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# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

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**Scientific Journal**
# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

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# EDITORIAL

Dear readers, this issue of INDECS contains the selected contributions to the international conference DECOS 2006, the details of which are given by the guest editor, Prof. Katalin Martinás in the following page.

The release of this issue marks the beginning of the INDECSA 2006 contest, the result of which will be given in the INDECS 5(1).

In order to contribute further to linking complex systems description with topics important for our world, INDECS 5(2) is devoted to *Modelling Related to Poverty and Human Development*, and will be guest edited by Prof. Joanna Bryson and Ivana Čače. Please find more details regarding that in the Announcement on page *iv* of this issue.

Zagreb, 20 December 2006

Josip Stepanić

# GUEST EDITORIAL

The DECOS 2006 conference was a continuation of the series of international workshops and conferences on Complex Systems in Natural and Social Sciences (CSNSS), which took place in Hungary (Matrafüred 1995 and 2002, Tata 1996, Budapest 1997 and 2003) in Poland (Kazimierz Dolny 1999, Zakopane 2000, Torun 2001) and in Croatia (Zagreb 2005). Its purpose was to bring together the interdisciplinary groups of researchers working on complex systems and nonlinear dynamics in sciences, economy and humanities; to provide a forum for exchange of new ideas, for discussing the emerging topics and also for gaining fresh insights into possible applications of methods of sciences to study of socio-economic systems.

We hope that this purpose has been fulfilled, at least partially, since definitive success cannot be expected in this "never-ending story" of interchange between real life and various research domains.

The conference was financially supported by the Croatian Ministry of Science, Education and Sport.

Articles for the Proceedings were accepted after affirmative evaluation by the Scientific Committee.

Budapest and Krakow, 15 January 2007

Katalin Martinás

*Interdisciplinary Description of Complex Systems* 4(2), iv, 2006

# ANNOUNCEMENT

The Journal INDECS announces the special issue *Modelling Related to Poverty and Human Development*, to be released as INDECS 5(2).

The motivation for that topic is to contribute to the Global Theme Issue on Poverty and Human Development, initiated by the Council of Science Editors. In particular, science journals throughout the world will simultaneously publish papers on this topic of worldwide interest – to raise awareness, stimulate interest, and stimulate research into poverty and human development.

Human Development and Poverty are situated within a complex web of factors of both local and global scale: environmental, political, economic, religious, personal and other. We are looking for papers that describe parts of this network and analyse the relationships between the many factors that play a part in poverty and human development.

We hope that you will participate in this initiative by submitting articles for the special issue, in order to help shed some light on these important issues.

Both papers presenting original research as well as viewpoint and review articles, are invited. We do not wish to pose strict limits on length, but we do encourage authors not to use more words than the topic requires.

Research methods we are interested in include (but are not limited to) simulations, agent based or otherwise, or other descriptive models of complex real-world systems. The target research areas could be any of a number of topics, that in some way relate to human development and poverty. For example, in no particular order: urbanisation, war, charity, over-fishing, trade, improved access to information, disease threats, recent liberalisation trends in utilities, public traffic, public health care, corruption, migration, human trafficking, energy demands. Insights in to the research methods themselves are also welcome, answering questions like: How to compare simulation results with data?

Prospective authors are invited to submit an extended abstract of approximately 500 words by 15 April, in order to enable us to give feedback in an early stage. The deadline for full papers is 1 June 2007, to allow sufficient time for review process. The common publication date for all the journals participating in this theme issue will be Monday, October 22, 2007.

Bath and Utrecht, February 1, 2007 Joanna Bryson and Ivana Čače

# LIST OF REFEREES

The following scholars, listed in alphabetic order, refereed manuscripts for the journal INDECS in 2006:

| | |
|---|---|
| Attila Grandpiere | Vladimir Mikuličić |
| Timotej Jagrič | Ning Nan |
| Helena Jeriček | Barbara Pabjan |
| Maja Jokić | Mirjana Pejić-Bach |
| Boris Kožnjak | Mohammad Rahmat Widyanto |
| Katalin Martinás | Yasufumi Takama |

Their contribution to the quality of the Journal's content is acknowledged.

Zagreb, 20 December 2006

Josip Stepanić

*Interdisciplinary Description of Complex Systems* 4(2), 63-79, 2006

# NON-EQUILIBRIUM ECONOMICS

**Katalin Martinás***

[1]Department of Atomic Physics, Eötvös Loránd University,
Budapest, Hungary

*Conference paper* *Received:* 15 October 2006. *Accepted:* 2 January 2007.

## SUMMARY

A microeconomic, agent based framework to dynamic economics is formulated in a materialist approach. An axiomatic foundation of a non-equilibrium microeconomics is outlined. Economic activity is modelled as transformation and transport of commodities (materials) owned by the agents. Rate of transformations (production intensity), and the rate of transport (trade) are defined by the agents. Economic decision rules are derived from the observed economic behaviour. The non-linear equations are solved numerically for a model economy. Numerical solutions for simple model economies suggest that the some of the results of general equilibrium economics are consequences only of the equilibrium hypothesis. We show that perfect competition of selfish agents does not guarantee the stability of economic equilibrium, but cooperativity is needed, too.

## KEY WORDS

equilibrium hypothesis, non-equilibrium economics, avoid the avoidable losses

## CLASSIFICATION

PACS: 01.40.gb, 01.55.+b

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## INTRODUCTION

There is a formal and important analogy between economics and thermodynamics, namely they are phenomenological theories. We must know the rules, they govern our life. From our experiences we formulate in the form of axioms or postulates the basic behaviour. This axioms serve the basis of the mathematical version of the theory. Thermodynamics have shown that the choice of axioms is not unique, and the resulting mathematical theories maybe different, too. The distinction of equilibrium, non-equilibrium and extended thermodynamics is an example for the possibility of different construction of the axiom system. Each of them represents a different model of the reality. In standard equilibrium thermodynamics the reality is reflected as a collection of equilibrium systems, and the changes are described as "quasi-static" processes. The extended thermodynamics does not assume the equilibrium. These thermodynamic approaches are different on the metaphysical level.

Economics formulates the basis of decision rule in form of postulates. The formulation is not unique, but on metaeconomic level the basic properties are the same .A standard axiom in economic theory holds that humans are self-interested. Economists recognize, of course, that the assumption is not literally true. Many argue, however, that it is good enough for explaining most important economic phenomena.

In the neoclassical model, the essence of what the economy does is sustain (or fail to sustain) an equilibrium. The equilibrium relies crucially on the assumption of a competitive environment where buyers and sellers take the terms of trades (prices) as a given parameter of the exchange environment. Each trader decides upon a quantity that is so small compared to the total quantity traded in the market that their individual transactions have no influence on the prices. That approach assumes that individuals choose actions based on the short-sighted evaluation of their consequences based on preferences that are selfish and exogenously determined. The Walrasian approach [1] represents economic behaviour as the solution to a constrained optimization problem faced by a fully informed individual in a virtually institution-free environment. The similarities of Walrasian approach and thermodynamics are investigated and explored elsewhere [2 – 7].

Nevertheless relaxation of the Walrasian assumptions confronts us with an embarrassment of riches. In the absence of some empirical restrictions or theoretical refinements, a paradigm will remain vacuous. Few empirical predictions will be forthcoming if individuals may be self interested or not depending on the person and the situation, if some interactions are governed by contracts, others by handshakes, and others by brute force, and if there exist multiple stable equilibrium. The need for empirical grounding of assumptions is nowhere clearer than in the analysis of individual behaviour, where the process of enriching the conventional assumptions about cognition and preferences can easily descend into ad hoc explanation unless disciplined by reference to facts about what real people do. It is not enough to know that self interest is not the only motive; we need to k which other motives are important under what conditions. These restrictions are most likely to come from one of the sources that undermined the Walrasian paradigm, namely the great advances in empirical social science stemming from new techniques in econometrics, the improvement in computational capabilities and data availability, experimental techniques, and continuing progress in quantitative history. Theory, too, can provide useful restrictions on the set of plausible assumptions and outcomes. The modelling of genetic and cultural evolution, for example, can help restrict the range of plausible behavioural assumptions by distinguishing between emotions, cognitive capacities, and other influences on behaviours.

In summary the modern economic paradigm is based on the unholy trinity of Solow, the "ERG", that is "equilibrium, rationality and greed". This unholy trinity was (and is) criticized, but to neglect one of the elements ruins the present economic theory. Nevertheless the equilibrium hypothesis is natural for economists. The necessity of equilibrium follows from the comment of George Soros, who wrote [8]: "In the absence of equilibrium, the contention that free markets lead to the optimum allocation of resources loses its justification. The supposedly scientific theory that has been used to validate it turns out to be an axiomatic structure whose conclusions are contained in its assumptions and are not necessarily supported by the empirical evidence. The resemblance to Marxism, which also claimed scientific status for its tenets, is too close for comfort."

Economists are trained that the understanding of equilibrium will lead to understanding of processes. That role of equilibrium is not justified by thermodynamics, it shows just the opposite. So for non-economists the equilibrium hypothesis is an oxymoron. As a physicist Ruelle wrote [9]: "Textbooks of economics are largely concerned with equilibrium situations between economic agents with perfect foresight. The textbooks may give you the impression that the role of the legislators and government officials is to find and implement an equilibrium that is particularly favourable for the community... The examples of chaos in physics teach us, however, that certain dynamical situations do not produce equilibrium but rather a chaotic, unpredictable time evolution. Legislators and government officials are thus faced with the possibility that their decisions, intended to produce a better equilibrium, will in fact lead to wild and unpredictable fluctuations, with possibly quite disastrous effects. The complexity of today's economics encourages such chaotic behaviour, and our theoretical understanding of this domain remains very limited."

As Veseth summarized [10] Ruelle's critique is natural for outsiders, but it is irrelevant for economists. Ruelle supposes that the aim of economics is to help a good economic policy. All the economists know that the theory is about the hypothetical ERG economics. Ruelle's point, however, is extremely important for outsiders. The equilibrium hypothesis of economists necessarily eliminates the possibility of not equilibrium when there is no particular reason to do so. So the forecasts of the equilibrium and equilibrium behaviour are not results but built in assumptions.

The second letter in "ERG" is "R", which is for rationality. Without rationality there is mathematical theory. Rationality makes economics to a "predictive" science. If agents are rational, then theories can predict their behaviour and the predictions can be evaluated. The hypothesis can be tested against real world data. There is no real forecasting power, but after the events the explanations can be made. Without rationality it is not possible. So without rationality, economics is not a mathematical science. Bowles wrote [11]: "In adopting the rationality axiom, neoclassical economics became part of a bigger project – the program of a grand unified theory of science based on the methodology of logical positivism. The desire to make economics a science is thus embedded in the rationality axiom. As a result, there is much to lose if irrational markets exist, and especially if they exist where they may restrict the largest market process of all – globalization."

As Thaler in 2001 wrote in the American Economic Association's Journal of Economic Perspective [12]: "Economics can be distinguished from other social sciences by the belief that most (all?) behaviour can be explained by assuming that rational agents with stable well defined preferences interact in markets that (eventually) clear. An empirical result qualifies as an anomaly if it is difficult to 'rationalize' or if implausible assumptions are necessary to explain it within the paradigm."

Greed is the last element. Rationality implies that the driving force of actions is the desire to obtain the more money, the more wealth, the more material possessions. The real governing rule is the greed. In modern economic theory greed is a code word for purposeful behaviour. Historically it is a new phenomena. Greedy individuals were considered to be harmful to society as their motives often appear to disregard the welfare of others. Further, greed was the synonym of avarice. So they were considered as hopeless people, who are not able to enjoy the richness of the life, they love only the money. Greed is listed as one of the Christian seven deadly sins. Nevertheless desire to increase one's material wealth has become acceptable in Western culture. The desire to acquire wealth has been understood as indispensable for economic prosperity. Many economic rationalists agree that greed is the only consistent human motivation. No one has been able to construct a society where communal altruism dominates individual greed. Chinese philosopher Lao Tzu wrote 2500 years ago: "There is no calamity greater than lavish desires, no greater guilt than discontentment and no greater disaster than greed."

If he is right, we have created a mighty sick world for ourselves. The acceptance and need for greed follows from the misunderstanding of the role of competition. Competition is a fundamental good in utilitarian economics. Competition is a process which ensures the maximum efficiency of the economy. The competition implies greed, so greed produces preferable economic outcomes most times and under most conditions. The resulting inequalities are the price for the perfect economy. Further, they maintain that altruism does not seem to be congruent with the way human beings are constructed.

## BASIC CONCEPTS

Economic activity is modelled as transformation and transport of commodities (materials) owned by the agents. Rate of transformations (production intensity), and the rate of transport (trade decisions) are defined by economic decisions made by the agents. There is a natural constraint for decisions. Balance equations for goods satisfy the law of mass conservation. The model developed here is an attempt to investigate the emergence and stability of economic equilibrium in a multi agent approach, or with other words the working of the invisible hand in a dynamical system approach. The basic assumption is that agents wish to reach a better economic state. The success depends on their skills. The desire to increase the economic well-being as a basic characteristic of economic decisions serves as a corner stone to the mathematical description of decisions. It leads to a welfare measure, $Z$. Welfare is a function of the goods and money possessed by the agent [13]. This function contains the economic characteristics of the agent. The first derivatives yield the economic values.

*An economic agent* (EA) is the smallest entity with an implicit or explicit decision-making rule. In most cases, the EA is either a firm or an individual. EAs are characterized by the scope of their activities, by their knowledge, their experiences and their belongings (goods and money). In a mathematical description, every stock can be listed, which can be effected by the economic activity of the agent, the ones, which effect the economic activity of the agent.

*Commodity or good* is a material or non-material object, which is denoted by $X_\mathrm{i}^\alpha(t)$ where $\alpha$ identifies the owner of the good. Superscript 0 represents nature, which is considered as an agent. Index i = 1, ...., $n$ is for the different goods and $t$ is for time. For material goods the balance equations read

$$\frac{\mathrm{d}X_\mathrm{i}^\alpha(\mathbf{r},t)}{\mathrm{d}t} = J_\mathrm{i}^\alpha(\mathbf{r},t) + S_\mathrm{i}^\alpha(\mathbf{r},t). \tag{1}$$

where $J_\mathrm{i}^\alpha(\mathbf{r}, t)$ is the flow term, usually the trade (ownership change) or transportation (location change) and $S_\mathrm{i}^\alpha(\mathbf{r}, t)$ is the source or sink term. It describes production, consumption or degradation. For money a similar equation holds. In the following subscript i = 0 represents money. The rules of the economic system (state) define the rules of money creation, that is for the source term is zero. For labour, the equation is similar, but there are two ways of modelling: namely the labour is a service, so the stock of labour is always zero. Or the labour potential is bought, and used, then labour is a commodity.

In economics the different stock changes are coupled. In trade the transfer of a good from agent $\alpha$ to agent $\beta$ is always accompanied by the transfer of money (or an other good) from agent $\beta$ to agent $\alpha$. The elementary event is not a change of stock, but a change of a bundle of stocks.

Let *Price* be the money given for a unit amount of good, *p*. A trade event of quantity *y* from good i for money $p{\cdot}y$ between EA $\alpha$ and $\beta$ is written as for agent $\alpha$:

$$X_i^\alpha(t+1) = X_i^\alpha(t) + y(t)\,, \tag{2a}$$

$$X_0^\alpha(t+1) = X_0^\alpha(t) - m\,, \tag{2b}$$

and for agent $\beta$

$$X_i^\beta(t+1) = X_i^\beta(t) - y(t)\,, \tag{3a}$$

$$X_0^\beta(t+1) = X_0^\beta(t) + m\,. \tag{3b}$$

Introducing a short-hand notation of activity vector, $q^{\alpha\beta}$, where $q_i^{\alpha\beta}$ is the quantity of the i[th] good going from agent $\beta$ to agent $\alpha$ that in the unit transaction. $y^{\alpha\beta}$, the intensity of the $\alpha\beta$ trade, and $y^{\alpha\alpha}$ is the intensity of the $\alpha^{th}$ agents's production. Then

$$\begin{aligned} X^\alpha(t+1) &= X^\alpha(t) + y^{\alpha\beta}\cdot q^{\alpha\beta} \\ X^\beta(t+1) &= X^\beta(t) + y^{\beta\alpha}\cdot q^{\beta\alpha} \end{aligned} \tag{4}$$

where the conservation laws demand for:

$$y^{\alpha\beta} = y^{\beta\alpha}\,, \tag{5}$$

and

$$q^{\alpha\beta} = -q^{\beta\alpha}\,. \tag{6}$$

As between two agents several type of trades are possible (the same good with other price, or different goods), a new index, j, is introduced to identify the transaction type between the agents, so $q_i^{\alpha\beta,\,j}$ tells that in the j[th] type unit transaction which quantity of the i[th] good goes from agent $\beta$ to agent $\alpha$ and $y^{\alpha\beta,\,j}$ is the intensity of the $\alpha\beta$, j trade. Total trade is written as:

$$X^\alpha(t+1) = X^\alpha(t) + \sum y^{\alpha\beta,j}\cdot q^{\alpha\beta,j}\,. \tag{7}$$

Consumption is given by the consumption vector $C^\alpha$, so the total change of goods is

$$X^\alpha(t+1) = X(t) + \sum_{\beta,j} Y^{\alpha\beta,j}\cdot q^{\alpha\beta,j} - C^\alpha\,. \tag{8}$$

Equation (8) describes time evolution of the economic system through the stock changes. The activity set $\{q^{\alpha\beta,\,j}\}$ describes the "hardware" of the economic system. The institutional and technological constructions. The activity parameters $\{y^{\alpha\beta,\,j}\}$ are the decisions of the agents. Constraints:

$$y^{\alpha\beta,j} = y^{\beta\alpha,j}$$

and the production intensity must not exceed the built in limit

$$y^{\alpha\alpha,j} \le y^{\alpha\alpha,j}{}_{\max}\,.$$

The stocks must be positive

$$X_1^\alpha \ge 0\,.$$

Dynamics is defined by the activity parameters $y$. In an agent based approach $y^{\alpha\alpha, j}$ is fixed (decided) by agent $\alpha$, while $y^{\alpha\beta, j}$ is decided by agents $\alpha$ and $\beta$.

## MATHEMATICAL MODEL OF DECISIONS

For the sake of simplicity, we assume that in a time moment $t$, one action is selected. All but one component of $y^{\alpha\beta, j}(t)$ are non-zero. Our ability to make trade and production decisions implies that we have the ability, to estimate whether an action is advantageous or disadvantageous for us. Every agent is characterised by an economic welfare function, and the symbol for it is $Z^\alpha = Z^\alpha(X^\alpha)$. Sign convention is selected so that $\Delta Z^\alpha > 0$ for preferred state, and $\Delta Z^\alpha < 0$ for loss-making transactions. $Z^\alpha$-function's partial derivatives

$$w_1^\alpha = \frac{\partial Z^\alpha\left(X^\alpha\right)}{\partial X_1^\alpha}, \tag{9}$$

are the $Z^\alpha$-value of the good i. Similarly

$$w_0^\alpha = \frac{\partial Z^\alpha}{\partial X_0^\alpha}, \tag{10}$$

is the marginal $Z^\alpha$-value of money, and

$$v_1^\alpha = \frac{\partial Z^\alpha}{\partial X_i^\alpha} = \frac{\partial Z^\alpha}{\partial X_0^\alpha} = \frac{\partial X_0^\alpha}{\partial X_i^\alpha}, \tag{11}$$

is the value of the i$^{th}$ good for agent α, measured in monetary units. In real exchanges the value must be higher than the price for the buyer. The value is subjective, it is decided by the agent. Economy works because the values are different.

On the basis of the above considerations, we may write the welfare change in a general form:

$$\mathrm{d}Z^\alpha = w_0^\alpha\left(v_1^\alpha \mathrm{d}X_1^\alpha + v_2^\alpha \mathrm{d}X_2^\alpha + \mathrm{d}X_0^\alpha\right). \tag{12}$$

Dividing both sides by the value of money:

$$\Delta W^\alpha = \frac{dZ^\alpha}{w_0^\alpha} = v_1^\alpha dX_1^\alpha + v_2^\alpha dX_2^\alpha + dX_0^\alpha, \tag{13}$$

where the right hand side is the stock change multiplied by the monetary value plus the money change, the economic wealth change.

The form of the welfare function, $Z^\alpha$ should be determined experimentally. As the results are yet unavailable we write down some simple wealth function and look for the economic behaviour expressed by them. We expect, that

- $Z^\alpha$ is a first-order homogeneous function of stocks,
- $Z^\alpha$ is an increasing function of $X_0^\alpha$, that is $w_0^\alpha > 0$,
- $w_0^\alpha$ is a decreasing function of $X_0^\alpha$, that is the value of money decreases with increasing stock of money,
- $v_i^\alpha > 0$, the values of goods are positive (This is not a general rule, since there maybe harmful stocks (wastes) with negative values, and they are then decreasing function of stocks). – $v_i^\alpha$ is decreasing fuction of stock, $X_i^\alpha$, that is $\partial v_i^\alpha/\partial X_i^\alpha < 0$ and – $v_i^\alpha$ is increasing fuction of money, $X_0^\alpha$, that is $\partial v_i^\alpha/\partial X_0^\alpha > 0$.

The last two properties are not necessarily valid. In case of cultural goods as for instance the higher stock leads to higher value for the agent. Nevertheless, in the present work we consider only normal goods, where the last two conditions apply, too.

