but its results are equally relevant for social networks of the same type [24]. These results show that scale-free networks are exceptionally resilient to the loss of a random node. This is not surprising if we remember that most of the nodes in a scale-free network possess only few connections. Loss of such nodes does obviously not have significant impact on network's functioning. More surprising, however, is the fact that scale-free networks may suffer, without being destroyed, random losses as serious as 80 % of their total number of nodes. Another surprising finding is that scale-free networks are often not destroyed even when their central node is removed. As already noted, redundancy and flexibility enable network's quick restructuring without losses of functionality. To destroy a scale-free network, one must simultaneously remove 5-15 % of its nodes, primarily those in central positions[9]. Only such simultaneous attack can destroy redundancies in network connections in the amount needed to prevent any possibility of network's recovery.

Let us check this last property for the network of airplane kidnappers and their supporters from Figure 5. Successive removal of central nodes from this network is schematically represented in Figure 10.

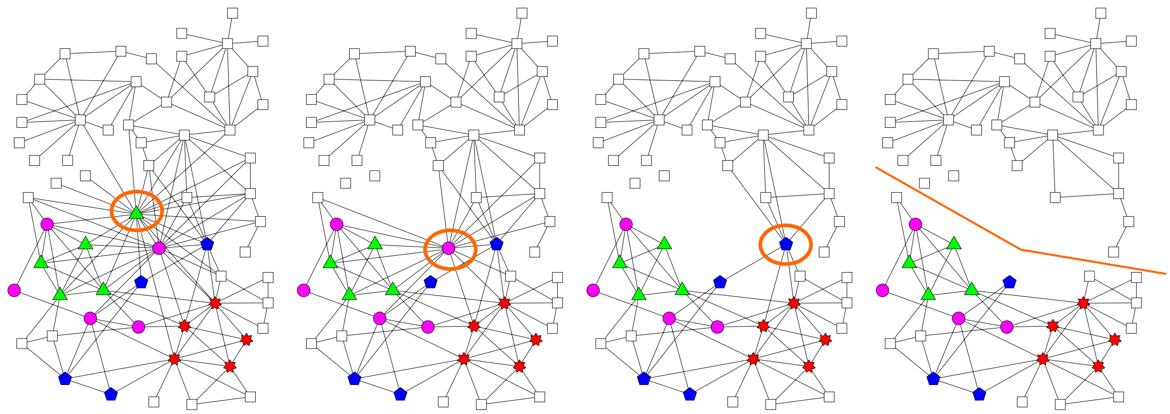

Figure 10. Network's destruction by successive removals of its central nodes: after three removals network disintegrates into two mutually disconnected parts.

As can be seen, three central nodes need to be removed in order to break up this network. This must be done almost simultaneously to prevent network's recovery. Three nodes make up 5 % of the total number of nodes in this network, which is in accordance with the theoretical results cited above. Taking into account that, for secrecy reasons, the number of connections in a terrorist network is kept near minimum, it is not surprising that this network is slightly more sensitive to removal of nodes than technical networks, i.e. that it already breaks down at the lower theoretical bound for percentage of nodes that need to be removed.

CONCLUSIONS

This article shows that terrorist organisations like al-Qaeda present no especially ingenious case of organisational design, but that there exist spontaneously emerging and nevertheless perfectly sensible regularities between the structure of such organisations and their manifest properties. We still need to learn how to detect these regularities and how to understand them better. These regularities are not typical of only terrorist organisations. Networked organisations are present in almost all areas of life and their influence on our social and economic life is important and probably still increasing. Sales networks, open source code community, anti-globalisation movement, and scientific research networks are only some of the examples that have recently attracted much attention in public and in scientific communities.

As we have argued here, in some circumstances these networked organisations have significant advantages over classical hierarchies. Understanding and knowledge gathered in studying network structures in one area can often be more or less directly applied to organising complex systems in some other area. For example, problems of organising a number of autonomous or semi-autonomous software agents to perform business transactions on Internet or some other computer network are in certain aspects similar to problems of organising networks such as al-Qaeda. As another example, in the "net-war" of the future, networked organisation would consist of platforms equipped with various sensors or arms, operating on a battlefield with considerable autonomy, and exchanging information with each other. The real strength of such systems would lay not so much in their number or firepower, as in flexibility and coordination emerging from their networked organisation.

Understandably, very often and especially in situations where strict control over all parts of organisation is necessary, where efficient use of resources is paramount, or where strict responsibility and traceability are critical, hierarchies will still find their application domain.

REMARKS

[1]Using the terminology of graph theory, which is fundamental for the network analysis, a network will be sometimes called a graph, members of the network will be referred to as nodes of the graph, while the connections between members will be referred to as links of the graph.

[2]A path is an alternating sequence of nodes and links, starting and ending with a node. The length of a path is defined as the number of links in it.

[3]This means that average degree of nodes is 4,9, where degree of a node represents the number of links coming out of the node.

[4]Connectedness of a given network is the ratio of actually existing number of links in this network and the maximal number of links that would be possible in a network with the same number of nodes, where each node would be linked to each other.

[5]Closeness of a node is an inverse of the average length of shortest paths from the given node to all other nodes of the network.

[6]Betweenness of a node is the number of shortest paths that go through that node, divided by the number of shortest paths in total.

[7]Degree, closeness, and betweenness of a node are varieties of centrality indicators. Each centrality indicator has its corresponding centralisation measure. The centralisation measure measures unevenness in distribution of centrality indicator's values over all network's nodes. Higher centralisation means higher unevenness.

[8]The term "scale-free network" originates from the fact that for such networks the ratio of number of nodes with degree k and the number of nodes with degree αk does not depend on the scaling factor k for any fixed $\alpha > 0$. This property is also related to the property of self-similarity, which indicates fractal structure of a network.

[9]The exact percentage is dependent on the amount of redundancy and, to some extent, on the choice of nodes to be removed.

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O MODELIRANJU KOMPLEKSNIH MREŽA S PRIMJENOM NA MODELIRANJE TERORISTIČKIH SKUPINA

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SAŽETAK

Na temelju dostupnih podataka o različitim organizacijama i mrežama članak istražuje neka ključna svojstva strukture terorističkih organizacija. Težište analize je na nižim razinama organizacijske hijerarhije, gdje je jasno vidljiva mrežna struktura s eksponencijalnom distribucijom broja lukova između čvorova mreža. Takve mreže rastu organički, vrlo su učinkovite u širenju informacija te robusne s obzirom na slučajne gubitke čvorova i na ciljane napade. Navedene osobine mreža ilustrirane su na primjeru terorističke mreže koja je izvela napade na New York i Washington 11. rujna 2001.

KLJUČNE RIJEČI

kompleksne mreže, struktura mreže, svojstva mreže

MANUSCRIPT PREPARATION GUIDELINES

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SUMMARY Concisely and clearly written, approx. 250 words.

KEY WORDS Not more than 5 key words, as accurate and precise as possible.

CLASSIFICATION Suggest at least one classification using documented schemes, e.g., ACM, APA, JEL, PACS.

TEXT Write using UK spelling of English. Preferred file format is Microsoft Word. Provide manuscripts in grey tone. For online and CD-ROM versions, manuscripts with coloured textual and graphic material are admissible. Consult editors for details.

Use Arial font for titles: 14pt bold capital letters for titles of sections, 12pt bold capitals for titles of subsections and 12pt bold letters for those of sub-subsections.

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