One of the simplest expressions satisfying all the required conditions is a logarithmic function:

$$Z^{\alpha} = \sum_{i} X_{i}^{\alpha} \lg\left( \frac{X_{0}^{\alpha}}{X_{1}^{\alpha} C_{1}^{\alpha}} \right). \tag{14}$$

where $C_i^{\alpha}$ are constants. The value functions will be

$$v_{i}^{\alpha} = w_{0}^{\alpha} \lg\left( \frac{X_{0}^{\alpha}}{X_{i}^{\alpha} C_{i}^{\alpha}} \right). \tag{15}$$

Wealth gain in process $\alpha\beta, j$ is:

$$\Delta W^{\alpha\beta,j} = \frac{1}{w_{0}^{\alpha}} \left[ Z^{\alpha}\left( X^{\alpha} + q^{\alpha\beta,j} \right) - Z^{\alpha}\left( X^{\alpha} \right) \right]. \tag{16}$$

Let $\alpha\beta, j$ be an exchange process of the good i. The wealth change can be written for agent $\alpha$ as:

$$\Delta W^{\alpha} = \frac{1}{w_{0}^{\alpha}} \left( \frac{\partial Z^{\alpha}(X)}{\partial X_{i}^{\alpha}} - p_{i}^{\alpha\beta,j} \frac{\partial Z^{\alpha}\left( X^{\alpha} \right)}{\partial X_{0}^{\alpha}} \right) y^{\alpha\beta,j} = \left( v_{i}^{\alpha} - p_{i}^{\alpha\beta,j} \right) y^{\alpha\beta,j}. \tag{17}$$

The principle of wealth increase tells that the agent buys if $i_i^{\alpha} > p_i^{\alpha\beta}$, and sells if $p_i^{\alpha\beta} > v_i^{\alpha}$. Wealth gain in unit process ($y^{\alpha\beta, j} = 1$)is:

$$\Delta W_{0}^{\alpha} = \left( v_{i}^{\alpha} - p_{i}^{\alpha\beta,j} \right). \tag{18}$$

The aim of economic activity is to increase the wealth, the expected wealth increase is the driving force which pushes the actors to act. The activity is a function of force, defined by the agent. In linear approximation

$$y^{\alpha\beta,j} = L^{\alpha\beta,j} \Delta W_{0}^{\alpha\beta,j}. \tag{19}$$

where $L^{\alpha\beta, j}$ is the activity coefficient and the driving force for the process is the wealth gain of the unit process.

**Price equation:** Trade is a transfer of a good from an agent to an other agent, accompanied by the opposite motion of another good or money. The quantity of good and money does not change during the trade. These conservation laws define the scope of possible trade transactions. The conservation law demands for unit process that

$$y^{\alpha\beta k} = y^{\beta\alpha k}. \tag{20}$$

that is

$$L^{\alpha\beta k} \Delta W^{\alpha k} = -L^{\beta\alpha k} \Delta W^{\beta k}. \tag{21}$$

It defines the price as:

$$p_{i}^{\alpha\beta} = \frac{L_{i}^{\alpha\beta} v_{i}^{\alpha} + L_{i}^{\beta\alpha} v_{i}^{\beta}}{L_{i}^{\alpha\beta} + L_{i}^{\beta\alpha}}. \tag{22}$$

Production is a process which transforms some goods and materials into a new form, but it involves always a material flow to and from the nature. There is raw material input and waste output. The wealth gain in a unit production is

$$\Delta W^{\alpha\alpha,j} = \sum_{i} v_{i}^{\alpha}(t) q_{i}^{\alpha\alpha,j}. \tag{23}$$

In the force law assumption the expected wealth increase is the driving force, which pushes the actors to act, that is

$$y^{\alpha\alpha,j} = L^{\alpha\alpha,j} \Delta W^{\alpha\alpha,j}.$$

where $L^{\alpha\alpha, j}$ is the activity coefficient for production. The production intensity has two natural upper limits. The built in capacity, defined by the capital stocks gives a natural limit, on the other hand the scarcity of inputs also defines an upper limit. The real economy works below this boundary ($y_{max}^{\alpha\alpha, j}$).

## EQUILIBRIUM IN A PURE EXCHANGE ECONOMY

In a pure exchange economy there is no production and consumption. An exchange of the $i^{th}$ good between agent α and agent β is feasible if the values are different, $v_i^\alpha \neq v_i^\beta$. Let $x_i^\alpha$ be the quantity what the agent α gets for price $p_i$, then after the exchange the values will change, as

$$\Delta v_i^\alpha = \left( \frac{\partial v_i^\alpha}{\partial X_i^\alpha} - p_i \frac{\partial v_i^\alpha}{\partial X_0^\alpha} \right) x_i^\alpha . \qquad (24)$$

A symmetric relation is valid for agent β. The net result is that the value difference decreases, and welfare of the agent increases. The process continues until the value difference diminishes. In the final, equilibrium state all the values will be the same, but the welfare (wealth) of the agents maybe different.

It is easy to show, that the equilibrium value and so the equilibrium price depends on the path, in the present model on the *L* parameters. The wealth gain of the agents also depends on the choice of parameter *L*. Details see in [14]. All equilibrium states are Pareto-optimal, but from economic point of view they are different, as the wealth distributions are different.

## TIME DEPENDENT SOLUTIONS

### MODEL ECONOMY

For the numerical solutions we selected a model economy, based on the text book examples of macroeconomics. Our minimum sectoral model of an economy has 3 economic agents, corresponding to sectors: agriculture, industry, and households. Agents decide the production intensity, and by the bargaining rule, they agree in the prices and traded quantities. The nature is considered infinite, the activity vectors, decision parameters do not change.

Time is discretized. One unit is called one cycle. The cycle consists of trade decisions. The agents make the trade decisions (price and quantity determination) all together. After they make independently the production intensity decision based on their stocks. In the final part the consumption happens, and if it exists, then the interest payment. From the results we plot for some cases the total production as a function of time (completed cycles) and the welfare of agents and the total welfare of economic system as a function of time.

The initial values, given to parameters and variables previously defined, are listed as:

- identification of agents – selected as industry ($\alpha = 1$), agriculture ($\alpha = 2$)and households ($\alpha = 3$),
- identification of goods: tools ($i = 1$), food ($i = 2$) and labour ($i = 3$),
- welfare Z-function of agents. We selected the logarithmic form for the Z-function of agent as in eq. 14. where the values of constants $C^\alpha$ are to be specified,
- the number of different technologies available for agent, selected as 1,
- $X_i^\alpha$ $i^{th}$ stock of agent α listed in Table 1.
- $q_i^{\alpha\beta}$ activity matrix of the agents: Production activity vectors were selected as listed in Table 2.
- *g* is a parameter. Selection $g = 1$ describes a reproductive economy. Value $q > 1$ implies an economy where the total quantity of goods is increasing, economic growth may appear. Technological change in this aggregate level is modelled as a change in *g*, as an efficiency development. The same input results in more output. That is, the production activity matrix is changed.
- $q^{\alpha\beta,j}$: trade activity matrix. It was selected as $q_i^{\alpha\beta,j} = \delta_{ij}$ and $q_0^{\alpha\beta,j} = -p_j^{\alpha\beta}$, that is there is a separate trade process for each good. The prices are defined in the bargaining process.

**Table 1**: Initial Stock vectors.

| | Money | Food | Tools | Labour |
|---|---|---|---|---|
| **Agriculture** | 1000 | 22,98 | 18,51 | 14,07 |
| **Industry** | 1000 | 22,05 | 19,34 | 14,16 |
| **Households** | 1000 | 21,97 | 18,86 | 14,73 |

**Table 2**: Production activity vectors $q^{\alpha\alpha}$.

| | Food | Tools | Labour |
|---|---|---|---|
| **Agriculture** | $g$ | -0,14 | -0,08 |
| **Industry** | -0,40 | $g$ | -0,36 |
| **Households** | -1,83 | -1 | $g$ |

- $C_i^{\alpha}$, the consumption vector of the agents, which were selected as:

$$C_1^{\alpha\alpha} = C_{10}^{\alpha} \text{ if } y^{\alpha\alpha} < 1. \tag{25}$$

$$C_1^{\alpha\alpha} = C_{10}^{\alpha\alpha} \cdot y^{\alpha\alpha} \text{ if } y^{\alpha\alpha} > 1. \tag{26}$$

  Selection of the $C$ terms: Agriculture (0,25; 0; 0), Industry (0; 0,06; 0) and Households (0; 0; 0.04),

- $L_i^{\alpha\beta}$ activity coefficient of the agents The $L$ parameter for all trades is assumed to be unity, viz. $L = 1$. Each agent trades with every other agent. The production parameters are: $L_1 = 0{,}352$, $L_2 = 0{,}288$ and $L_3 = 0{,}352$.

## REPRODUCTIVE ECONOMY

The above model economy is an equilibrium economy. The initial state is reproduced. A slight modification of the initial stocks reveals two important properties of the system. The price equilibrium appears shortly, so for a short time the equilibrium seems to be stable, nevertheless wealth difference appears and increases with time. The rich will be richer, the poor become poorer, until the poor looses everything. We considered economic death of the agent when the stocks are less then a critical level. It is the collapse of the economic system. The initial stocks were modified randomly with the rule:

$$X_1^{\alpha} = X_1^{\alpha} \cdot \frac{3-a}{2}$$

where $a$ is a random number in the [0, 1] interval. In Figure 1 the total production of the economy is plotted as a function of time, where total production is the sum of the production intensity of the individual agents.

The system apparently finds the equilibrium after the first 20 cycles. There is a relatively stable production, or economic equilibrium for the first 4500 cycles. This equilibrium is not a perfect equilibrium. Wealth differences, shown in Fig. 2, continuously increase, until the instability of the economic system is reached[1].

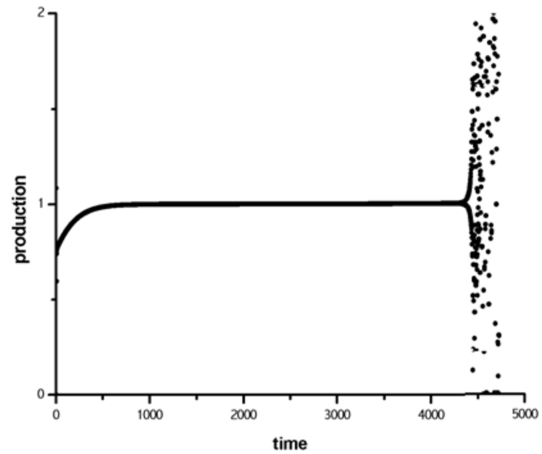


**Figure 1.** Production as a function of time for near equilibrium initial stocks.

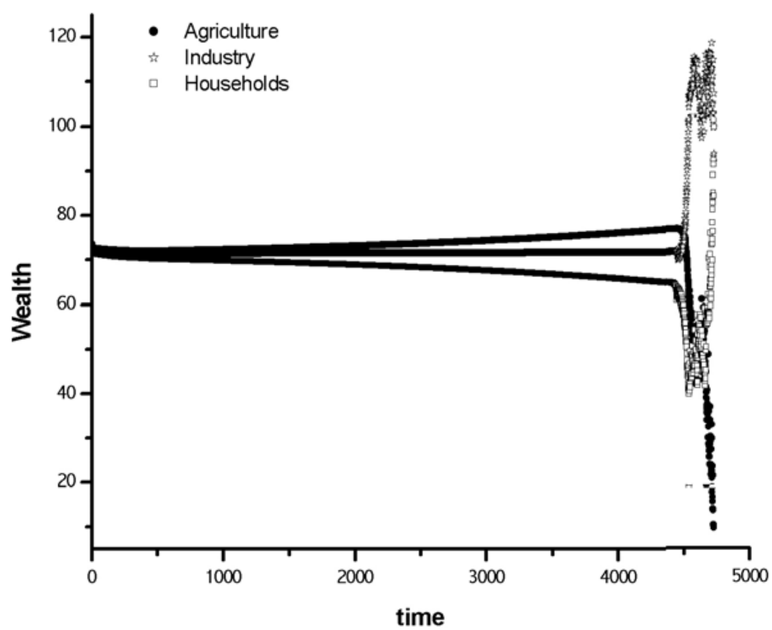


**Figure 2.** Wealth of agents as a function of time.

## GROWTH

The production activity with net gain is necessary for the increase of production activity, but it is not sufficient, as the previous results show. Money has to be increased, too, as otherwise the economic driving forces decrease. As a first approach to the problem, the interest was introduced. Every agent gets an interest payment for their money sock in each cycle. So the money is increased as

$$X_0^\alpha(t+1) = X_0^\alpha(t)\left(1+\frac{r}{1000}\right). \tag{27}$$

The results show that *r* defines the growth rate, while *g* defines the stability range of quasi-equilibrium growth. In the next figures we show the results for different interest rates at $g = 2$.

## Constant money

Constant money, that is $r = 0$ leads to a chaotic reproductive economy. Fig. 3 shows that total production starts at 14, but after 50 cycles the average production decreases to approximately 0,5. The economy founds the working path as a nearly reproductive economy. The production seems to show a chaotic behaviour until the collapse, which appears at 48 000 cycles.

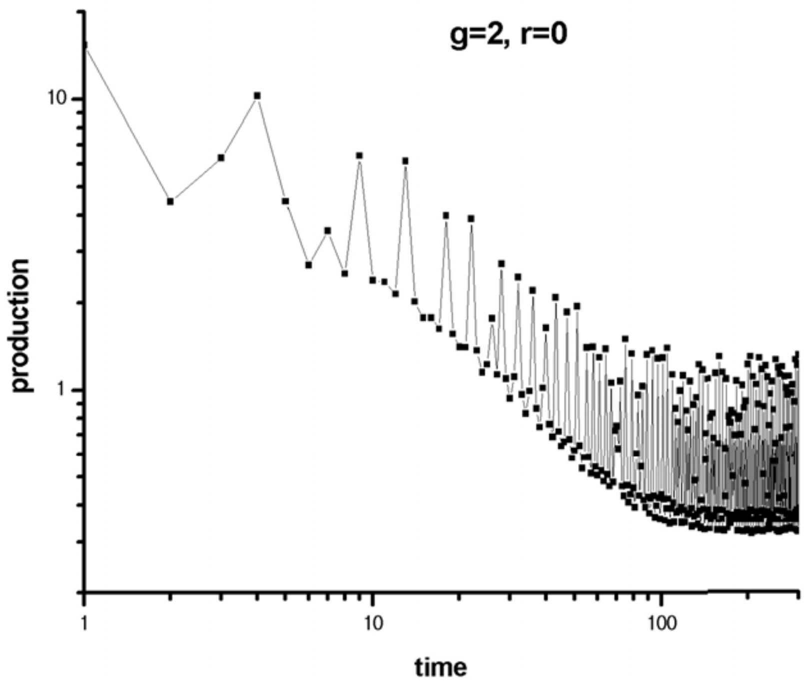


**Figure 3.** Production as a function of time for $g = 2$ with constant money ($r = 0$)

The production for the whole time interval is shown in Fig 4.

The technical change did not improve the total welfare. As it is shown in Fig 5. the total welfare is decreasing in time. It is interesting to note, that just before the collapse a relatively huge (but instable) welfare growth appears.

## Very low interest rate, *r* = 0,005

This small interest rate increases the life time of economy to 62 000. Further, the disappearance of the chaos appears. At between 37 000 and 40 500 an almost perfect oscillations replaces a chaos. At 53 500 a new ordering appears, but the collapse finishes the economy at 62 000, Fig 6.

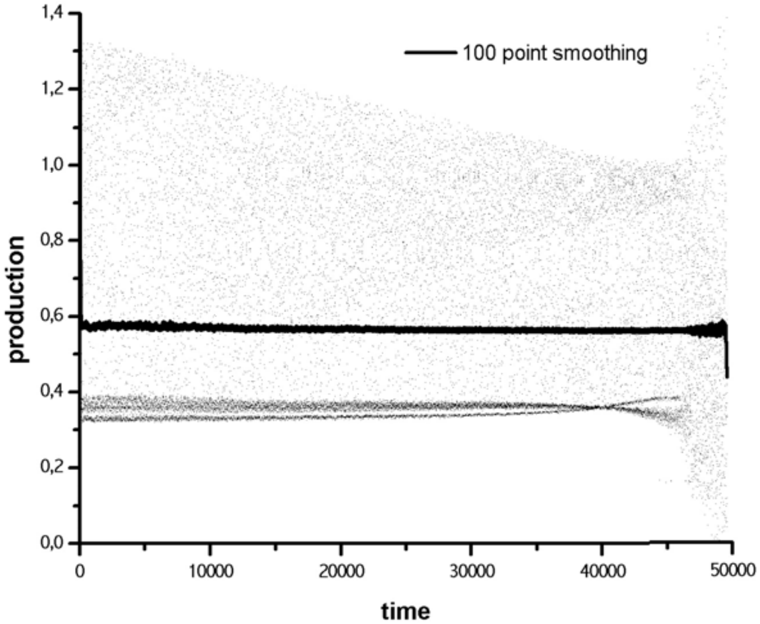


**Figure 4.** Production as a function of time for g = 2 with constant money ($r = 0$).

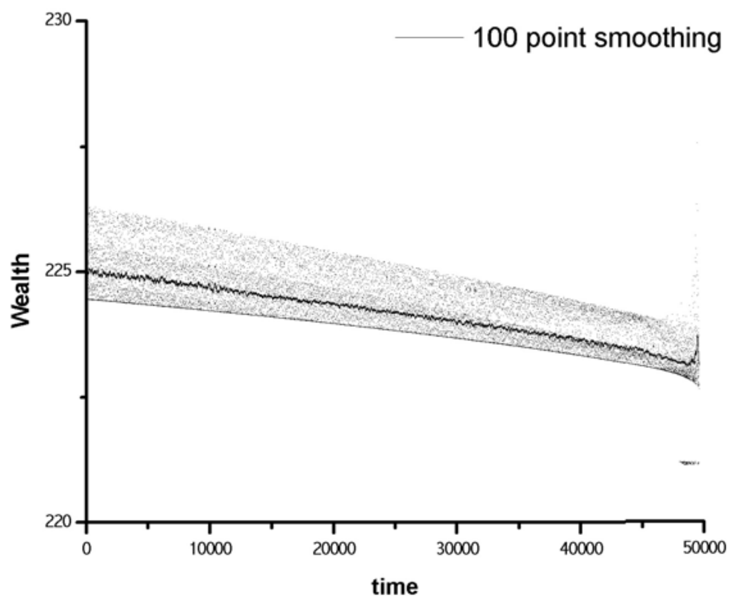


**Figure 5.** Wealth as a function of time for $g = 2$ with constant money ($r = 0$).

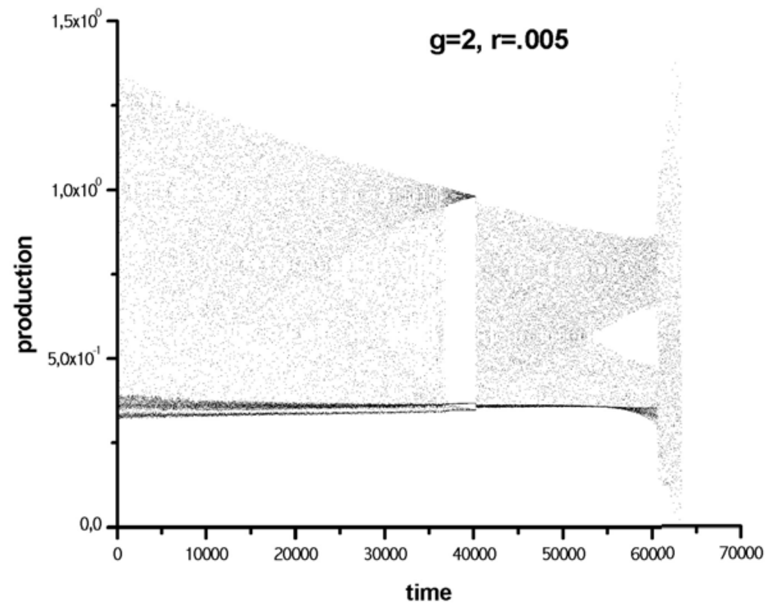


**Figure 6.** Production as a function of time for $r$ = 0.

The Fig. 7 shows the smoothed results.

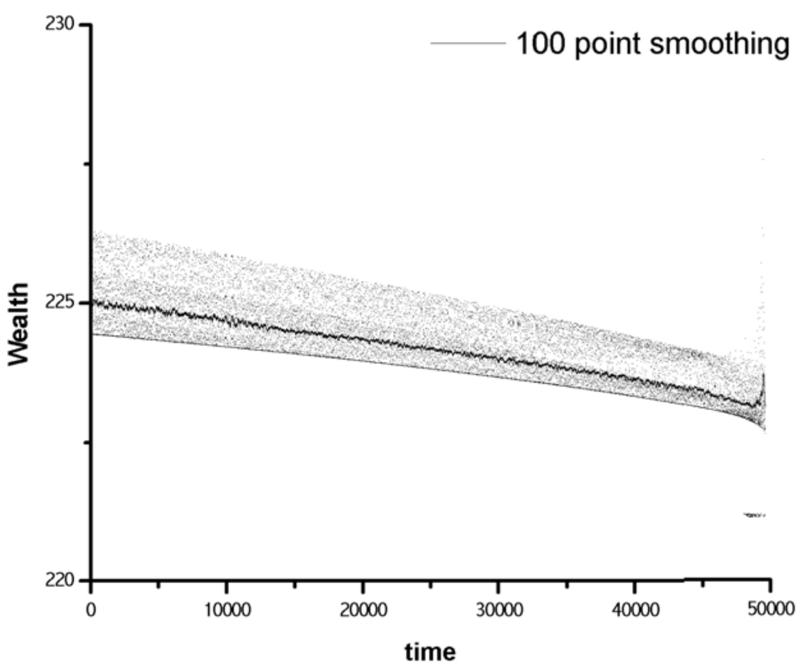


**Figure 7.** Production as a function of time for $r$ = 0,005.

### Low interest rate *r* = 0,013

The economy finds the equilibrium growth path if the interest rate is higher than a critical value (depending on *g*). The time needed to achieve the equilibrium path decreases with the interest rate. Nevertheless this equilibrium growth has a finite lifespan, which is decreasing with increasing interest rate. For each g belongs an *r*, where the equilibrium growth seems to have an infinite lifetime. In the following figures some results are plotted.

The critical interest rate is in the 0,0015 – 0,0025 range. First we show the result for *r* =0,0013 in Fig. 8, its detail in Fig. 9, and then the result for *r* = 0,0025 in Fig. 10.

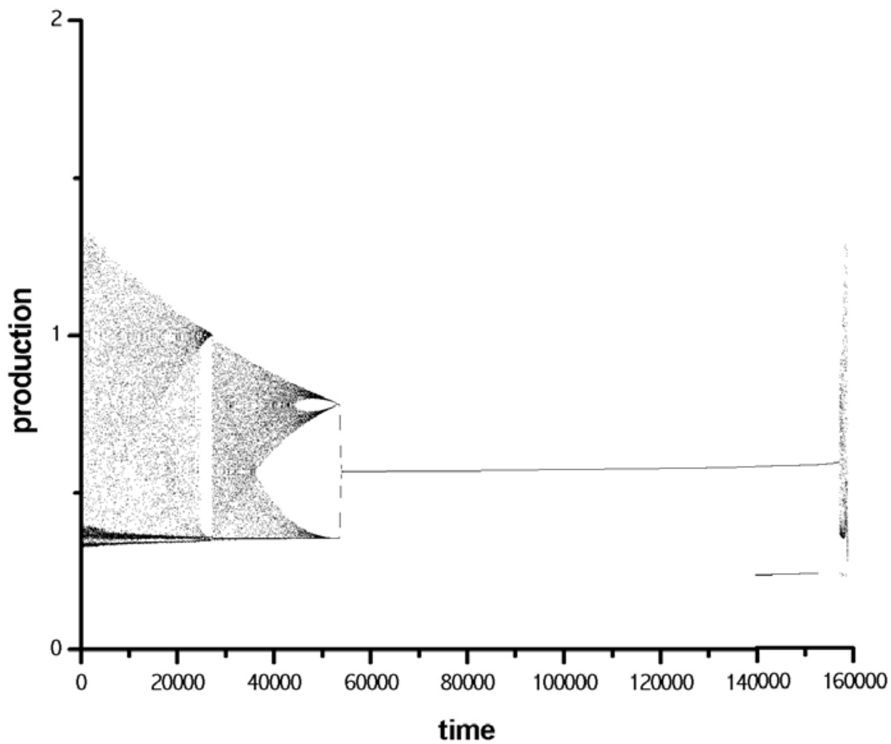


**Figure 8.** Production as a function of time, *r* = 0,0013.

On the curve 3 distinct regime can be distinguished. In the first (0 < i < 50 000), the emergence of order. The technological factor means that we start our economy out of equilibrium. The interaction of agents (through the trade) leads to this very slow equilibration. The first period is magnified in Figure 9. It resembles to a chaotic path.

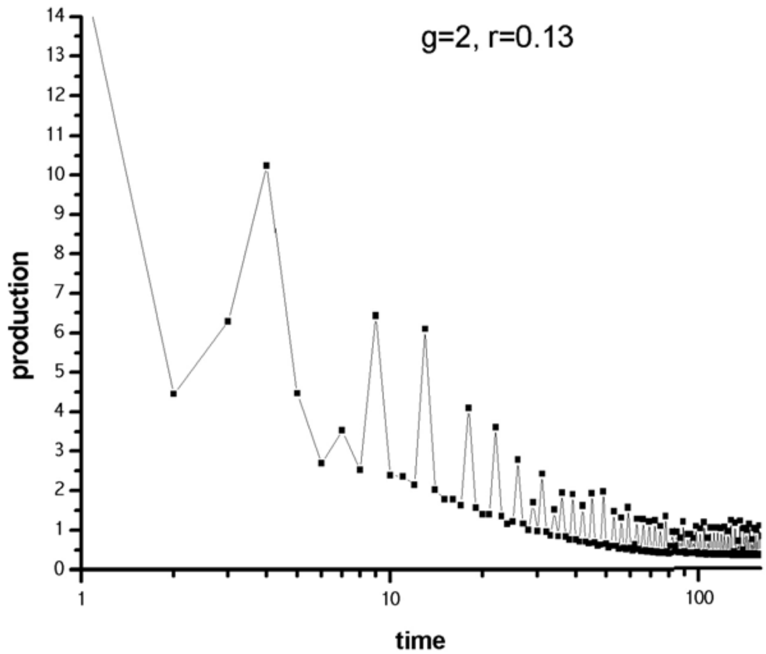


**Figure 9.** Production as a function of time, $r$ = 0,013.

## Stable Growth

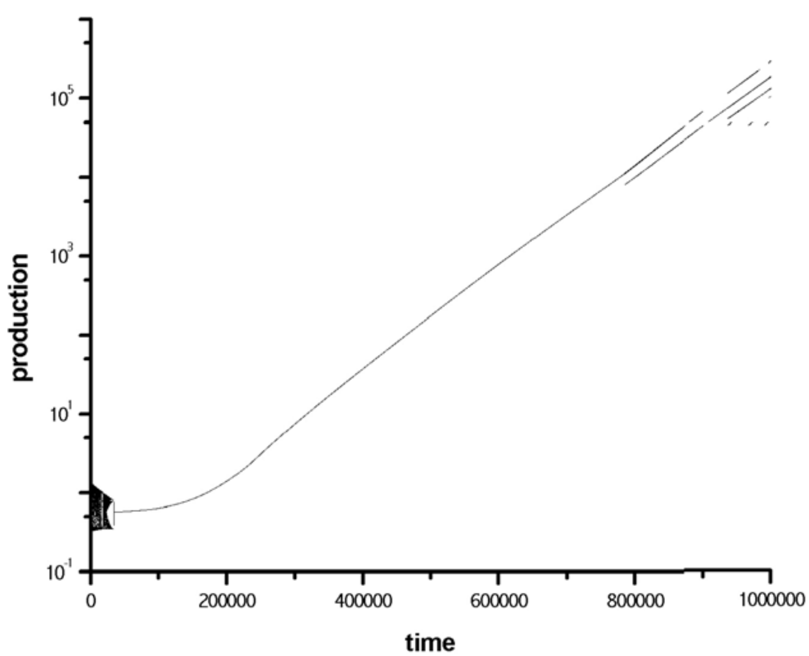


**Figure 10.** Production as a function of time, $r$ = 0,025.

## CONCLUSIONS

The modern economic paradigm is based on the unholy trinity of Solow, the "ERG", that is "equilibrium, rationality and greed". This unholy trinity has been criticized, but here we outlined a non-equilibrium foundation of economic theory, where the maximization (rationality) is replaced by the "Avoid the avoidable losses" rule, the greed assumption is not needed and the equilibrium is the question of dynamics.

The results of numerical solutions show that this rule is sufficient to ensure the emergence of market prices and the stable working of an economy, nevertheless for longer run the system is unstable, because of the emergence of wealth differences.

## ACKNOWLEDGMENT

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## REMARK

[1]It is worthwhile mentioning that the stability of equilibirum can be ensured by a "social law". Introducing a social wealth redistribution, namely the richest give some money to the poorest, gives a longevity to the system.

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# NERAVNOTEŽNA EKONOMIJA


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## SAŽETAK


Mikroekonomski okvir dinamičke ekonomije, temeljen na učesnicima, postavljen je u materijalističkom pristupu. Naznačeno je aksiomatsko utemeljenje neravnotežne mikroekonomije. Ekonomska aktivnost je modelirana kao transformacija i transport dobara (materije) koje posjeduju učesnici. Stope transformacije (intenzitet proizvodnje) i transporta (trgovine) definirane su putem učesnika. Ekonomska pravila odlučivanja izvedena su iz opaženog ekonomskog ponašanja. Nelinearne jednadžbe su za modelnu ekonomiju rješene numerički. Numerička rješenja za jednostavni model ekonomije upućuju na zaključak kako su neki od rezultata iz opće ravnotežne ekonomije posljedice samo hipoteze ravnoteže. U članku se pokazuje kako potpuna kompeticija sebičnih učesnika ne garantira stabilnost ekonomske ravnoteže, nego je potrebna i kooperativnost.



## KLJUČNE RIJEČI

hipoteza ravnoteže, neravnotežna ekonomija, izbjeći gubitke koje se može izbjeći

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# EMERGENCE IN THE FORESIGHT

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## SUMMARY

The aim of this paper to present some items of emergence in the foresight that should be meant and handled scientifically. It highlights three resources of social emergence. One of them is originated from the development of shared future views at a community level from the individual future shapes. The second one is originated from the relations among the foresight and its social environment and the outer conditions. The third one is originated from the double loop of discourse between subjective – inter-subjective and human – non-human world. The paper reasons that a meta-theory and an extended participatory approach should be needed to research, study and give meaning complex social emergence in the foresight.

## KEY WORDS

futures studies, foresight, social emergence

## CLASSIFICATION

ACM Categories and descriptors: J.4 [Computer Applications]; Social and behavioral sciences

APA: 3040, 4010

JEL: A10

PACS: 87.23.Ge, 89.75.-k

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## FORESIGHT CONCEPTIONS

The word "foresight" has various definitions and interpretations. Some of the most frequently employed definitions are listed below. Foresight is:

- a human ability or capability like the ability to learn, to think or to love,
- a thought concerned with the individual's own future,
- a subject of discourses among individuals and their communities, or among individuals, about the future of their communities,
- a segment of Futures Studies, which deals with the shaping process of community future-views,
- a special activity in the service of policy making.

Futures Studies literature quotes Slaughter's definition as a starting point. *He emphasises the human ability aspect of foresight.* He says that foresight is, "...a universal human capacity which allows people to think ahead, consider, model, create and respond to future eventualities. Founded on the rich and inclusive environment of the human brain-mind system which, crudely put, has sufficiently complex neural 'wiring' to support an extended mode of perception whose main functions are proactive and facilitating." [1]. Slaughter also extends both the concept and application of foresight to the community level and to a longer time-span than is needed to survive the immediate environmental effects [2, 3]. He says that, "...the foresight needs to be deployed at social and organisational levels ..." and, "... to extend from the here-and-now to a wider temporal span and from simple person-to-person interactions to systemic ones mediated by a range of powerful technologies. Hence we are challenged to exercise our foresight capabilities in new ways." [2: p.44 and p.56].

*Foresight in Futures Studies* (FFS) began with the analysis of the future orientation of everyday people [4] and was followed with the analysis of the future orientation and the expectation of the youth generation [5]. Foresight activities in critical Futures Studies concentrate on the comparative analyses of community future-views using a hermeneutic approach. Through critical evaluation futurists strive to enrich social discourse about individual and community future views with the latest points of view. (For example see [6] [7] and [8].) Participative workshops techniques have become dominant in the present FFS. They are used in Futures training programmes (For examples see [9] and [10].) and self-developing courses for Future sensitive specialists and civil groups (For example see [11] and [10].). Giving comparative analyses of future ideas of different types and dealing with the foresight as a development of future views at a community and a personal level FFS stimulates people to explore their own future ideas and to pay attention them in their everyday activities.

There is another type of foresight concepts too, which is developed directly form the needs of social practice. This has emerged first in the technology foresight, then in the regional one, and it can be seen in the organisational or corporate or social foresight [12]. Let us name this type of foresight the *Praxis Foresight* (PF). PF is the effort to assess future conditions based on current conditions and trends, "... a process by which one comes to a fuller understanding of the forces shaping the long-term future which should be take into account in policy formulation, planning and decision making." [13: p.43]. The aims of PF are:

- creating social future information to support decision preparation process,
- encouraging participants to reflect the future,
- putting stakeholders together to form collective or shared vision of the future. i.e. to develop consensus future at community level.

Since the main aims of PF are to get shared futures views and to influence policy decisions, it also includes a number of normative assessments of how to reach a future state that is considered desirable at community level.

Both types of foresight conceptions emphasis that the future is essentially unpredictable, because the society has not laws as the nature has. *Human expectations, efforts and actions cause the changes of society*. For this reason the foresight process is as important as its outcome. Putting up participants as stakeholders of the future, to share their future visions and teaching participants how to think flexibly are as important as the specific findings derived from these efforts. Foresight managers focus mainly on the expectation of the participants' mental models so that they can better interpret any social future conditions arisen in the course of consensus building.

In spite of the similarities the distinction between the two foresight concepts is reasonable because of their different social functions [14]. The main difference appears in the answer for the following question: Does it associate with policy making or not? PF always associates with policy making, but FFS does not it frequently. The other difference between them is that FFS focuses on the foresight of individuals while PF focuses on the foresight at a community level.

Since foresight concepts are still under development, certain criticism hit them in the literature. Some of them are mentioned bellow to support the development of the theoretical basis of foresight concepts. We agree with the following criticism, because it concerns both two foresight concepts. „However, the vast majority of foresight literature focuses either on providing descriptions and critiques of various methodologies, or on reporting the „newsworthy" results derived from the use of a particular methodology. By contrast, very little information is available that details the actual experiences organisations have into conducting foresight activities. Clearly, the success of any foresighting program will rest not only on the specific method being used, but also on the details of how the program is conducted." [15: p.3].

Additionally the human perception in foresighting has some limitation as follows

- participants judge future probabilities in the basis of their ad-hoc ideas, for example they overestimate the future impact of published trends against the no published ones,
- participants tend to overestimate the likelihood of low probability events and underestimate the probability of very likely events,
- participants may distort the representativeness of events, by focusing on details of their desirable futures.

Both foresight concepts often focus on one area and neglects important factors that can significantly affect the future outcomes. Foresighting often occurs in a vacuum and no attention is paid to the other associated factors that affect future. *Foresight sometimes fails testing desirable futures or shared future views by their chances of occurrence*. FFS believes that it is sufficient to develop the future sensitivity of participants' thinking. PF believes that it is sufficient to build the consensus future into the policy making to fulfil it.

There are also other limitations at group level. Since foresighting is a consensus building process, it tends toward the centre. Extreme views are often dropped, in order to get at a position that everyone can agree with. Participants often reflect the popular dominant opinion, „the spirit of times", in other words. This limitation is not so firm in the case of FFS, because it does not strive to get one consensus future. It allows getting different future visions at subgroup levels. Contrary, the PF does strive to get consensus future. Consequently PF can be seen as a special case of FFS in this aspect.

Later we use foresight in Slaughter's view, because it involves wider range of topics potentially than FFS or PF. *To actualise this general definition and to avoid the imperfectness of FFS and PF we ad to it the preferred conditions and social environment in which the foresight can function in a contemporary society*. The preferred conditions and social environments of this special activity can be characterised by the specifications are as follows:

- Foresight has a greater chance to fulfil its aims in a democratic society where the critique of the status quo and the future perspectives is an everyday activity of the citizens and their communities. It is the best, when the discourse is „power free".
- Individuals should not only passively bear the consequences of the foresight of others or the community foresight, but also actively participate in the social foresighting independently of its specific forms. Foresight can have organised and not organised forms.
- Foresight should include the ability and intention of fulfilment.
- Foresight should also include experience, knowledge, reflection and self-reflection.
- The creation and employment of foresight should serve the harmony between the individual and the individual and the world around him or her.
- Both individual and community foresight has also interconnections and -actions to the non-human worlds entities.

Applying Slaughter's foresight concept together with its social function and environment makes possible to give meaning a wide range of emergent phenomena.

## TOPICS OF EMERGENCE BY FORESIGHT CONCEPTS

Both FFS and PF and our foresight concept too give great importance to *the development of shared future view or views at community level from the individual future shapes*. This issue can be meant as one type of social emergence because it constitutes the interaction between the micro and the macro level of society. The discourse among individuals about the community future is the main form of the integration of individual and community foresight. Therefore a number of participation based foresight tools have been developed and successfully applied. These include the Delphi method, public Delphi, story telling and workshop techniques. Some of these focus on individuals and others on communities [16]. It is also obvious that foresight can be managed by participatory techniques. This time subjective – inter-subjective discourse among social stakeholders is emphasised in bringing to the fore and on shaping the desirable future of human communities. This future compels people to act and take responsibility.

This issue is well known in the recent foresight theories and practice. *But there are other issues which can* be *seen as sources of social emergence* but which are failed or overshadowed in the recent foresight theories and practice. These originate from *the relations and interactions among the foresight and its social environment and outer non-human conditions in which foresight develops and fulfils or does not fulfil after all*. The interactions can trigger social emergent events, for example new civil movements, changes in behavioural or organisational patterns. New ideas about them could be born in the process of foresight.

The forms of these relations and interactions are dialogue [17] or discourses within human spheres and between human and non-human world. In the development of a future idea not only desires, traditional values, moral consideration and fantasy *but experience and knowledge on the nature, the society and the technical world have significant role as well*. Recently these concerns are overshadowed because of failure and bias of forecasting. Namely, in forecasting every issue is approached as a fact or would-be fact of objective reality, therefore there is no place for the human action and choice. This forecasting approach to the social future has proven one-sided and unsuccessful at the same time. Against this we can not state that the social future can be formed freely by own desire or human action. There are some constraints and possibilities coming from the environment or resulted from our earlier actions which influence the space of our future actions. The possible future alternatives on the experience and the knowledge – among them the results of science – concerning to the above mentioned constraints and possibilities should be also considered during the foresight both at a community and a personal level. These future issues should also be rehabilitated in the frame of foresight. Our recommended foresight conception makes it possible by paying attention social function and environmental concerns of foresight activity.

Beside the above mentioned there is another very interesting and slightly studied source of emergence. This can be developed from *the double loop of discourse between subjective – inter-subjective and human – non-human world*. The double loop means a continuously interweaving among human desire, will and critical and limited knowledge about the outer and the inner world concerning the future. The different meanings of this interweaving can be another resource of social emergence from which complex alternative future shapes can be formed out. Complex alternative futures and discourse about them are needed to make ripe the shifts of social and individual life. Foresight should also involve this type of future discourse. The recommended extended foresight concept gives space for building and discussion of complex alternative futures through its self-reflective characteristics and striving to get harmony between the inner and outer world.

At this stage I do not wish to enter into the philosophical debate whether there is such a thing as objective reality or there is only reality as interpreted by human being. Accepting the statement of modern science concerning of it that awareness and knowledge constitutes constantly changing human interpretation. The genuinely interesting question for science and foresight lies in how this knowledge is created or constructed, how much of it stems from human stimuli and impulses, how much from those coming from the outside world, what role do they each play, and how do the two sides intertwine in the process of interpretation or construction concerning the future. From the point of view of time *human perception* seizes not only the moment of right now as a point in time, but can still see what has just happened and is thus also aware of the moment that has just passed. It is aware, furthermore, of passing from now to a new now and, looking ahead, anticipates it. An attentive consciousness and an attentive life impressions and perceptions make for the anticipation of the new now. All human perception is always characterized by an original future intention passing from the past to now, which is invariably linked to experience intentions rooted in the past [18]. *Perception is total in as much as it happens not only within certain time limits, but is filled with content as well even when we recall something or apply it to the future. Content entails a mental state, but it must always have a trigger or a source as well.*

Husserl, the father of phenomenology, *studied perception as a process unfolding within human beings, i.e. always in relation to the way human being reacts to the outer world or to the inner reality.* This offers two conclusions. One is that perception is the starting point of all interpretation, discourse and human communication, while *it is reflexive and self-reflexive as well.* The other is that, given the nature of perception and interpretation, *it possesses an internal and external dynamic, which merge into complex dynamics.* To put it in simpler terms, he or she who perceives and interprets, what is perceived and interpreted and their end-product, the perception and the interpretation, are all present in the process of perceiving and interpreting, each one of them as well as their interaction being variable.

*The continuous dynamics of perception, awareness and interpretation* can be the subject of philosophy, social and natural sciences and foresight alike. The *link* among them *is human being, the active individual who is an integral part of an undertaking to create reality* [19]. Human being, therefore, is an individual with relative independence and autonomy or, as the specialist literature describes him or her, someone with the characteristics of an agent. He or she realizes the individual and collective sources of his actions in local and partial activity within the field of his or her own agency. He or she activates some collection of the social sources of interpretations made available by the situation, and at the same time he himself or she herself becomes active within the interpretations, putting forth at least relating the interpretations to himself or herself and using new interpretations in his or her own and in his or her community's life, and thus contributes to increasing the social sources of interpretations [20, 21]. From the point of view of our subject this leads us to the conclusion that *only human being can have foresight, even if it relates to the future of his or her community or the outer world*. Our second conclusion is that *the outer world as well as the relationship between the outer world and human being are also important for* "*an attentive consciousness and an attentive life*".

I think that socio-cultural constructionism and critical social sciences, among them the conceptions of FFS and PF as well, broke with the latter relation. Recurring to Habermas' train of thought, social constructionists often refer to society being "lifeworld", i.e. inter-subjective relations, which must be freed from the power of the world of the instrumental mind, i.e. the relationship of subject and object [22]. They tackle this freeing or liberation by transposing the subject-object relation (the relation between human being and the outer world, in our terminology) into a relation between subjects. This, however, does not follow from the logic of Habermas. What does follow from it is that the "instrumental rationality", knowledge of the outer world, serves lifeworld. Habermas, therefore, is no subjectivist and voluntarist in the absolute sense of the word, but someone who claims that man's inner and outer world can be shaped through the critique of what already exists. *Foresight may be one form of critique, to which both types of relations of them may belong if it builds on the foresight capacity at the communitiy level (of both types of relations), the foresight capacity of the individual and if it considers as its subject, besides inter-subjective relations, subject-object relations as well as their interaction.*

The richness of human perception is valid together with the above mentioned limitation. During history experience was collected and science was developed to decrease the limitation of human perception. Consequently the foresight should also take the opportunity of them. I think that not even foresight can avoid the subject of complex dynamics. The question that arises is how foresight deals with this subject.

## TOWARDS TO THE META-THEORY IN THE FORESIGHT

Laszlo, Malaska, Mannermaa, Hideg thinks [23 – 26] that the general evolutionary theory (GET) can be the suitable theoretical and methodological background to study issues of emergence in Futures Studies and in the foresight process. The reason of it, that the subject of the GET is to extend meaning of evolution and to study the emergent characteristics of evolution. Despite the reasonability of GET the foresight is unwilling to use the evolutionary way of thinking, and methods and/or tools based on GET in a wide range. I think that GET plays a role in this reluctance, too.

Recently a new trend of GET was developed. This is the fully human meaning of evolution as it named by Loye [27, 28]. It means that the socio-cultural evolution or the human evolution has some characteristics which are very different from the characteristics of the evolution of non-human systems. In the socio-cultural evolution the pattern of "survival of the fittest" is not exclusive. Other patterns as mutuality, cooperation, love, moral sensitivity, consciousness, creativity and capability for a choice of destiny in a world in which choice of destiny is an option are influencing the socio-cultural evolution and the future of evolution as a whole dominantly. The cause of that is the fact that humans and their societies have developed co-evolutionary with a brain and mind system [29]. Consequently, societies have their own specifically human evolutionary aims. Societies are more complex than other systems because their basic elements are individuals with their knowledge, intentions, wills, morals and foresights.

The statement that the society can be seen as a new level of evolution can be acceptable. *But the conclusion that* the *development of society can become independent of the world being out of society can not be acceptable.* This non acceptance is proved by rough damages of the natural environment and the biological background of society caused by environmental pollution, hunger and other so-called global problems. If the foresight disregards of them or handles them only on desire or moral consideration, it can not present realisable future solutions of these problems and the socials issues, too. Therefore *a new meta-theory is needed for foresight activity* to bring together human desire, morals and creativity and experience, knowledge about the outer and inner world of human being. It could be said that the complexity theory could be adapted to the meaning of foresight. This *adaptation can not be solved because of insufficient future contents of complexity theory at this time.* This paper does not also sign on the theory building but recommend an approach through which it can contribute to the further development of the theoretical and methodological basis of foresight.

For this propose *applying an extended participatory approach is recommended*. Participatory approach as a form of human communication has been developed and used in the foresight activity in a wide range. This approach is used to communicate social stakeholders about their desire futures and/or to get a consensus future world view. Using participatory approach in this way is pure subjective as it was pointed out above. The main weakness of it is that presumption according to which shaping desirable futures and/or consensus future explicate their fulfilment. Nevertheless, the participatory approach can be reform. Beside stakeholders of society we can interpret certain systems of the nature and the artificial world as "stakeholders" who have significant impacts to the future of society and the foresight of social stakeholders, as well. Inversely, the foresight of social stakeholders has also significant impact to "the foresight of stakeholders of nature and artificial world". "The foresight of stakeholders of nature and artificial world" could consist of their dynamic characteristics as we understand, know or suppose about them. These interconnections and –actions can be imagined as a drama in which there are different characters and the drama constitutes a discourse among different characters. Playing a "future drama" the interactions and -corrections of foresight of different stakeholders could be carried out.

This "future drama" could be also loaded up with interactively used models on dynamic behaviour of system-models of different type and social scenarios. In these models the foresight of social stakeholders could be the one element and the others could be "the foresights of the nature and artificial world" i.e. the possible future states or time paths of non-human systems emerging from the interaction of "foresights of different stakeholders". The future states or time paths of systems could be interpreted as possible futures which are also filtered by understanding, knowledge and desire of social stakeholders in this way. Construction of "future drama" should need transdisciplinary models in which applied models, their interconnections and -actions among the human perceptions, experience, scientific knowledge, future expectations, options and human actions concerning to actual foresight issues should be also mapped. Both moments of adaptations and goal-orientations of social level of evolution could be also expressed in the foresight process as a "future drama" in this way.

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# IZVIRANJE U PREDVIĐANJU

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## SAŽETAK

Članak prikazuje neke cjeline izviranja u predviđanju koje treba znanstveno razmatrati. Ističe tri podloge socijalnog izviranja. Jedna od njih se očituje u razvoju zajedničkih pogleda na budućnost zajednici na temelju individualnih oblika budućnosti. Druga se očituje u relacijama između predviđanja, njene socijalne okoline i vanjskih uvjeta. Treća se očituje u dvostrukoj petlji diskursa između subjektivnog i nesubjektivnog te svijeta ljudi i svijeta bez ljudi. U članku se zaključuje da su metateorija i prošireni pristup sudjelovanja potrebni za istraživanje, proučavanje i razumijevanje kompleksnog socijalnog izviranja u predviđanju.

## KLJUČNE RIJEČI

proučavanje budućnosti, predviđanje, socijalno izviranje

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# AID COMPETITION – A SIMPLE AGENT-BASED MODEL OF COMPETITION FOR FINANCIAL AID

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## SUMMARY

The research into the effectiveness of financial aid is gaining momentum lately. Some say it is ineffective, some say it could be effective, but all seem to agree that some of its aspects are currently unmapped.

This article aims at showing a rather simplistic agent-based model that might hint at a possible useful approach of the issue. It will be shown, that the donor policies do influence stability, convergence and the path of economic growth.

## KEY WORDS

aid, agent-based modelling, competition

## CLASSIFICATION

JEL: C65, I38

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## INTRODUCTION

The issue of financial aid is getting more and more into focus these days. The International Monetary Fond releases study after study discussing the theoretical and empirical problems arising in their day-to-day practices.

The general idea behind financial aid is to induce economic growth in the recipient nation. There can be various reasons for wanting that: humanitarian (improving the quality of life, health, life expectancy of the population) economic (generating market, developing the extraction of natural resources) and social (prevent migration). Most of these reasons emphasize the need to increase the per-capita output of the recipient nation, even though some of these goals are easier to achieve using a different approach. Thus the theory of financial aid developed from the theory of economic growth.

In the 1950s-70s many western nations actively engaged in the aiding movement. Billions of dollars were spent on improving the productivity of low income nations, but the expected expansion never happened. Starting from the 1990s economists started to question the efficiency and even the effectiveness of financial aid. Many studies, e.g. [1 – 4], showed that financial aid has dubious effects (at times it even hinders growth).

The goal of this paper is to build up a simple and efficient agent-based modeling framework, that makes it possible to investigate the relationship between donor policies and the resulting economic growth. It will be shown, that it is possible to create a relevant model using even the simplest possible agent-based approach. It will be demonstrated, that the proper donor policies can lead to converging macrosystems and better life expectancy across multiple regions.

## AGENT BASED SIMULATION

Agent-based simulation is a bottom-up approach to building models. It focuses on modeling the low level entities, and then drawing the conclusions after seeing how they interact in the modeling environment.

### POSSIBLE APPROACHES

Since financial aid is such a complex issue, agent based modeling can be utilized in various ways. These approaches differ mostly in their depth – and consequently, in their speed and execution cost. The three models discussed are: (1) the high-level Nation model, (2) the low-level Family model, and (3) the integrated Hierarchical model.

#### Nation model

The Nation model, or “Agents as nations” model basically focuses on the national level. The agents in this simulation approach are the members of the aiding process. The modeled activities  are the interactions of the nations. Since the internal workings of every agent can be as complex as necessary, the end result can contain information about the internal states of the nations, too.

The agents in this approach face macroeconomic decisions. How much value should be spent on consumption? How much of the savings should be channeled to investment? Does the government intervene? How much aid should be given/how should the received aid be used?

This approach is advantageous in may ways. It provides for fast simulations, since the processing power requirements are relatively small. The results which can be obtained from this model are easy to interpret, since the agents are the modeled nations themselves, so their rvariables can be read directly. It is easy to introduce a form of decision making process to the national level that mimics "planning", that might lead to unforeseeable outcomes. Last but not least, using a Nation model one can observe the relationships among different countries. Which national strategy could lead to a dominant position? Which strategy could lead to the highest amount of aid received?

The significant problem of this model is that it is oversimplified. The internal workings of the agents are – no matter how detailed – given as macromodels, so this proves no true micro-macro synthesis.

### Family model

The Family model, or Agents as Families focuses on the "atomic" entity within a society. This could either be interpreted as the individual or as the family. In case of most short-run models this distinction is irrelevant, as these agents face decisions about consumption, savings and work. If, however, the simulation deals with health care issues or individual consumer choices, families cannot be equated with individuals. A further problem is, that long run models also have to deal with mortality and birth. (Models based on "families" are better at dealing with communal consumption, since they do not have to worry about the formation of "micro-societies". Individual based models are better at dealing with birth and death.)

The Family model is significantly more detailed than the Nation model. It provides a detailed micro-macro synthesis: the relationships modeled are microeconomic in nature (the "agents" decide how they are spending their time, "where" they work, "what" they buy, etc.) These models are capable of dealing with various different corporate sectors in the economy, thus does not have to use a unique product – or a unique price level within the nation. Sectoral shocks can be introduced/modeled, which enables the observation of phenomena like the development of dutch disease.

A further advantage is, that "thinking" can easily be simulated in this environment. Thinking is, once again, at the agent level – but at this time agents are individuals/families, firms, government entities (or even banks), not nations. This allows for more natural approach to modeling thought, what will enhance the model's verifiability.

This model is not without its share of problems, however. It requires a large amount of agents to run, thus will consume huge amount of processing power. This leaves simulations more costly in terms of processing power, time and money, making experimentation in this environment both slower and pricier. The other issue is even more worrisome: since only a single nation is modeled, international relationships (like financial aid) are not incorporated. (An example of these kinds of models is the Scandia National Laboratories' ASPEN model, that simulated the US economy and three of its sectors, supported by government and the bank sector [5].)

### Hierarchical model

The hierarchical model is an attempt to blend the multiple-nation approach of the Nation model with the detail of the Family model.

The Nation model focuses on the fact that its entities are models of nations, whereas the Family model models nations. It is, then, easy to see that by "plugging in" one or more Family models into the Nation model, the accuracy of some aspects of the high-level Nation model can be increased. One could, for example, model aid donors and aid beneficiaries with simple macroeconomic models, and replace one beneficiary with a family model. This approach would slightly increase the complexity of the Family model, but would enable the nation to engage in international trade and receive financial aid. Another possibility would be to include a family model to the donors. This could help evaluate the positive effects of giving aid. The most detailed picture could be gained if all nations were modeled with Family models, but that would slow down modeling speed considerably.

### Comparison of the different approaches

It is easy to see that there is a strong trade off between detail and processing speed among the various modeling approaches. There is no obvious "good" or "bad" among them, one has to choose the appropriate approach for a given task. The following table summarizes the relevant attributes of the different approaches.

**Table 1.** Comparison of different approaches to modelling of financial aid.

| | Nation model | Family model | Hierarchical model |
|---|---|---|---|
| Detail | - | + | ++ |
| Ease of extraction of results | ++ | - | - |
| International connections | + | - | + |
| Speed | ++ | - | -- |
| Complexity | - | + | ++ |

## MODELING ENVIRONMENTS

The problem with agent-based modeling is that its rather computer-intensive. There are various environments to help the social scientist, but these usually place severe restrictions on the models themselves. For this reason it is important to be able to quickly review the various environments helping the modeling process.

### Mathematical frameworks

The most simplistic agent based models can be depicted as a set of equations, in essence as a symbolic model. In this case simulation can be done using some general mathematical framework[1]. The advantage of this approach is, that it allows the social scientist to use known tools, resulting in a flatter learning curve. The disadvantage is also obvious: this leaves the agent-based approach a thought experiment, and disables most of the opportunities presented by agent based modeling.

### Agent-based modeling frameworks

These frameworks are designed especially for aiding the development of agent-based models. They usually require some degree of programing knowledge, but allow for rapid development of agent based models. The usual trade-off is  between ease of use and model complexity.

### Programing languages

If the concern is the runtime of the simulation, then using a programing language to develop the agent-based model can be the ideal solution. This, of course, requires significant programing skills, but can potentially lead to the smallest possible execution time.

### Selected environment

The "Agents as nations" model uses the Repast modeling environment, which is a modeling framework designed for social scientists. It is rather straightforward to use with sufficient programing knowledge, and provides excellent support for model creation and display.

## THE NATION MODEL OF AID

The declared goal of the model is to be able to show the relationship between the aiding process and the state of the recipient nations. In order to do so, the model needs to be able to show international connections, so the Family model is unsuitable. Aiming for ease of use and development, the Nation model was chosen, since it can show all the necessary information, but remains small and well-contained at the same time.

### THEORETICAL BACKGROUND

The model incorporates two kinds of nations: aid donors and aid beneficiaries. It focuses on the important aspect of the aid relations, so there is no trade among the nations, no consumption in any nation – the model essentially disregards how will income be utilized. A simple assumption can justify this: the aid beneficiaries produce only non-tradable goods. This means, that regardless of their output, they will consume it all, since their product is not sold on the international marketplace. This might seem a strong assumption, but can be valid for a number of low income countries[2].

These agents interact with each other only through the aid distribution process: donors provide aid, whereas the beneficiaries use it to purchase goods and services from the international marketplace.

### Donor model

In this model, the donors are kept as simple as possible. Since we only worry about how their decisions and policies affect the recipients, there is no actual need to simulate their internal structure. From the model's point of view, donors are "infinite sources" of aid. Part of this aid gets distributed evenly among the recipient nations (to prevent "aid starvation"; even the worst performers should get a chance to buy tradable goods – which can only be bought from financial aid, since the recipients produce only nontradable products). The other part of the aid will get distributed according to the policies of the donors. Considering that the purpose of the model is to determine the effect of the donor policies, it is important that they should be easily to change.

### Beneficiary model

The beneficiaries are modeled with a simple, production-oriented macroeconomic model. They use three resources (labour, capital and land, although land will remain unused in this version of the model), and have a Cobb-Douglas type production function

$$F(K, L, A = \alpha K^{\beta} L^{\gamma} A^{\delta},\ \beta + \gamma + \delta = 1.$$

The model does not deal with consumption (assumes that production is always at the potential level). This implicitly assumes a neoclassical model, where consumption and savings only determine the distribution of production[3].

The model emphasizes production, and thus factors of production. The current version uses labor and capital, and treats them both as endogenous variables. In case of the capital goods, this means that although they have an initial stock, their amount will change in time. The model handles amortization (a parameter described in years, meaning the average time a capital good becomes unusable). As opposed to the standard neoclassical model, these low income nations are completely unable to produce capital goods, thus investment will not be backed by savings, rather a portion of the financial aid can be diverted and utilized for this purpose.

The other factor of production, labor, changes with time, as well. The growth of the labor force is described with the fertility rate $r_{\text{fertility}}$. Its counterpart is the mortality rate $r_{\text{mortality}}$, that shows the percentage of the population that dies by the end of the year. These ratios are fixed for a nation, but they can decide to spend a portion of aid on medication, that will reduce the mortality rate.

These two constraints determine the decisions a nation faces. They get to allocate the received financial aid between medication and capital goods. One helps replace the capital stock, the other makes people live longer. The capital goods are purchased at a price, the medication's cost is unity (in essence, the price of the capital is a relative price between medication and capital).

Capital changes according to the following equation:

$$K_{\text{new}} = K_{\text{old}}\left(1 - \frac{1}{\text{amortization\_years}}\right) + \frac{a_{\text{K}}}{P_{\text{K}}}$$

where $a_{\text{K}}$ is the amount of aid spent on capital, $P_{\text{K}}$ is the price of capital. It is clear to see, that increasing the average durability of the investment goods (the amortization years) the capital stock would deplete slower. Also, allocating more aid on capital or a reduction in the price of capital would speed up capital accumulation.

Labor is governed by the following rule:

$$L_{\text{new}} = L_{\text{old}} \cdot \exp\left[r_{\text{fertility}} - r_{\text{mortality}}\left(1 - \frac{a_{\text{M}}}{a_{\text{M}} + L_{\text{old}} / eff_{\text{med}}}\right)\right]$$

where $a_{\text{M}}$ is the amount of aid spent on medication. The variable $eff_{\text{med}}$ is the medication effectiveness, basically describing the effectiveness of medication to reduce mortality rate. With unity medication effector, if the country spends one unit of aid for each member of its population, they effectively halve the mortality rate. Spending twice that amount cuts the mortality rate by 2/3, etc. If the medication effector is 0, the medication has hardly any effect on the population. If it is infinity, the population becomes immortal.

## Model parameters

In order to make the model easier to use, a number of parameters got their graphical controls. Some of these parameters deal with the system as a whole, others determine the workings of the aid donor.

The parameters used in the model level:

- Capital cost: describes the relative cost of capital to medication. Increasing this value would make it more expensive to invest in capital goods.
- Average amortization time: this variable sets the durability of the goods used in the production process. Increasing its value makes capital deplete slower.
- Medication effectiveness: increasing this value will make a unit of medicine do more work.

The parameters of the donor(s):

- Distributed aid: The amount of aid the donor can distribute annually.
- Aid to be distributed evenly: this percentage of the aid gets distributed evenly among the

aid recipients. This allows, that no matter the chosen aiding policy, no nation will be left without the opportunity to prosper.

- Donor behaviour: This is the most important parameter, since we want to evaluate its effect on the output and population of the recipient nations. Notable goals include: output per capita, percentile change in output levels, change of the output level, level of the output and population count.

The parameters for the beneficiaries:

- Initial population: this effectively sets the labor force. Distinction between the two is unnecessary, since consumption is disregarded.
- Initial land: this will not change in this version of the model, only incorporated to make extension easier.
- Initial capital: the starting capital stock
- Fertility and Mortality rates: determine the labor growth rate.

## PROGRAMING DECISIONS

The most important programing decision was the extensive use of interfaces. The various elements in the model connect through pre-defined ways, and this makes the model easy to extend.

The most important interface is that of the decision rules. Any object implementing this interface can be used in the model as a possible "rule" for the aid donors to choose how much money would the various possible recipient countries receive. Other interfaces include the one defining the aid donors, and another describing aid recipients.

For easier use, some parameters of the donor(s) are wired to the main console. This makes causing global changes in the system easier to implement.

## NOTABLE RESULTS

Even though the model used is oversimplified, a number of significant results are obtained from it.

In the comparative experiments three distinct nations were used. Nation "RED" was the least developed, had low capital stock, and high mortality rate. The fertility rate and the population count was highest here. The "BLUE" nation was the opposite: had relatively high capital stock and comparatively low population, with low fertility and even lower mortality rate. The "GREEN" nation was the middle ground, possessing both average capital stock and population, having intermediate fertility and mortality rates. The initial resource allocation was determined, so that all nations would have the same level of output.

A single donor was used, thus only one donor policy was present at a time. This setup allows to examine how a particular policy affects the recipient nations.

### Convergence and stability

Probably the greatest result of the model is that convergence and stability can be achieved. In case the donors choose to award more aid for higher positive change to output, the beneficiaries start to focus on their output levels. This causes the overall levels of production to converge. This has the side effect of stabilizing population, thus stabilizing per-capita GDP stabilizes, as well.

The aid distributed among the nations fluctuate, Fig. 1. Initially the higher population of the RED nation allows it for fast expansion through heavy investment in capital stock, but its growth hits a curb and after that the "average" GREEN nation soars. Throughout all this BLUE has stable growth, until it flats out at its potential output.

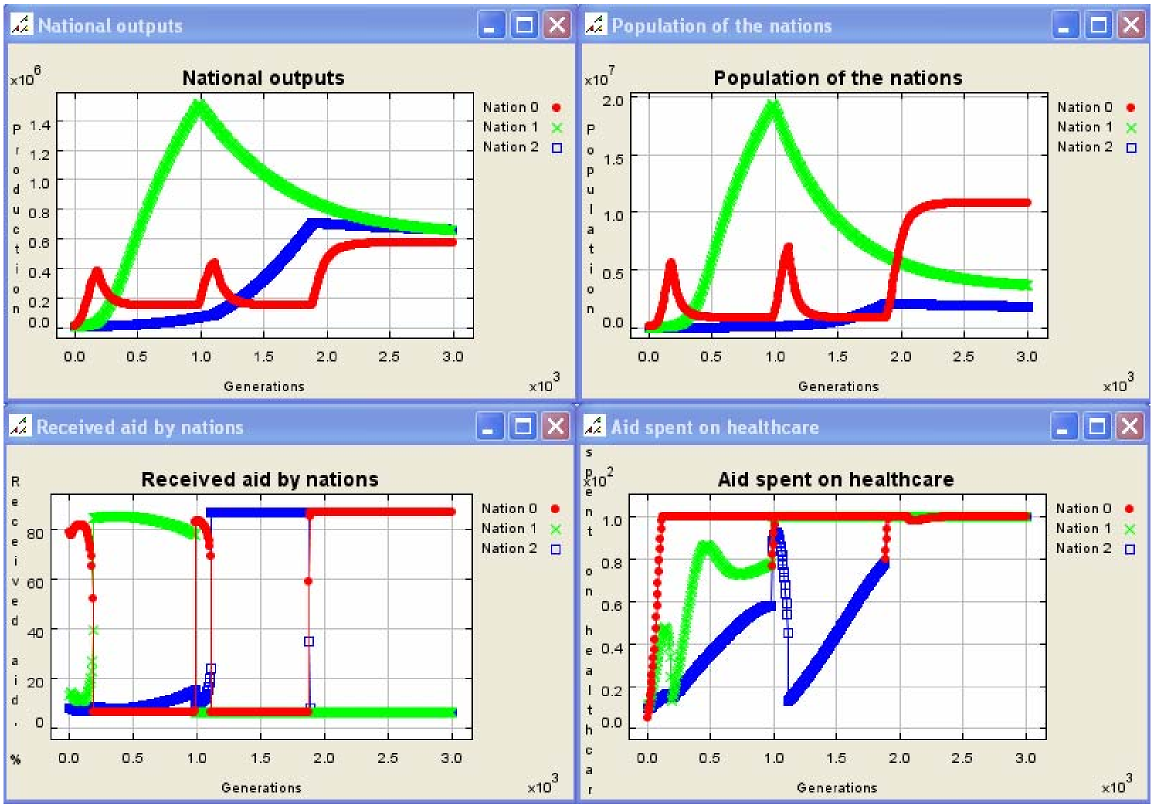


**Fig. 1.** Time dependence of aid in case of tri nations.

The aid utilization is clearly beneficial for humanitarian reasons. Following its own goals of maximizing production, RED ends up spending nearly all of its aid on healthcare very fast. The other nations adopt this policy later on. It is also important to notice that all nations periodically need investments to replenish their capital stocks, but after that they will return to finance medication.

It is also important to note, that the system stabilizes even if the chosen policy is to maximize the change in the per-capita output. The exact path of the convergence will be different, for obvious reasons, but the end results are comparable. Per-capita outputs do not converge but they stabilize, as before. Population levels stabilize, and the nations have the same order as before, Fig. 2.

The amount of aid received fluctuates here, too. In this scenario the BLUE country can take a good head start, since its low population grants it a high per-capita output, thus ensuring more aid in the early times. The utilization of the aid shows a clear focus on humanitarian benefits. In those countries where the living conditions are bad, a heavy investment in healthcare becomes dominant at a very early stage. Even the relatively advanced BLUE country converges slowly to the all-healthcare policy.

## Problem of local maxima

The model also shows the problem of the “greedy” algorithm of decision making. If the countries always do what is best for them in the short run, they might miss out on long run opportunities, Fig. 3.

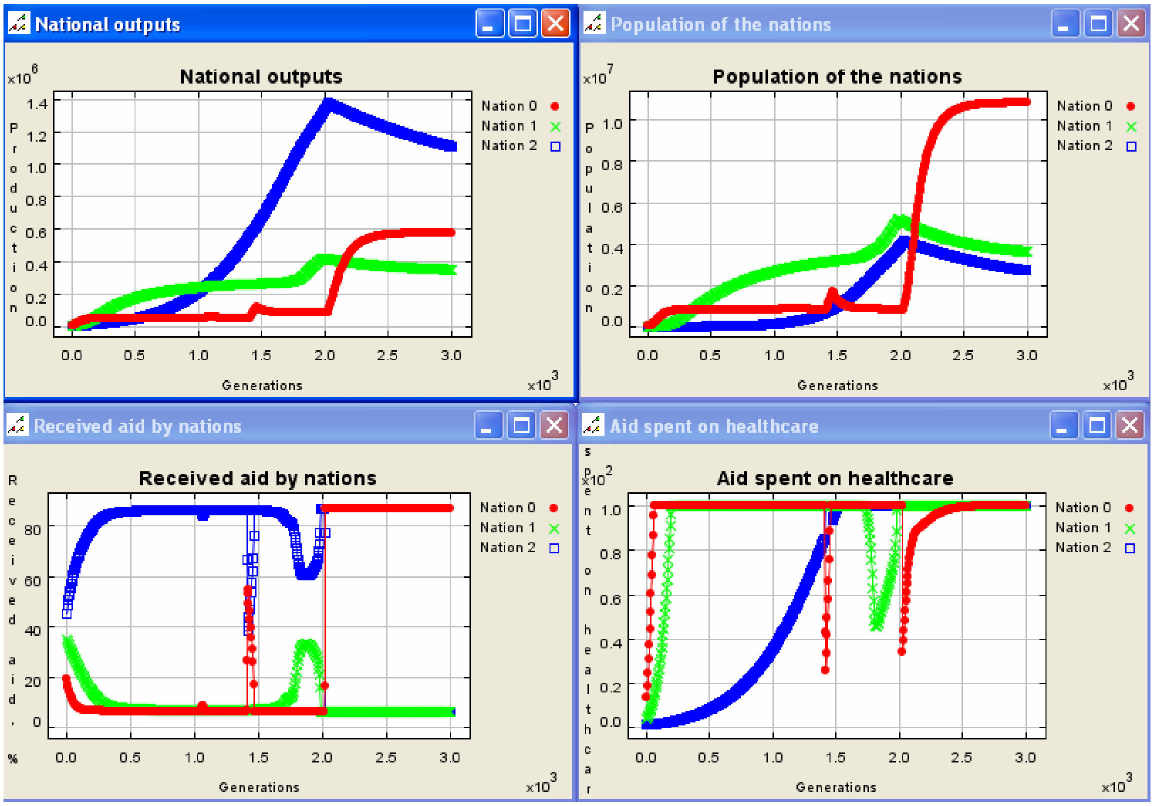


**Fig. 2.** Time dependence of aid.

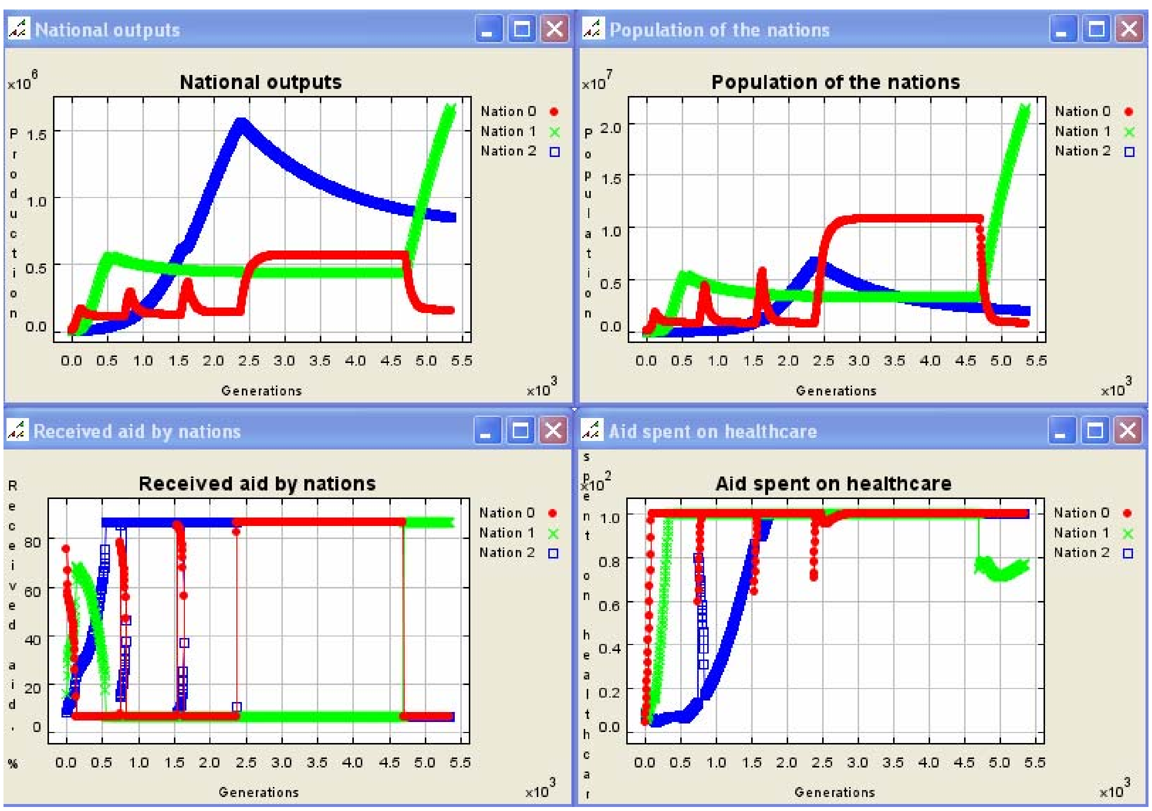


**Fig. 3.** Consequences of "greedy" algorithm of decision making.

In this experiment, the target was the percentile change of total output. This lead to a weird anomaly:

- it is obvious, that the nations perform better, when they spend heavily in capital (in case of the RED country's three booms, an investment surge is apparent. BLUE moved away from investment, and its production fell. When GREEN finally decided to focus on investment, both its population and production skyrocketed),
- despite this, all nations prefer to invest heavily in healthcare.

It is quite likely, that if the decision making was not simply rule-based, but some sort of "memory" or "experimentation" was built into the model, it would be able to avoid this problem of "local maxima"[4].

## Strategy forming

A completely unexpected but welcome result was the parallel to the real-life situation, where the beneficiaries "outsmart" the donors. By knowing the decision making of the aid donors, recipients make their decisions to gain more aid, and not to achieve the initial goals. This might lead to following the donor's intention to the letter, and at the same time causing great damage to other aspects of the economy.

The example scenario looked at what happens if the donor wants great growth in population, when medication is rather ineffective, Fig. 4.

It is clear, that by setting the population goal the donor more or less achieved its aim. It stabilized the population levels in two countries, and the population stabilized even in RED, although after a significant demographic catastrophe. For obvious reasons all nations spend all of their money on healthcare, trying to extract as much aid from the donor as possible. The

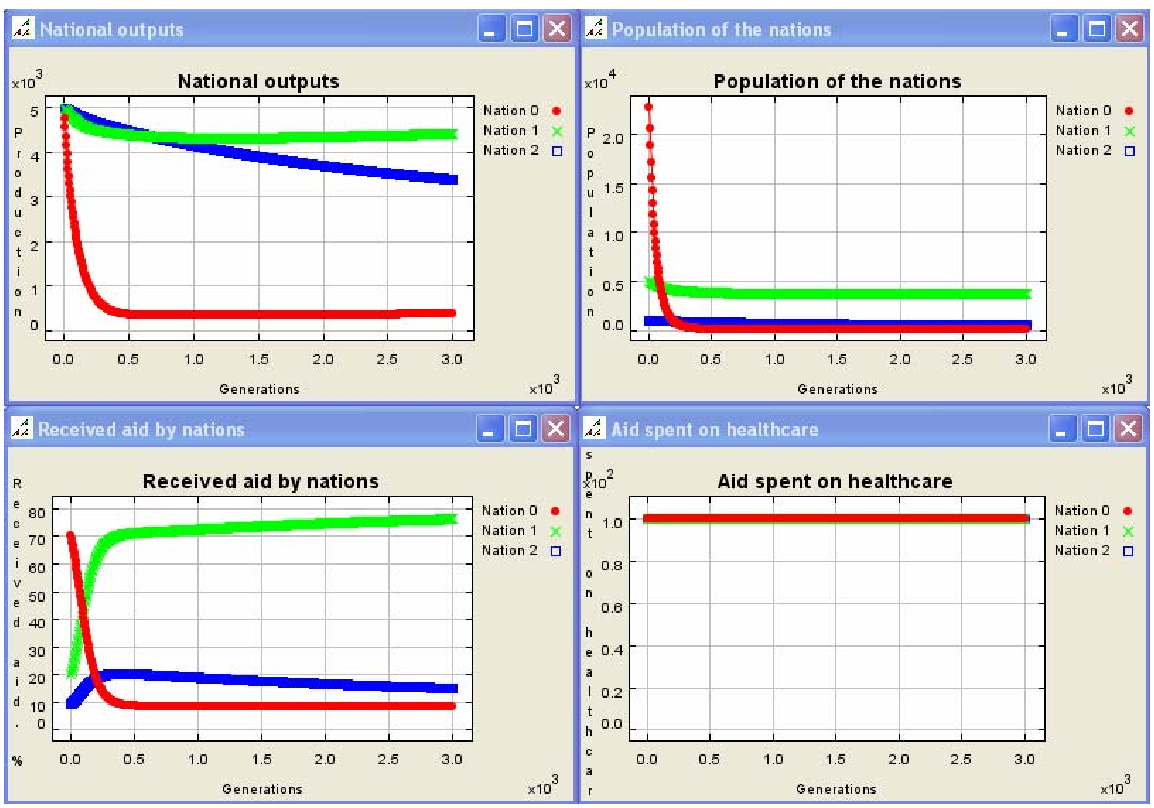


**Fig. 4.** Time dependence of aid with ineffective medication.

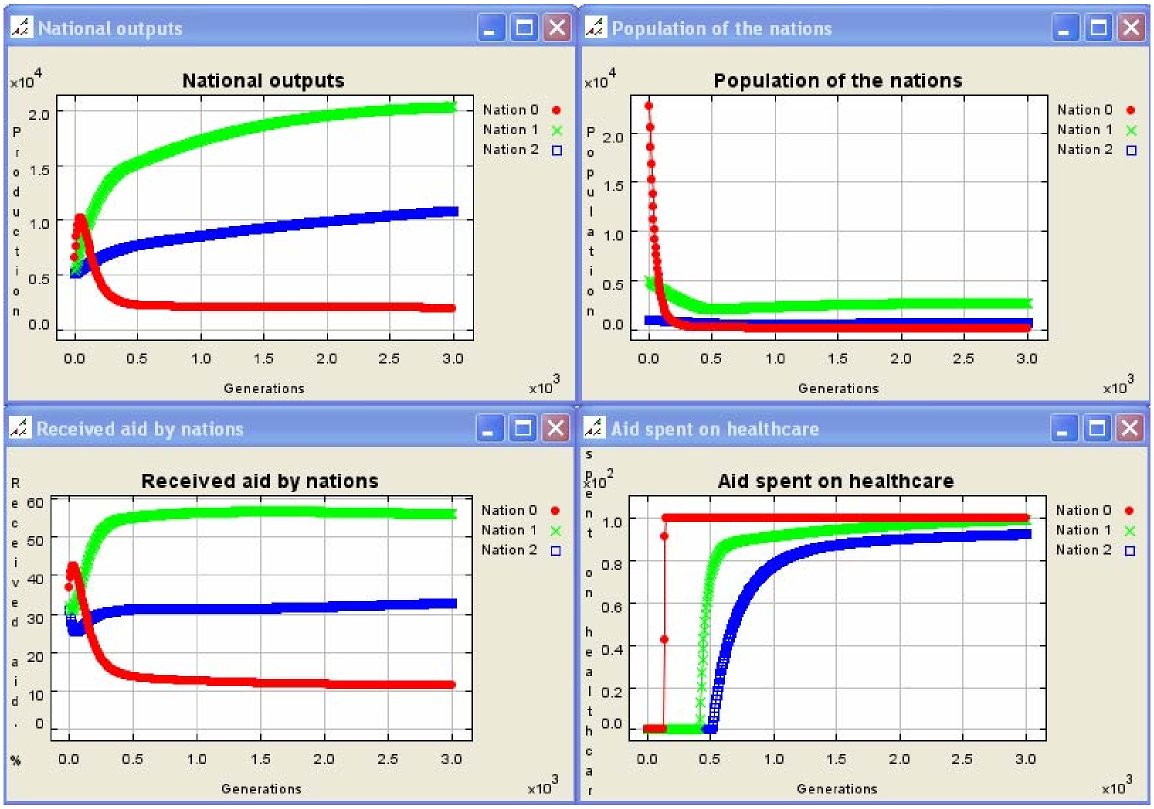


**Fig. 5.** Time dependence of aid with set goal.

side effect is, that the output decreased in all nations, and there is no sign of recovery, only stability. In addition to that, both aid distribution and utilization are distorted.

Should the aid donor choose a different approach, the end result might be more beneficial overall. In the same basic setup, setting an output growth goal will propel the nations to invest in capital initially. This stabilizes their output, and after a short while their production statistics start to improve. This approach results in more even aid distribution and utilization. Of course even this approach cannot prevent the disaster in RED, but that is a given; in case of a strong infection (high mortality rate) only effective medication could help, and the initial condition was, that only mediocre medication is available, Fig. 5. But in this scenario at least the economy of the nations survive, allowing for greater possibility of recovery and development.

## POSSIBILITY FOR EXTENSION

Even without moving out from the “Nation model” approach, this model can be meaningfully extended in a number of ways.

1. Parametrized production functions could be used for the aid beneficiaries. This would allow to better depict different levels of improvement, as resource processing would not be the same across the board.
2. More donors could be simultaneously examined. This would enable conflicting goals. How would the beneficiaries react? Would each nation find the donor best “suited” for its own goal? Would the donor’s goals cancel each other out? Would it mean greater or lesser growth in the beneficiaries?
3. The beneficiaries could have different goals. This could allow for even stronger strategy forming. How would the differing donor and beneficiary goals interact?

4. Some feedback between population growth rate and output could be included. It could be in the form of some kind of logistic curve, so that higher output would not always have the same effect on fertility rates.
5. Smarter decision making process could be included for all parties. This could come in many forms (genetic algorithm, neural networks, swarms etc.) but would lead to different, and probably more relevant results.
6. Warfare could be included with the introduction of land to the production set. The nations could spend some of their aid on weapons, and then could use those weapons to wage wars on the other beneficiaries. The wars would reduce the population in both countries, and would increase the supply of land in one country at the expense of the other.

## CONCLUSIONS

Even using a simple, high-level agent-based model of financial aid, notable results were achieved. It was shown, that the policies the donors set to distribute financial aid affect the beneficiary nations significantly. This means, that the donors have a great responsibility when providing financial aid, since they have the power to prevent (or even cause!) catastrophes, and can create higher output and even better quality of life.

## REMARKS

[1]Loosely based on [6].

[2]It is worthy to note, however, that while it is possible that a nation's entire output would be nontradable, such a nation would probably not attract financial aid. As suggested before, financial aid is mostly given to increase the output of the recipient nation – but if that output cannot be used anywhere else, this increment would only be a form of humanitarian aid.

[3]For the precise definition of the used neoclassical model, see [7].

[4]Memory can be easily modeled trough the usage of neural networks, whereas "experimentation" can be simulated with genetic algorithms.

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# AID COMPETITION – A SIMPLE AGENT-BASED MODEL OF COMPETITION FOR FINANCIAL AID

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## SAŽETAK

Istraživanja učinkovitosti financijske pomoći u zadnje se vrijeme intenziviraju. Prema jednom tumačenju financijska je pomoć neučinkovita, prema drugom može biti učinkovita, ali u oba tumačenja neki njeni vidovi sada nisu jasni.

Članak predočava pojednostavljeni model temeljen na učesnicima koji može doprinijeti temi. Pokazano je da pristupi donora utječu na stabilnost, konvergenciju i put ekonomskog rasta.

## KLJUČNE RIJEČI

pomoć, modeliranje putem učesnika, kompeticija

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# OPTIMAL PROCESSES IN IRREVERSIBLE MICROECONOMICS

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## SUMMARY

In this paper we consider optimal trading processes in economic systems. The analysis is based on accounting for irreversibility factor using wealth function concept.

## KEY WORDS

microeconomics, non-equilibrium thermodynamics, irreversibility

## CLASSIFICATION

PACS: 05.70.Ln, 28.60.+s

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## INTRODUCTION

In the last decades macro system theory has been extended to economic systems, see [1 – 3]. The crucial role here is played by the concept of resource value for a subsystem and the concept of exchange kinetics that is based on the differential of resource's value estimate by two economic systems. This technique makes it possible to determine the optimal behaviour of an economic intermediary operating in an irreversible economic system. In this framework economic intermediary is similar to a heat engine in thermodynamics. It controls its intensive variables (prices it orders to buyers and sellers). A direct economic exchange is always irreversible. However an exchange via an intermediary can be reversible, if the price for a resource used by an intermediary is intuitively close to resource's value estimate by a subsystem. In this case the rate of exchange will be intuitively close to zero. It is worth noting that if an exchange with the given rate is carried out via an intermediary then its irreversibility is lower than irreversibility of a direct exchange. If the duration of exchange or its rate is constrained then the problem of finding what are the prices an intermediary has to order to buyers and sellers in order to obtain maximal profit.

We denote the cash holding of an intermediary as $M$ and the value of its assets as $F$.

## PROFITABILITY AND CONDITIONS OF MINIMAL DISSIPATION

We denote profitability as the maximal amount of cash that can be extracted from the system subject to given conditions. The system is denoted as a number (possibly one) of economic intermediaries and their environment. The constraints here play an important role. They can include the constraints on the final states of some of the economic agents, conditions for the intensive variables of the system, constraints on the exchange duration and others. These constraints reduce the profitability. If a system does not have an environment with constant estimates this definition of profitability will still be valid.

The problem of finding the profitability does not have a solution for some systems without constraints. For example, profitability for a system with more than one economic reservoir (market) is unlimited, because an intermediary operating between them can generate an infinite profit. Note that profitability for a system with one reservoir, given initial state of an intermediary and no constraints on the duration of exchange represents an economic analogy of exergy in thermodynamics, which is widely used in engineering.

Let us consider an economic system with $k$ economic agents. Each agent has resource's inventory $N_i$ (i = 1, 2, …., $k$) and cash holding $M_i$. Resource's value estimate $p_i$ depends on $N_i$ and $M_i$. Economic reservoir (system with constant resource value estimate $p$) can be one of subsystems.

We assume that the system is closed with respect to resources. When there is contact between i-th and j-th subsystem then resource and capital flows between them $n_{ij}$ and $m_{ij}$ occur. Resource flow is directed from the system where its estimate is lower to the system where this estimate is higher. The capital flows in the opposite direction.

If a system contains an intermediary (economic agent) then its objective is to organize resource exchange in such a way that it extracts capital $M$ from the system. We will assume here that economic subsystems cannot exchange resources directly but only via an exchange though an intermediary. This intermediary regulated this exchange by setting up the prices for buyers and sellers. It controls this price setting to maximise $M$. The flows of buying and selling depend on the price $c_\nu$, offered by the intermediary to the ν-th subsystem and on the resource i value estimate for this subsystem $p_{i\nu}$. Thus,

$$n_{i\nu} = n_{i\nu}(p_i, c_i),\ n_{i\nu} = 0,\ p_\nu = c_\nu,\ \mathrm{sign}(n_{i\nu}) = \mathrm{sign}(p_\nu\ \ c_\nu).$$

We denote the flow directed to the intermediary as positive and from it as negative. The flow is a monotonically increasing function of $c_i$. The intermediary does not produce anything, it just resells what it purchased earlier.

It is clear that the flow of capital

$$m_\nu(p_\nu, c_\nu) = -\sum_i c_{i\nu} n_{i\nu}(p_i, c_i)\,.$$

The evolution of resource and capital inventories in the ν-th subsystem is described by the equations

$$\dot{N}_{i\nu} = -n_{i\nu}(p_\nu, c_\nu)\,,\ N_{i\nu}(0) = N_{i\nu 0}.$$
$$\dot{M}_\nu = \sum_i c_{i\nu} n_{i\nu}(p_i, c_i)\,,\ M_\nu(0) = M_{\nu 0}.$$

As a rule, estimates $p_{i\nu}(N_\nu, M_\nu)$ monotonically decrease when $N_i$ increases and $M_\nu$ is fixed. These estimates also are non-decreasing functions of $M_\nu$ for fixed $N_\nu$.

Next we will calculate how much money can be extracted by an intermediary over an infinity period and over a finite period of time for a system that includes economic reservoir and which lacks it.

## PROCESS DURATION IS NOT CONSTRAINED

### System with one reservoir

The profit from reselling of a resource can be only extracted if initially the system is in a non-equilibrium state. That is, if vectors of resources' estimates $p_\nu(0)$ for different subsystems have different values. The trading stops in equilibrium when estimates for all subsystems become equal to reservoir's estimates

$$p_{i\nu}(\overline{M}_\nu, \overline{N}_\nu) = p_i^0,\ (i = 1, \ldots, n \text{ and } \nu = 1, \ldots, m). \tag{1}$$

The maximum of the extracted profit corresponds to a minimum of the combined capital of the economic intermediary and reservoir

$$\sum_{\nu=0}^{m} \overline{M}_\nu \to \min\,. \tag{2}$$

To achieve that objective, the intermediary buys resource using the lowest prices (estimates) from economic subsystems with estimates of the i-th resource below $p_i^0$, and resells it using the highest prices (estimates to economic subsystems with estimates higher than $p_i^0$. Both buying and selling processes are reversible and the increment of the combined wealth function is equal zero. The initial stocks of capital of economic subsystem is given and the increment of the capital of economic reservoir is

$$\Delta M^0 = \sum_{\nu=1}^{m} \sum_{i=1}^{k} [\overline{N}_{i\nu} - N_{i\nu}(0)] p_i^0\,.$$

We have $m \times n$ conditions (1), and $m$ reversibility conditions to find the state of the system in equilibrium

$$S_\nu(\overline{M}_\nu, \overline{N}_\nu) = S_\nu[M_\nu(0), N_\nu(0)] = S_\nu,\ \nu = 1, \ldots, m. \tag{3}$$

Equations (1) and (3) determine the equilibrium stocks of resource and capital. The extracted capital is equal to the difference between combined final and initial capital of the system minus reservoir's capital increment

$$E_\infty = \sum_{\nu=1}^{m} [M_\nu(0) - \overline{M}_\nu] - \Delta M^0\,. \tag{4}$$

### System without reservoir

In this case the condition of equilibrium (1) still holds but the vector of equilibrium estimates $p^0$ is unknown. It is to be found form the condition that intermediary does not accumulate resource

$$\sum_{\nu=1}^{m}[\overline{N}_{i\nu} - N_{i\nu}(0)] = 0\,,\ \mathrm{i} = 1,\ \ldots,\ n. \tag{5}$$

The system (1), (3), (5) determines $(m + 1)\cdot n$ subsystem's state variable in equilibrium. Naturally, equilibrium in a system with an intermediary $\overline{N}_0$, $\overline{N}$ is different from equilibrium for direct exchange. The maximum of the extracted capital is

$$E_\infty = \sum_{\nu=1}^{m}[M_\nu(0) - \overline{M}_\nu]\,. \tag{6}$$

### Example

We consider an economic system which consists of an economic reservoir, a passive economic subsystem with finite capacity and an intermediary. Subsystem's wealth has the following form

$$S = M^{1/3}N_1^{\,1/2}N_2^{\,1/6}.$$

The initial inventories of both resource and capital are

$$M(0) = 150,\ N_1(0) = 20,\ N_2(0) = 30.$$

and the equilibrium prices for economic reservoir are

$$p_1^{\,0} = 5,\ p_2^{\,0} = 2.$$

The equilibrium states of subsystem equilibrium estimates are found from (1) and (3). The maximal amount of capital extractable from the system is determined by (4) and is equal to $M = 20{,}95$. Suppose that reservoirs' prices are scaled with the coefficient $k$. The dependence of $E_\infty(k)$ is shown in Figure 1. Let us emphasise that unlike the case of direct contact, the equilibrium state does not depend on the exchange kinetics for an exchange via an intermediary.

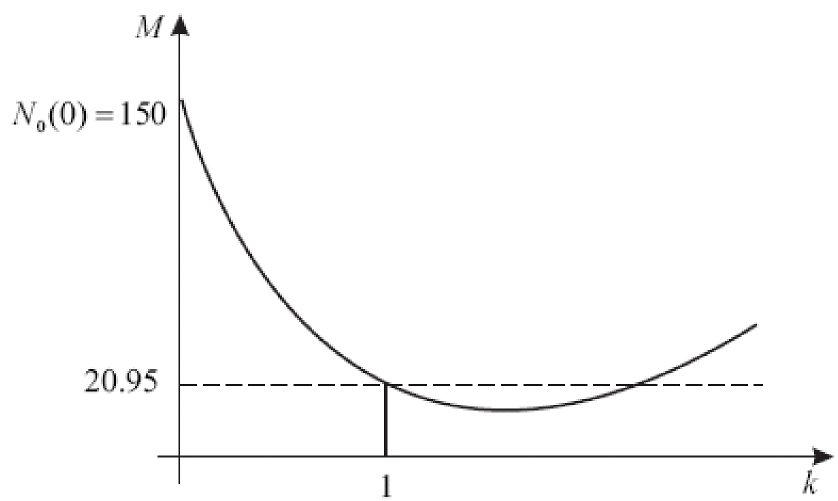


**Figure 1.** The dependence of intermediary's profit on the economic reservoir's price scale.

## LIMITED DURATION OF EXCHANGE

We assume that the duration of exchange $\tau$ is given. In this case an intermediary is forced to increase the prices offered to sellers above the equilibrium estimates $p_i$. It is also forced to reduce the prices it offers to buyers below equilibrium estimates. This leads to an irreversible losses and reduces the amount of capital it is able to extract from the system. The maximal possible value of this capital $E_\infty$ turns out to be lower than $E_1$. Their difference

$$\Delta E = (E_\infty - E_\tau) > 0,$$

describes the irreversibility of the trading.

We shall call the capital loss above the capital loss in equilibrium (reduction in system's profitability) the *capital dissipation*

$$\sigma = n(p, c)(p - c). \tag{7}$$

For scalar exchange the dissipative losses are determine as

$$\Delta S(\tau) = \int_0^\tau \sigma(t)\mathrm{d}t = \int_0^\tau n(p,c)(p-c)\mathrm{d}t\,. \tag{8}$$

### Condition of optimality for trading

We consider an exchange between an intermediary and a finite-capacity economic subsystem. We want to find out how to control the price offered to buyers c in order to sell in given time $\tau$ the given amount of resource $\Delta N$ and to obtain the maximal price for it. It is clear that the optimal prices offered to the sellers would obey to the same conditions. In both case the capital of the finite-capacity subsystem $M(\tau)$ must be minimal possible.

The problem takes the following form

$$\overline{M} = M(\tau) \to \min_{c(t)}\,. \tag{9}$$

subject to

$$\overline{N} = N(\tau) = N_0 - \Delta N\,. \tag{10}$$

$$\frac{\mathrm{d}M}{\mathrm{d}N} = -c\,. \tag{11}$$

$$\int_0^\tau \mathrm{d}t = \int_{\overline{N}}^{N_0} \frac{\mathrm{d}N}{n[p(N,M),c]} = \tau\,. \tag{12}$$

We can substitute the independent variable d$t$ with d$N$ using the kinetic dependence

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -n(p, c),$$

on the interval (0, $\tau$) with a non-zero flow $n$. In the problem (9) – (12) it is required to find such function $c^*(N)$ that the increment of the economic intermediaries capital is minimal.

The conditions of optimality of this problem are given by the Maximum principle. It is formulated in terms of the problem's optimization Hamiltonian

$$H = -\psi c + \lambda \frac{1}{n[p(N,M),c]}\,.$$

The maximum principle consists of the equations of motion, the equation for the adjoint variable

$$\frac{\mathrm{d}\psi}{\mathrm{d}N} = -\frac{\partial H}{\partial M} = -\lambda \frac{\partial n/\partial p(\partial p/\partial M)}{n^2[p(N,M),c]}\,,\ \psi(\overline{N}) = 0\,, \tag{13}$$

and the condition on maximum on $c$ of the Hamiltonian (which is a convex and differentiable function)

$$\frac{\partial H}{\partial c} = -\psi + \lambda \frac{\partial n / \partial c}{n^2[p(N,M),c]} = 0\,.$$

After eliminating $\psi$ using (13) we obtain the condition of optimality for trading condition of minimal dissipation for resource exchange

$$\frac{d}{dN}\left[\frac{\partial n/\partial c}{n^2(p,c)}\right] = \frac{\partial n/\partial c(\partial p/\partial M)}{n^2[p(N,M),c]} = 0\,. \tag{14}$$

which determines $c(N, M)$ up to the constant. This constant is to be found from the equality (12).

If resource estimate $p$ depends on its stock $N$ only ($\partial p/\partial M = 0$), then the condition (14) becomes simpler

$$\frac{\partial n/\partial c}{n^2(p,c)} = \text{const}\,. \tag{15}$$

Thus for

$$n(p, c) = \alpha(c - p), \tag{16}$$

from (15) it follows that the optimal price for given finite time $\tau$ is

$$c_\tau^*(N,\overline{N}) = p(N) - \frac{\overline{N} - N}{\alpha\tau}\,. \tag{17}$$

and the profit from trading is

$$E_\tau(\overline{N}) = E_\infty(\overline{N}) - \frac{(\overline{N} - N_0)^2}{\alpha\tau}\,. \tag{18}$$

where $E_\infty$ is the capital from trading for infinite long period $\tau \to \infty$ using equilibrium prices $c(N) = p(N)$. The function $E(\tau)$ is shown in Figure 2. For $\tau < \tau^0 = \Delta N^2/(\alpha E_\infty)$ the intermediary is forced to charge the seller less than it charges the buyer. For $\tau < 2\tau^0$ the average rate of profit $e(\tau) = E(\tau)/\tau$ is maximal and equal to

$$e^* = \frac{\alpha}{4}\left[\frac{E_\infty(\overline{N} - N_0)}{\overline{N} - N_0}\right]^2. \tag{19}$$

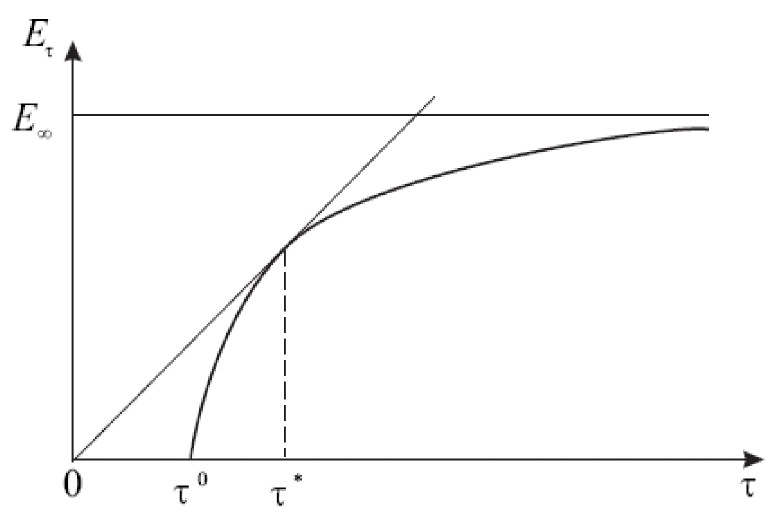


**Figure 2.** Dependence of trading profitability of its duration.

A given rate of trading flow corresponds to each $\tau$. In particular the flow which corresponds to $\tau = \tau^0$ occur when trading is not profitable. For a liner dependence of flow of price estimate differential the rate of exchange flow that maximizes profit is two times higher than the rate of non-profitable, equilibrium trading.

For the above considered example the irreversible losses are

$$\Delta E(\tau, \overline{N}) = \int_0^\tau \alpha [p(N) - c(N)]^2 \, \mathrm{d}t = \frac{(\overline{N} - N_0)^2}{\alpha \tau},$$

thus

$$E(\tau) = E_\infty(\overline{N}) - \Delta E(\tau, \overline{N}) = E_\infty(\overline{N}) - \int_0^\tau n(p,c)(p-c)\mathrm{d}t \,. \tag{20}$$

Equation (20) holds for an arbitrary $n(p, c)$. Indeed after substitution of $\mathrm{d}t$ with $\mathrm{d}N$ the integral in (8) can be rewritten as follows

$$\Delta E(\overline{N}) = E(\overline{N}) - E(N_0) = \int_{N_0}^{\overline{N}} [p(N) - c_\tau(N, \overline{N})]\mathrm{d}N \,.$$

The capital extracted is

$$E(\tau, \overline{N}) = \int_{N_0}^{\overline{N}} c_\tau(N, \overline{N})\mathrm{d}N \,, \; E_\infty(\overline{N}) = \int_{N_0}^{\overline{N}} p(N)\mathrm{d}N \,. \tag{21}$$

Equation (20) follows from comparing these two equations. Thus, the optimal trading processes are minimal dissipation processes and the condition (14) is the condition of minimal dissipation.

## Extracting maximal profit in a system with a few intermediaries

We assume that an intermediary operates in a system that is closed with respect to resource. Intermediary facilitates the exchange between subsystems in order maximize the amount of capital it extracts from the system. The problem here can be decomposed into the problem of optimal trading by an intermediary with a single subsystem. The trading is optimal if the price $c$ and the estimate $p$ obey the conditions of minimal dissipation (14), (15) for any moment when trading takes place. The trading volumes $\Delta N_i$ for each of $m$ subsystems are to be chosen optimally. The following condition holds

$$\sum_{i=1}^{m} \overline{N}_i = \sum_{i=1}^{m} N_{i0} \,. \tag{22}$$

We can view a reservoir as one of subsystems with the estimate $p_-$ that is independent on its stocks of resource and capital. Therefore for any dependence $n(c, p_-)$ the optimal price $c$ for trading on this market must be time independent.

Thus, the problem of extracting maximal capital from a closed microeconomic system in a given time is reduced to a two staged process. During the first stage $m$ problems (9) – (12) about the optimal trading with each of the subsystems with given initial and final resource stocks ($N_{i0}$ and $N_i$) are solved. During the second stage the optimal $N_i$ are found from the condition

$$\sum_{i=1}^{m} E_i(\tau, \overline{N}_i) \Longrightarrow \max_{\overline{N}_i} \,. \tag{23}$$

subject to (22). The optimality conditions for the problem (22) and (23) take the form

$$\frac{\partial E_i(\tau, \overline{N}_i)}{\partial \overline{N}_i} = \Lambda \,, \quad \mathrm{i} = 1, \ldots, m,$$

with $\Lambda$ to be found from (22).

After taking into account (21) we obtain

$$\frac{\partial E_i(\tau,\overline{N}_i)}{\partial \overline{N}_i} = c_{i\tau}(\overline{N}_i,\overline{N}_i) + \int_{N_{i0}}^{\overline{N}_i} \frac{\partial c_{i\tau}(N_i,\overline{N}_i)}{\partial \overline{N}_i} \mathrm{d}N_i = \overline{c}_{i\tau}(\overline{N}_i) \,. \tag{24}$$

The first term in the right hand side is the optimal price at time $\tau$. The second term is the correctional one. It is determined by the averaged sensitivity of the optimal price to the volume of trading. The condition of the optimal choice of trading volumes takes the form

$$\overline{c}_{i\tau}(\overline{N}_i) = \Lambda\,, \quad \mathrm{i} = 1, \ldots, m. \tag{25}$$

## Example

Consider the system where for each subsystem

$$p_i = h_i/N_i. \tag{26}$$

$$n_i(c,p) = \alpha_i(c_i - p_i)\,. \tag{27}$$

Suppose that trading time is not constrained. From (21) and (26) we get

$$E_{i\infty}(\overline{N}_i) = h_i \int_{N_{i0}}^{\overline{N}_i} \frac{\mathrm{d}N_i}{N_i} = h_i \ln\frac{\overline{N}_i}{N_{i0}}\,,$$

with i = 1, …, $m$ in previous equations. For time-constrained exchange after integrating (21) we obtain

$$E_i(\tau,\overline{N}_i) = h_i \ln\frac{\overline{N}_i}{N_{i0}} - \frac{(\overline{N}_i - N_{i0})^2}{\alpha_i \tau}\,. \tag{28}$$

The condition (25) for the optimal choice of $N_i$ takes the form

$$\overline{c}_{i\tau}(\overline{N}_i) = p_i(\overline{N}_i) - 2\frac{\overline{N}_i - N_{i0}}{\alpha_i\tau} = \Lambda\,. \tag{29}$$

The problem becomes much simpler when all subsystems have constant estimates $p$ = const. The condition of optimality (29) then is reduced to the equations

$$p_i - 2\frac{\overline{N}_i - N_{i0}}{\alpha_i\tau} = \Lambda \rightarrow \Delta N_i = \frac{\alpha_i\tau}{2}(p_i - \Lambda)\,. \tag{30}$$

From (22) it follows that $\Lambda$ is equal to the averaged value of resource estimate

$$\Lambda = \frac{\sum_{i=1}^{m}\alpha_i p_i}{\sum_{i=1}^{m}\alpha_i}$$

$$\overline{N}_i^* = \frac{\tau\alpha_i}{2}\left( p_i - \frac{\sum_{\nu=1}^{m}\alpha_\nu p_\nu}{\sum_{\nu=1}^{m}\alpha_\nu} \right) + N_{i0} \tag{31}$$

after substitution of $\overline{N}_i^*$ into (28) we obtain $E_i(\tau, \overline{N}_i^*)$ – the maximal amount of capital that can be extracted from the system in given time $\tau$. The profitability of the system is

$$E_\tau^* = \sum_{i=1}^{m}\left[ p_i(\overline{N}_i^* - N_{i0}) - \frac{(\overline{N}_i^* - N_{i0})^2}{\alpha_i\tau} \right].$$

After taking into account (31) we obtain for estimates that are independent from resource's stocks

$$E_{\tau}^{*} = \frac{\tau}{4}(p_i^2 - \Lambda^2).$$

In most of the cases the estimates are reduced when resource stock increases and profitability is a monotone convex function of trading (purchasing) time.

## CLASSIFICATION OF RESOURCE-EXCHANGE LAWS BY THEIR MINIMAL DISSIPATION CONDITIONS

The law of resource exchange for an economic system is determined by the function $n(x, u)$. The conditions of minimal capital dissipation, (14) and (15), are expressed in terms of this function. In this section we will demonstrate that it is possible to single out classes of exchange kinetics that have similar optimal trading regimes.

### CONDITION CONSTANT DISCOUNT IS OPTIMAL

This problem has the form

$$I = \int_0^{\tau} n(c, p)\mathrm{d}t \to \min_c .$$

subject to constraints

$$\int_0^{\tau} n(c, p)\mathrm{d}t = \Delta N, \qquad n > 0,$$
$$\dot{N} = -n(c, p), \qquad N(0) = N_0.$$

The estimate $p(N)$ here is the given function. The conditions of optimality for this problem have been obtained above.

Let us find $n(p, c)$ for which the optimal trading price $c$ for any moment of time $t$ is equal to the estimate of this resource in the subsystem $P$ up to a constant ($\varphi(p, c) = c - p = \text{const}$). From the above-derived conditions of minimal dissipation it follows that for an irreversible exchange

$$F = \frac{1}{n^2(p,c)}\frac{\partial n}{\partial c} = \text{const.},\ n(c, p) = 0,\ c = p. \tag{32}$$

It was shown that in order for the optimal discount to be constant it is necessary and sufficient that the following equation holds

$$\frac{F_c}{F_p} = \frac{nn_c c - 2n_c^2}{nn_c p - 2n_p n_c} = \frac{\varphi_c}{\varphi_p}. \tag{33}$$

This condition determines the exchange kinetics for each dependence $\varphi$ of the optimal price on resource estimate.

From (33) it follows that constant optimal discount corresponds to such dependencies $n(p, c)$ that

$$n(c, p) = \frac{M(c-p)}{1 + R(p)M(c-p)}. \tag{34}$$

Here $M(c - p)$ and $R(p)$ are arbitrary functions of $p$ and $c - p$, and $M(0) = 0$. We denote this discount as $c - p = \delta$. The expression (34) takes the form $n(c, p) = \mu(\delta)/[1+R(p)\mu(\delta)]$. Because

$$\int_0^{\tau} n(c, p)\mathrm{d}t = \Delta N .$$

$$\mu(\delta) = \frac{\Delta N}{\int_0^\tau \frac{dt}{1 + R[p(t)]\mu(\delta)}}. \qquad (35)$$

This condition determines the optimal discount $\delta$. The irreversibility of the trading is described the integral

$$\Delta E = \int_0^\tau \delta n(c,p)\mathrm{d}t = \delta \Delta N.$$

The average dissipation (average trading costs) is

$$\bar{\sigma} = \frac{\Delta E}{\tau} = \frac{\delta \Delta N}{\tau}. \qquad (36)$$

Equations (35) and (36) determine irreversibility of trading for any function $n(c, p)$ of the form (34).

## CONDITION WHEN OPTIMAL FLOW IS CONSTANT

The condition of optimality (32) is reduced to the condition that the flow $n(p, c)$ on the optimal solution $c^*(p)$ is constant when the left hand side of (32) depends on $n$ only

$$F(p,c) = \varphi[n(p,c)],$$

where $\varphi(\cdot)$ is an arbitrary function, or, which is the same, when $\partial n/\partial c$ is some function of $n$

$$\frac{\partial n}{\partial c} = n_c(p,c^*) = \varsigma[n(p,c^*)],\ \forall p. \qquad (37)$$

The following statement holds: *the optimal trading is the constant flow trading if and only if the resource exchange can be represented in the following form*

$$n(c,p) = (c-p)M(c-p). \qquad (38)$$

Here, $M(\cdot)$ is an arbitrary non-negative function of price differential. The optimal dependence $c^*(p)$ is determined by the condition

$$(c^* - p)M(c^* - p) = n^* = \frac{\Delta N}{\tau}. \qquad (39)$$

## EXAMPLE

We define

$$n(c,p) = \alpha \cdot \mathrm{arctg}(c-p),\ c > p.$$

Since (38) holds for this function, the optimal dependence of the price on time $c^*(t)$ is

$$c^*(t) = p(N^*) + \mathrm{tg}\frac{\Delta N}{\alpha\tau},$$

and the optimal profit

$$N^*(t) = N_0 - \frac{\Delta N}{\tau}t.$$

The optimal exchange flow is constant and equal to $\Delta N/\tau$. Proof: from (37) it follows that

$$\frac{n_{cc}}{n_{cp}} = \frac{n_c}{n_p} \Rightarrow \frac{\partial}{\partial c}\ln\left|\frac{n_c}{n_p}\right| = 0 \Rightarrow \frac{n_c}{n_p} = r(p),$$

where $r(\cdot)$ is an arbitrary function. We get the equation for $n$

$$n_c - r(p)n_p = 0,\quad n(p,c) = 0,\quad p = c. \qquad (40)$$

The characteristics equation is

$$\dot{c} = 1, \; \dot{p} = -r(p).$$

We get

$$c(t) = c_0 + t, \quad \mu(p) = t - t_0, \tag{41}$$

where $\mu(p)$ is an arbitrary differentiable function such that $d\mu/dp = -1/r(p)$. After elimination of $t$ from (41), we get the first integral of the equation (40)

$$\mu(p) - c = t_0 - c_0 = \text{const.},$$

thus the common solution is

$$n(c, p) = M[\mu(p) - c].$$

After taking into account that $n(p, c) = 0$ for $c = p$ we obtain the class of resource exchange laws (38) for which it is optimal to trade using constant exchange flow.

## TRADING WITH A NUMBER OF SUBSYSTEMS WITH PRICE DISCRIMINATION

In previous text we considered the problem when an intermediary was able to offer different prices to different buyers and sellers. In some cases it cannot do that but has to offer the single price to all buyers and another single price to all sellers. Naturally this reduces its profit.

We denote the prices offered to sellers and buyers as $c_1(t)$ and $c_2(t)$ correspondingly. The composition of trading partners for an intermediary is determined by exchange laws. All subsystems at any moment of time $t \in [0, \tau]$ can be divided into three category: sellers (from the intermediary) ($p_i(t) < c_1(t)$); buyers ($p_i(t) > c_2(t)$); and neutral. The intermediary does not contact them because it is not profitable for it ($c_1(t) \le p_i(t) \le c_2(t)$).

This problem becomes much simpler when subsystems are reservoirs (have constant estimates). In this case $c_1$, $c_2$ and $p_i$ are constants that maximise the profit.

The dependence of the exchange flow on $c_1$ can be written as

$$n_+(c_1, p_i) = \sum_{\nu=1}^{j} n_\nu(c_1, p_\nu), \tag{42}$$

where summing is done on all reservoirs with estimates lower than $c_1$. Similarly

$$n_-(c_2, p_i) = \sum_{\nu=1}^{n} n_\nu(p_\nu, c_2). \tag{43}$$

Here $p_i$ is the minimal estimate higher than $c_2$. The rate of profit must be maximal

$$s = [c_2 n_-(c_2, p) - c_1 n_+(c_1, p)] \to \max_{c_1, c_2}, \tag{44}$$

subject to non-accumulation of resource by the intermediary

$$n_+(c_1, p) = n_-(c_2, p) = n. \tag{45}$$

Equation (45) allows us to express $c_1$ and $c_2$ in terms of $n$. Substitution of these dependencies in (44) leads to unconditioned optimisation problem with respect to $n$. Substitution of its solution $n^*$ back into $c_1^*$ and $c_2^*$ determine division of the reservoirs into buyers sellers and neutral non traders.

Let us specify the dependencies $n_\nu$

$$n_\nu = \alpha_\nu (c - p_\nu),$$

and rewrite (45) as two equalities

$$n_{+} = \sum_{\nu=1}^{j} \alpha_{\nu} (c_1 - p_{\nu}) = n ,$$

$$n_{-} = \sum_{\nu=i}^{n} \alpha_{\nu} (p_{\nu} - c_2) = n .$$

After denoting

$$M_1(j) = \sum_{\nu=1}^{j} \alpha_{\nu} p_{\nu}, \quad M_2(i) = \sum_{\nu=i}^{n} \alpha_{\nu} p_{\nu}, \quad A_1(j) = \sum_{\nu=1}^{j} \alpha_{\nu}, \quad A_2(i) = \sum_{\nu=i}^{n} \alpha_{\nu},$$

we obtain

$$c_1(n,j) = \frac{n + M_1(j)}{A_1(j)}, \quad c_2(n,i) = \frac{M_2(i) - n}{A_2(i)} . \tag{46}$$

The objective (44) takes the form

$$s = n[c_2(n,i) - c_1(n,j)] \to \max . \tag{47}$$

For fixed $n$ the values of i and j are to be found from the conditions of maximum of $c_2$ and minimum of $c_1$, respectively.

The condition of minimum of $c_1$ on j yields

$$p_{j+1} > \frac{n + M_1(j)}{A_1(j)} > p_j . \tag{48}$$

Similarly, for maximum of $c_2$ on i we get

$$p_i > \frac{M_2(i) - n}{A_2(i)} > p_{i-1} . \tag{49}$$

The maximum of (47) on n, subjected to (46), (48) and (49) determines the maximal rate of capital extraction in a system with common prices. For a convex $s$ we obtain

$$c_2(n^*, i) - c_1(n^*, j) = n^* \left( \frac{\partial c_1}{\partial n} - \frac{\partial c_2}{\partial n} \right)_{n=n^*} . \tag{50}$$

For two economic agents (j = 1, i = 2) the problem becomes very simple. The optimal prices offered by an intermediary to buyers and sellers at any moment of time $t$ obey the following equations

$$c_1 = \frac{2\alpha_1 p_1 + \alpha_2 (p_1 + p_2)}{2(\alpha_1 + \alpha_2)}, \; c_2 = \frac{2\alpha_2 p_2 + \alpha_1 (p_1 + p_2)}{2(\alpha_1 + \alpha_2)},$$

and the limiting rate of profit extraction is

$$s^*(t) = \frac{\alpha_1 \alpha_2 (p_2 - p_1)^2}{4(\alpha_1 + \alpha_2)} .$$

If resource estimates $p_\nu$ for both economic agents depend on time then the optimal solution $c_1^*(t)$ and $c_2^*(t)$ is determined by these equation for every moment of time $t$.

Trading in a competitive market: Let us consider the system where a number of economic intermediaries compete to trade with a single finite-capacity economic subsystem in a given time (Figure 3). We assume that resource estimate p depends only on its stock $N$ and does not depend on the subsystem's capital. We assume that function $p(N)$ is known.

The expenses

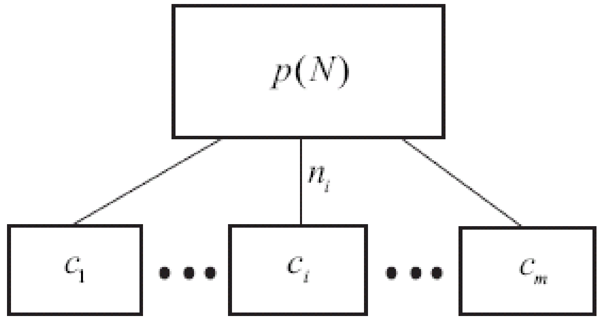


**Figure 3.** The structure of an economic system with a number of economic intermediaries and a finite-capacity subsystem.

$$\Delta S = \sum_{i=1}^{m} \int_0^{\tau} n_i(c_i, p)(c_i - p)dt \to \min_{c_i} . \tag{51}$$

are, for the given trading volume,

$$\int_0^{\tau} n_i(c_i, p)dt = \Delta N_i , \; i = 1, \ldots, \mathrm{m}. \tag{52}$$

The stock of resources is described by

$$\frac{dN}{dt} = -\sum_{i=1}^{m} n_i(c_i, p), \; N(0) = a > \sum_{i=1}^{m} \Delta N_i . \tag{53}$$

We denote

$$b = a - \sum_{i=1}^{m} \Delta N_i$$

and substitute the resource stock $N$ as a new independent variable. The problem then takes the form

$$\int_b^a \frac{\sum_{i=1}^{m} (c_i - p)n_i}{\sum_{i=1}^{m} n_i} dN \to \min_{c_i} , \tag{54}$$

$$\int_b^a \frac{n_i}{\sum_{i=1}^{m} n_i} dN = \Delta N_i , \; \mathrm{i} = 1, \ldots, m, \tag{55}$$

$$\int_b^a \frac{1}{\sum_{i=1}^{m} n_i} dN = \tau . \tag{56}$$

The Lagrange function of the problem (54 – 56) is

$$L = \frac{1}{\sum_{i=1}^{m} n_i \left[ \sum_{i=1}^{m} (c_i - p + \lambda_i)n_i - \zeta \right]} . \tag{57}$$

The necessary conditions of optimality become

$$\frac{\partial L}{\partial c_i} = 0 \Rightarrow -\frac{1}{\left(\sum_{i=1}^{m} n_i\right)^2}\frac{\partial n_i}{\partial c_i}\left[\sum_{i=1}^{m}(c_i - p + \lambda_i)n_i - \zeta\right] + \frac{1}{\sum_{i=1}^{m} n_i}\left[n_i + (c_i - p + \lambda_i)\frac{\partial n_i}{\partial c_i}\right] = 0 .$$

Thus, for all i the following conditions hold

$$\frac{n_i(c_i, p)}{\frac{\partial n_i}{\partial c_i}} + c_i + \lambda_i = p + \frac{\sum_{i=1}^{m}(c_i - p + \lambda_i) - \zeta}{\sum_{i=1}^{m} n_i(c_i, p)}, \; i = 1,\ldots, m, \tag{58}$$

For given dependence $p(N)$ these conditions, jointly with (55) and (56), determine the optimal solution of the problem $c(N)$.

## INTERMEDIARY OPERATING BETWEEN TWO FINITE-CAPACITY SUBSYSTEMS

Until this point we considered trading in a closed economic system. We will now consider trading in an open system in a stationary or cyclic regime.

### Economic reservoirs with constant prices

We consider the system with intermediary which maximizes its profit by trading with two economic reservoirs. It can establish contacts with reservoirs in turn by controlling not only prices it offers but also the timing of contact. Or intermediary can contact both reservoirs simultaneously and trade continuously. We denote the prices (estimates) of two reservoirs as $p_1$ and $p_2$, with $p_1 < p_2$. Exchange kinetics is given by

$$g(\overline{p}, p)(\overline{p} - p). \tag{59}$$

The estimate $p$ here can take to values, $p_1$ and $p_2$, and the prices offered by the intermediary for buying and selling p are the unknowns.

### Maximal rate of profit

The objective of the intermediary is to achieve maximal rate of capital extraction per cycle.

#### *Sequence of buying and selling*

We now consider the case when intermediary buys and sells from each of the reservoirs (markets) in sequence. The rate of profit is

$$N = \frac{1}{T}\int_0^T pg(\overline{p}, p)dt = \overline{pg(\overline{p}, p)} \to \max . \tag{60}$$

The intermediary here sells on the second market everything it buys on the first market,

$$\frac{1}{T}\int_0^T g(\overline{p}, p)dt = \overline{g(\overline{p}, p)} = 0 . \tag{61}$$

This is an averaged nonlinear programming problem with one constraint (61). The Lagrange function of the corresponding non-averaged problem is

$$L = pg(\overline{p}, p) - \lambda g(\overline{p}, p). \tag{62}$$

We denote $L$ as $L^0$ for $p = p^0$. We require that each of $L^0$ attains maximum on $p$, and get

$$\frac{dg(p_v, p)}{dp}(p-\lambda)+g(p_v, p)=0\,,\ v=1,2. \tag{63}$$

These equations determine the basic values $p_v^*(\overline{p}_v, \lambda)$. Their substitution into $L^0$ yields $L_1^*(\overline{p}_1, \lambda)$ and $L_2^*(\overline{p}_2, \lambda)$. The optimal $\lambda^*$ is determined by the condition

$$\max_v L_v^*\left(\overline{p_v}\lambda \to \min_\lambda\right). \tag{64}$$

Thus, the optimal prices for buying and selling are

$$p_1 = p^*\left(\lambda^*, \overline{p_1}\right),\ p_2 = p^*\left(\lambda^* \overline{p_2}\right).$$

Note that $p_1 > \overline{p_1}$ and $p_2 < \overline{p_2}$ .

Suppose that $\alpha$ in (59) depends only on the reservoirs estimate. That is, for $\overline{p} = \overline{p}_1$, $\alpha = \alpha_1$ and for $\overline{p} = \overline{p}_2$, $\alpha = \alpha_2$. Then $L$ takes the form

$$L = \alpha\left(\overline{p}\right)\left(\overline{p}-p\right)\left(p-\lambda\right).$$

The conditions (63) become

$$-\alpha_v\left(p-\lambda\right)+\left(\overline{p_v}-p\right)=0\,,\ v=1,2,$$

and

$$p_v^* = \frac{\overline{p_v}+\lambda}{2},\ v=1,2. \tag{65}$$

Substitution of these $p$ into $L$ gives

$$L_1 = \alpha_1\left(\overline{p_1}-\frac{\overline{p_1}+\lambda}{2}\right)\left(\frac{\overline{p_1}+\lambda}{2}-\lambda\right)=\alpha_1\left(\frac{\overline{p_1}-\lambda}{2}\right)^2,$$
$$L_2 = \alpha_2(\overline{p}_2-\lambda)/2. \tag{66}$$

Since $p_1^* > \overline{p}_1$, $p_2^* < \overline{p}_2$, the function $L_2^*$ decreases when $\lambda$ increases. At the same time $L_1^*$ increases. The minimum on $\lambda$ of the maximum of these two function is attained at the point where $L_1^* = L_2^*$:

$$L_1^*(\lambda)=L_2^*(\lambda)\Rightarrow\sqrt{\alpha_1}\left(\overline{p_1}-\lambda\right)=-\sqrt{\alpha_2}\left(\overline{p_2}-\lambda\right).$$

The functions $L_1^*$ and $L_2^*$ are shown in Figure 4. Their maximum is denoted with bold line. The minimum of $\max_i[L_1^*(\lambda)]$ on $\lambda$ is achieved at $\lambda^*$. Therefore,

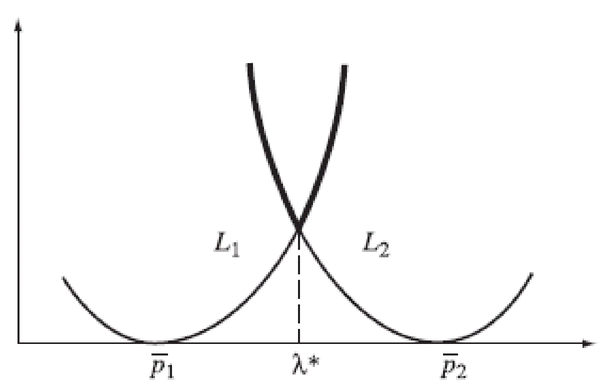


**Figure 4.** Characteristic dependence of maximum on $p_1$ and $p_2$ of the Lagrange function.

$$\lambda^* = \frac{\overline{p_1}\sqrt{\alpha_1} + \overline{p_2}\sqrt{\alpha_2}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}}. \tag{67}$$

Substitution of (65) (67) into (61) gives

$$\gamma_1\alpha_1\left(\overline{p_1} - p_1^*\right) + \gamma_2\alpha_2\left(\overline{p_2} - p_2^*\right) = \frac{\gamma_1\alpha_1}{2}\frac{\sqrt{\alpha_2}\left(\overline{p_1} - \overline{p_2}\right)}{\sqrt{\alpha_1} + \sqrt{\alpha_2}} + \frac{\gamma_2\alpha_2}{2}\frac{\sqrt{\alpha_1}\left(\overline{p_2} - \overline{p_1}\right)}{\sqrt{\alpha_1} + \sqrt{\alpha_2}} = 0.$$

Thus

$$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{\alpha_2}{\alpha_1}},\ \gamma_1 = \frac{\sqrt{\alpha_2}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}},\ \gamma_2 = \frac{\sqrt{\alpha_1}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}}, \tag{68}$$

for $\alpha_1 = \alpha_2$, $\lambda^* = (\bar{p}_1 + \bar{p}_2)/2$. The optimal prices for buying and selling are obtained after substitution of $\lambda^*$ into (65). For $\alpha_1 = \alpha_2$

$$p_1^* = \frac{3\overline{p_1} + \overline{p_2}}{4},\ p_2^* = \frac{3\overline{p_2} + \overline{p_1}}{4}.$$

The rate of profit here is

$$\eta = \frac{p_2^*}{p_1^*} - 1 = \frac{3\overline{p_2} + \overline{p_1}}{3\overline{p_1} + \overline{p_2}} - 1.$$

This is lower than $\eta_0 = \bar{p}_2 / \bar{p}_1 - 1$, for reversible buying and selling at reservoir prices.

The limiting rate of capital extraction is

$$N^* = \frac{\sqrt{\alpha_1\alpha_2}}{4\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)}\left[\sqrt{\alpha_2}\left(\overline{p_2^2} - \lambda^{*2}\right) + \sqrt{\alpha_1}\left(\overline{p_1^2} - \lambda^{*2}\right)\right], \tag{69}$$

with $\lambda^*$ to be found form (67).

One of possible constraints is the average over the cycle flow of capital spent by the intermediary to buy resource. This flow is given by the formula

$$U = p_1 g\left(\overline{p_1}, p_1\right)\gamma_1. \tag{70}$$

For (59)

$$U^* = \alpha_1\frac{\lambda^{*2} - \overline{p_1^2}}{4}\frac{\sqrt{\alpha_2}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}}. \tag{71}$$

If the flow of capital is constrained, eg. $U \leq \cdot U^{\max} < U^*$, then the intermediary has to add the equality (70) into its optimization problem as an additional constraint. Its profit $N$ here will be lower than $N^*$. If $U^{\max} \geq U^*$ then it does not make sense for an intermediary to spend all its capital and it will use only its fraction equal to $U^*$. Maximum of $N$ here corresponds to the maximal rate of profit.

### *Simultaneous buying and selling*

For continuous trading the intermediary has to select the prices it offers to buyers and sellers $p_1$ and $p_2$ in such a way that it rate of profit

$$\overline{N} = p_2 g_2\left(\overline{p_2}, p_2\right) + p_1 g_1\left(\overline{p_1}, p_1\right) \tag{72}$$

is maximal subject to selling everything it buys

$$g_1\left(\overline{p_1}, p_1\right) + g_2\left(\overline{p_2}, p_2\right) = 0. \tag{73}$$

This is a standard nonlinear programming problem. Its solution gives the conditions for optimal prices

$$\frac{g_1\left(\overline{p_1}, p_1\right)}{\dfrac{\partial g_1}{\partial p_1}} + p_1 = \frac{g_2\left(\overline{p_2} p_2\right)}{\dfrac{\partial g_2}{\partial g_2}} + p_2 . \tag{74}$$

If $g_\nu = \alpha_\nu(\overline{p}_\nu - p_\nu)$ ($\nu = 1, 2$), then the condition (74) takes the form

$$2p_1 - \overline{p_1} = 2p_2 - \overline{p_2} .$$

This condition jointly with (73)

$$\alpha_1\left(\overline{p_1} - p_1\right) = -\alpha_2\left(\overline{p_2} - p_2\right),$$

allows us to obtain the optimal buying and selling prices

$$p_1^* = \frac{\alpha_1 \overline{p_1}}{\alpha_1 + \alpha_2} + \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{\overline{p_2} + \overline{p_1}}{2} , \tag{75}$$

$$p_2^* = \frac{\alpha_2 \overline{p_2}}{\alpha_1 + \alpha_2} + \frac{\alpha_1}{\alpha_1 + \alpha_2} \frac{\overline{p_2} + \overline{p_1}}{2} . \tag{76}$$

The rate of profit is

$$\overline{N^*} = \frac{\alpha_1 \alpha_2 \left(\overline{p_2} - \overline{p_1}\right)^2}{4(\alpha_1 + \alpha_2)} . \tag{77}$$

It is easy to see that if $\alpha_\nu$ are equal then $\overline{N}^*$ is two times higher than $N^*$ in (69). This is natural because the exchange flows are the same and selling takes place during half of the cycle. If $\alpha_1 \neq \alpha_2$ then $\overline{N}^* / N^* < 2$.

### *Vector exchange*

When intermediary contacts reservoirs in turn and there is no constraint of the rate of spending the problem (60), (61) takes the form

$$N = \sum_{i=1}^{n} \overline{p_i g_i\left(\overline{p}, p_i\right)} \to \max_{p,\overline{p}} , \quad \overline{g_i\left(\overline{p}, p\right)} = 0 , \text{ i} = 1, \ldots, \text{n}.$$

This is an average nonlinear programming problem with $n$ constraints. The number of its basic solutions does not exceed $n + 1$. In order to solve it we first solve an auxiliary problem

$$L = \sum_{i=1}^{n} \left[g_i\left(\overline{p}, p\right)\left(p_i - \lambda_i\right)\right] \to \max_{p,\overline{p}} \min_{\lambda} , \quad \overline{p} = \left(\overline{p_1}, \overline{p_2}\right).$$

This problem becomes much simpler if $L$ turns out to be strictly convex on $p$. The prices offered by the intermediary here can be found by solving equations

$$\sum_{i=1}^{n} \frac{\partial g_i}{\partial p_\nu}\left(p_i - \lambda_i\right) + g_\nu\left(\overline{p_j}, p\right) = 0 , \quad \text{j} = 1, 2, \quad \nu = 1, \ldots, n.$$

If $g_i = g_i(\overline{p}_i, p_i)$, the problem with vector resource can be decomposed into $n$ independent sub-problems similarly to (60) and (61). All the results derived above hold. Note that (67) holds here for every $\lambda_i$, and the expression (69) for $N^*$ contains sum on i. Each term in this sum is positive. The profit rate can be found here using the formula

$$\eta = \frac{N^*}{\gamma_1 \sum_{i=1}^{n} g_i\left(\overline{p_{i1}}, p_i\right) p_i}.$$

The same is true for the problem where intermediary contacts reservoirs simultaneously.

The characteristic features of vector exchange become important when the rate of capital spending is constrained below the value which corresponds to $N^*$. This case is considered in the next paragraph.

## Intermediary operating between two finite-capacity economic subsystems

In intermediary trades with finite-capacity subsystems instead of markets-reservoirs then the prices it offers must depend on time. The problem of obtaining the maximal profit here in a given time is decomposed into three problems: the problem of optimal trading (optimal buying and optimal selling) with a single subsystem, and the problem of adjusting of the optimal buying and optimal selling by selecting optimally their common parameters.

It is important here that buying and selling here obeys the condition of minimal capital dissipation.

We consider a cycle where resource is first bought by the intermediary and is then sold by it. We denote the capacities of two subsystems with which the intermediary trades as $C_1$ and $C_2$. These parameters link the stock of resource with its estimate by the subsystem $p_i(t)$ (i = 1, 2). The estimates are defined as $(dp_i/dt)C_i = - dN_i/dt$. The price of the intermediary is denoted as $p(t)$. Furthermore, $\tau_i$ (i = 1, 2) are the durations of contact between intermediary and two sub-systems. It is required here to find such $\tau_1$ and $\tau_2$ that the total duration of the trading cycle was equal to the given $\tau$. The other control variable here is the volume of trading $\Delta N$. Intermediary maximises its profit per cycle

$$I = \int_0^{\tau_2} p(t) g(p_2, p) dt - \int_0^{\tau_1} p(t) g_1(p, p_1) dt \to \max. \tag{78}$$

subject to constraint

$$\tau_1 + \tau_2 = \tau. \tag{79}$$

$$\frac{dp_1}{dt} = \frac{g_1(p, p_1)}{C_1}, \ \frac{dp_2}{dt} = -\frac{g_2(p_2, p)}{C_2}, \ p_i(0) = \overline{p_i}, \tag{80}$$

$$\int_0^{\tau_1} g_1(p, p_1) dt = \int_0^{\tau_2} g_2(p_2, p) dt = \Delta N. \tag{81}$$

Suppose the exchange kinetics is linear

$$\begin{aligned} g_2(p_2, p) &= \alpha_2(p_2 - p), \\ g_1(p, p_1) &= \alpha_1(p - p_1) \end{aligned}. \tag{82}$$

Here from the conditions of minimal dissipation it follows that for each of the half-cycles it is optimal to maintain the constant flow of resource. As the result the equilibrium prices for buying and selling are

$$p_1^*(t) = \overline{p_1} + \frac{\Delta N}{\tau_1 C_1} t, \ t \in [0, \tau_1], \tag{83}$$

$$p_2^*(t) = \overline{p_2} - \frac{\Delta N}{\tau_2 C_2} t, \ t \in [0, \tau_2]. \tag{84}$$

The price offered exceeds $p_1^*(t)$ by $\delta_1 = \Delta N/(\tau_1 \alpha_1)$ and is $\delta_2 = \Delta N/(\tau_2 \alpha_2)$ below $p_2^*(t)$. Thus, the prices offered by the intermediary for buying and selling are

$$p^*(t) = \overline{p_1} + \frac{\Delta N}{\tau_1}\left(\frac{t}{C_1} - \frac{1}{\alpha_1}\right),$$
$$p^*(t) = \overline{p_2} - \frac{\Delta N}{\tau_2}\left(\frac{t}{C_2} + \frac{1}{\alpha_2}\right). \tag{85}$$

Substitution of the dependencies (85) into the objective (78) allows calculating the dependence of profit on $\tau_1$, $\tau_2$ and $\Delta N$, and to find its maximum on these variables subject to (79). We get

$$I = \Delta N\left(\overline{p_2} - \overline{p_1}\right) - \Delta N^2\left(\frac{1}{\tau_1\alpha_1} + \frac{1}{\tau_2\alpha_2} + \frac{1}{2C_1} + \frac{1}{2C_2}\right). \tag{86}$$

The first term corresponds to the profit from equilibrium exchange. The second term describes losses due to finite time and capacities.

Quantities $\tau_1$ and $\tau_2$ are found by solving the problem

$$\left(\frac{1}{\tau_1\alpha_1} + \frac{1}{\tau_2\alpha_2}\right) \to \min / \tau_1 + \tau_2 = \tau\,. \tag{87}$$

Its solution can be reduced to the condition

$$\tau_1^* = \tau\frac{\sqrt{\alpha_2}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}}\,,\ \tau_2^* = \tau\frac{\sqrt{\alpha_1}}{\sqrt{\alpha_1} + \sqrt{\alpha_2}}\,,$$

which gives

$$\frac{1}{\alpha_1\tau_1^*} + \frac{1}{\alpha_2\tau_2^*} = \frac{\sqrt{\alpha_1} + \sqrt{\alpha_2}}{\tau}\left(\frac{1}{\alpha_1\sqrt{\alpha_2}} + \frac{1}{\alpha_2\sqrt{\alpha_1}}\right). \tag{88}$$

The optimal trading volumes $\Delta N$ is found by maximizing $I$ on $\Delta N$ subject to (88)

$$\Delta N^* = \frac{\left(\overline{p_2} - \overline{p_1}\right)\tau}{\tau\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + 2\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)\left(\frac{1}{\alpha_1\sqrt{\alpha_2}} + \frac{1}{\alpha_2\sqrt{\alpha_1}}\right)}. \tag{89}$$

The maximal profit is

$$I^* = \frac{\left(\overline{p_2} - \overline{p_1}\right)^2\tau}{4\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)\left(\frac{1}{\alpha_1\sqrt{\alpha_2}} + \frac{1}{\alpha_2\sqrt{\alpha_1}}\right) + 2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\tau}. \tag{90}$$

The longer is the cycle the lower is the rate of profit $N^* = I^*/\tau$.

For simultaneous buying and selling $\tau_1$ and $\tau_2$ should be chosen in such a way that the following condition holds for any $\theta \in [0, \tau]$

$$\int_0^\theta g_1(p_1, p_1)dt \geq \int_0^\theta g_2(p_2, p_2)dt\,. \tag{91}$$

When $\tau_1$ and $\tau_2$ increase, the flow of resource in the optimal process decreases. Since the total amount of traded resource $\Delta N$ during its buying and selling is the same, as a rule $\tau_1 \leq \tau_2$. In particular, for linear exchange kinetics, (91) becomes an equality $\tau_1 = \tau_2 = \tau$. The prices offered to sellers and buyers obey (85), and the profit is given by (86) and (85) after substituting $\tau_1$ and $\tau_2$ with $\tau$. The optimal profit is

$$\Delta N^* = \frac{\overline{p_2} - \overline{p_1}}{\frac{2}{\tau}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right) + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)}, \tag{92}$$

$$I^* = \frac{\left(\overline{p_2} - \overline{p_1}\right)^2}{\frac{4}{\tau}\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right) + 2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}. \tag{93}$$

## Limiting rate of profit

If profit $M$ is below $I^*$, then the intermediary may want to achieve this profit using minimal amount of its capital. The problem becomes

$$I_1 = \int_0^{\tau_1} pg(p_1, p)dt \to \min,$$
$$\int_0^{\tau_1} p_2 g(p_2, p)dt - \int_{\tau_2}^{0} pg\left(p_1^p\right)dt = M. \tag{94}$$

This problem is very similar to the previous problem. The same conditions of minimal dissipation must hold here for half-cycles of buying and selling.

The problem of adjusting these half-cycles takes the form

$$I_1^*(\tau_1, \Delta N) \to \min_{\Delta N, \tau_1, \tau_2}, / \, I_2^*(\tau_1, \Delta N) - I_1^*(\tau_2, \Delta N) = M\ . \tag{95}$$

This gives the same expressions for the optimal $\tau_1^*$ and $\tau_2^*$ as above in the problem (87). The optimal volume of buying is

$$\Delta N^*(\lambda) = \frac{\lambda\left(\overline{p_1} - \overline{p_2}\right) - \overline{p_1}}{\lambda\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{2}{\alpha_1 \tau_1^*} + \frac{2}{\alpha_2 \tau_2^*}\right) + \left(\frac{1}{C_1} + \frac{2}{\alpha_2 \tau_2^*}\right)}.$$

Substitution of $\Delta N^*(\lambda)$ into (95) gives equation for $\lambda^*$.

### *Constraints on the rate of spending*

If intermediary is constraint on how much it can spend buying his stock then it is optimal for it to maximize its profit N subject to the given expenses. Since the profit rate is defined as

$$\eta = \frac{N}{U},$$

the problem is equivalent to maximization of $\eta$ subject to fixed $U$ or $N$.

Consider a cycle when intermediary-monopolist contacts in turns with two markets and the exchange kinetic is given by (82). The problem of optimal trading here takes the form

$$N = \left[\alpha_2 \gamma_2 \left(\overline{p_2} - p_2\right)p_2 - \alpha_1 \gamma_1 \left(p_1 \overline{p_1}\right)p_1\right] \to \max, \tag{96}$$

with constrained expenses

$$U = \alpha_1 \gamma_1 \left(p_1 - \overline{p_1}\right)p_1 \le U^{\max} \tag{97}$$

and non-accumulation of the resource by the intermediary

$$\alpha_1 \gamma_1 \left(p_1 - \overline{p_1}\right) = \alpha_2 \gamma_2 \left(\overline{p_2} - p_2\right), \tag{98}$$

with $\gamma_1 + \gamma_2 = 1$, $\gamma_i \ge 0$, i = 1, 2.

The condition (97) can be rewritten in the following form

$$\alpha_1\left(p_1-\overline{p_1}\right)^2\left(2p_2-\overline{p_2}\right)=\alpha_2\left(\overline{p_2}-p_2\right)^2\left(2p_1-\overline{p_1}\right). \tag{99}$$

This condition jointly with (97), (98) determines $p_1$, $p_2$, $\gamma_1$ and $\gamma_2$.

If buying and selling occur simultaneously then the cost constraint uniquely determines $p_1$ and $p_2$ and the optimal profit $N$. For (82) we get

$$p_1=0{,}5\overline{p_1}+\sqrt{0{,}25\overline{p_1}^2+\frac{U^{\max}}{\alpha_1}},\ p_2=\overline{p_2}-\frac{U^{\max}}{p_1\alpha_2}, \tag{100}$$

$$N=U^{\max}\frac{p_2}{p_1}=U^{\max}\left(\frac{\overline{p_2}}{p_1}-\frac{U^{\max}}{p_1^2\alpha_2}\right). \tag{101}$$

*Optimal choice of trading composition*

If an intermediary can trade in a number or resources (vector exchange) then it can optimise its performance by controlling both prices offered and composition of its trading

$$N=\sum_{i=1}^{n}N_i\left(U_i\right)\to\max \Big/ \sum_i U_i=U^{\max}, U_i\geq 0. \tag{102}$$

Here $U_i$ are the expenses from purchasing i-th resource. The dependence of $N_i$ on $U_i$ is determined by (100) and (101) after substitution of $U_{max}$ into them.

If $N_i(U_i)$ are strictly convex functions and their derivatives at the coordinate origin are infinitely large, the optimal $U_i$ are positive and obey equations

$$\frac{dN_i}{dU_i}=\lambda,\ i=1,\ \dots,\ n.$$

After taking into account (100) and (101) they take the form

$$\frac{\overline{p}_{2i}}{p_{1i}}-\frac{2U_i}{p_{1i}^2\alpha_{2i}}+\frac{U_i}{2\alpha_{1i}}\left(\frac{2U_i}{\alpha_{2i}p_{1i}^3}-\frac{\overline{p}_{2i}}{p_{1i}^2}\right)\frac{1}{\sqrt{0{,}25\overline{p}_{i1}^2+U_i/\alpha_{1i}}}=\lambda, \tag{103}$$

with $i=1,\dots n$, $p_{1i}=0{,}5\overline{p}_{1i}+\sqrt{0{,}25\overline{p}_{1i}^2+U_i/\alpha_{1i}}$ and $\sum_{i=1}^{n}U_i=U^{\max}$ .

Solution of the system (103) determines the optimal composition of the resources for buying and their optimal prices.

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# OPTIMALNI PROCESI U IREVERZIBILNOJ MIKROEKONOMIJI

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## SAŽETAK

U radu se razmatra proces optimalnog trgovanja u ekonomskim sustavima. Pristup se temelji na uzimanju u obzir faktora ireverzibilnosti putem koncepta funkcije bogatstva.

## KLJUČNE RIJEČI

mikroekonomija, neravnotežna termodinamika, ireverzibilnost

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# OPTIMAL SEPARATION SEQUENCE FOR THREE-COMPONENT SYSTEMS

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## SUMMARY

In this paper the problem of finding the optimal separation sequence for a three-component mixture in a two-stage separation system is considered. Two solutions are obtained. The first minimizes the energy used, subject to a given flow rate of the input mixture, by selecting optimal separation sequence and by distributing the contact surfaces between the first and second stages optimally. It is also shown that the input flow rate of a heat-driven two-stage separation system is bounded and that this bound (the maximal possible rate of heat-driven separation) depends on the separation sequence used. The closed-form expression for this dependence is obtained.

## KEY WORDS

separation, three-component systems, zeotropic mixture

## CLASSIFICATION

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## INTRODUCTION AND PROBLEM FORMULATION

Separation processes consume large amounts of energy. The lower bound on the amount of energy required is given by the reversible minimal work of separation $A^0$ [1]. It depends on the amount of mixture, its composition and composition of the output mixtures and is equal to the increment of the mixture's free energy. For mixtures that can be considered as nearly ideal gases or nearly ideal solutions the free energy (Gibbs energy) of one mole of the i-th component in the j-th flow is equal to its chemical potential

$$\mu_{\mathrm{ij}}(T, P_{\mathrm{j}}, x_{\mathrm{ij}}) = \mu_{\mathrm{i}}^0(T, P_{\mathrm{j}}) + RT \ln x_{\mathrm{ij}} . \tag{1}$$

Here $R$ and $T$ are the universal gas constant and mixture's absolute temperature; $x_{\mathrm{ij}}$ is the concentration of the i-th component in the j-th flow measured in molar fractions; $\mu_{\mathrm{i}}^0(T, P_{\mathrm{j}})$ is the chemical potential of the pure i-th component (known for most of substances) and $P_{\mathrm{j}}$ is the pressure in the j-th flow.

The reversible work of separation for $N_0$ moles of $k$ component mixture with concentration $x_{\mathrm{i}0}$ (i = 1, ... ,k) (if the temperature and pressure of the mixture are equal to the temperatures and pressures of the $m$ output flows of the separation system) is [2]

$$A^0 = RN_0T\left[\sum_{j=1}^{m}\gamma_j \sum_{i=1}^{k} x_{ji} \ln x_{ji} - \sum_{i=1}^{k} x_{i0} \ln x_{i0}\right], \tag{2}$$

where $\gamma_{\mathrm{j}}$ is the fraction of the input mixture that is separated into j-th flow with vector of concentrations $x_{\mathrm{j}} = (x_{\mathrm{j}1}, \ldots , x_{\mathrm{jk}})$. In the particular case when the input mixture is separated into pure components the number of components is equal to the number of flows $m = k$, and the fraction of the j-th flow is equal to the concentration of the corresponding component in the input mixture: $\gamma_{\mathrm{j}} = x_{\mathrm{j}0,}\, x_{\mathrm{jj}} = 1$ and $x_{\mathrm{ji}} = 0$ for $\mathrm{i} \neq \mathrm{j}$.

Therefore the first term in the square brackets in (2) is equal zero and

$$A^0 = -RN_0T\sum_{i=1}^{k} x_{i0} \ln x_{i0} . \tag{3}$$

If instead of the amount the molar rate of the input mixture $g_0$ is given together with output rates $g_{\mathrm{j}} = \gamma_{\mathrm{j}} g_0$, then the same formulas can be used to derive the power of reversible separation for an incomplete separation

$$p^0 = -Rg_0T\left[\sum_{j=1}^{m}\gamma_j \sum_{i=1}^{k} x_{ji} \ln x_{ji} - \sum_{i=1}^{k} x_{i0} \ln x_{i0}\right]. \tag{4}$$

and for the complete separation into pure substances

$$p^0 = -Rg_0T\sum_{i=1}^{k} x_{i0} \ln x_{i0} . \tag{5}$$

The reversible estimates (4), (5) are realised if the rate $g_0$ is infinitely close to zero or if the heat and mass transfer coefficients are infinitely large, that is, if the size of separation apparatus is infinitely large. These estimates are proportional to the rate $g_0$, depend on the compositions of the input and output flows only and do not depend on the separation sequence. Therefore these estimates do not allow us to compare different separation sequences and to choose the best sequence.

Real processes occur in finite-sized apparatus with finite rate. Irreversible losses, which increase the power required for separation, play an important role here. These losses depend on the exchange kinetics and on input/output flows' compositions and rates. These losses depend nonlinearly on the rate of the input mixture $g_0$. Irreversible losses are different for separation sequences. They allow us to compare different variants with each other. Irreversible losses for separation using mechanical power are different to irreversible losses for heat-driven separation. For separation that uses mechanical energy (membrane separation, centrifuging, short-cycle absorption etc.) most of irreversibility is due to mass-transfer kinetics. For heat-driven separation that uses heat energy (distillation, boiling, drying, absorption-desorption cycles where the working solution changes temperature) irreversible losses are due to both mass transfer and heat-exchange accompanying transformation of heat energy into the work of separation.

When the number of component increases, the number of possible separation sequences increases dramatically. The problem of optimal separation sequence attracted substantial interest (see review [3]). It can only be solved if dependence of irreversible energy losses at each stage of separation for all process design as function of its rate and sizes are known.

We propose a much simpler approach in this paper. It provides coarser estimates and relies on mass and heat transfer coefficients only. Nevertheless, it allows one to find the ways to improve separation efficiency by distributing optimally contact surfaces between stages.

Estimates for the minimal power of separation in mechanical systems and for minimal amount of heat required in heat-driven separation systems with given rate were obtained in [4 – 7] under the following assumptions:

1. temperatures of the input molar flow $g_0$ and output molar flows $g_j$ (j = 1, ... ,m) are equal to the same temperatures $T$,
2. mass transfer flows depend linearly on the chemical potentials' difference. For the i-th substance that is transferred from the flow $g_0$ to the flow $g_j$,
$$g_{ij} = \alpha_{ij}\Delta\mu_{ij}\,, \text{ (i = 1, …, k and j = 1, …, m).} \tag{6}$$
Here $\alpha_{ij}$ is the effective (that takes into account the area of contact surface) mass transfer coefficient for transfer of the i-th component into j-th flow, $\Delta\mu_{ij}$ is the difference of chemical potentials for i-th component in the input mixture and in j–th flow (the driving force of mass transfer)
$$\Delta\mu_{ij} = \mu_{i0}(T, P_0, x_{i0}) - \mu(T, P_j, x_{ij})\,, \tag{7}$$
3. the laws of mass transfer in heat-driven separation systems are linear
$$q = \alpha\Delta T\,, \tag{8}$$
where $\alpha$ is the heat-transfer coefficient for the whole heat-exchange surface, $\Delta T$ is the temperature difference between the working body and the mixture which is being heated.

The flows structure for mechanical and heat-driven separation systems m = 2 are shown in Figure 1.

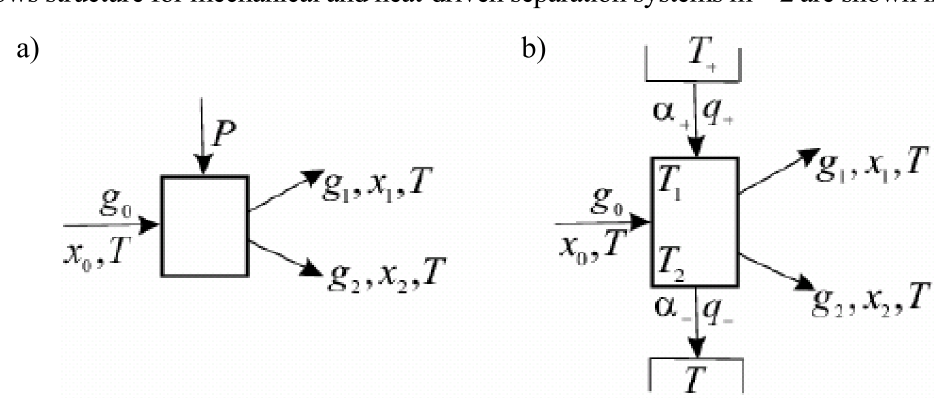


**Figure 1.** Flows' structures in a) mechanical and b) heat-driven separation systems.

Under these assumptions it was shown [4 – 7] that the power required for separation in mechanical system can't be lower than

$$p_1 = p^0 + g_0^2 \sum_{j=1}^{m} \gamma_j^2 \sum_{i=1}^{k} \frac{x_{ij}^2}{\alpha_{ij}} = p^0 + \Delta p , \tag{9}$$

for separation into flows with compositions $x_j$ and

$$p_2 = p_0^0 + g_0^2 \sum_{i=1}^{k} \frac{x_{i0}^2}{\alpha_i} = p^0 + \Delta p . \tag{10}$$

for separation into pure components. In (10), $p_0$ and $p_0^0$ corresponds to (4) and (5), respectively. The first term is proportional to the feed rate $g_0$ of the mixture that is separated, and the second term, due to process' irreversibility, is proportional to the $g_0^2$.

For heat-driven separation the heat flow $q_+$, removed from the hot reservoir with the temperature $T_+$, cannot be lower than

$$q_v^+ = \frac{p_v}{\eta(p_v, \overline{\alpha})} . \tag{11}$$

Here $v = 1$ corresponds to separation into flows with given compositions, and $v = 2$ corresponds to separation into pure components, while $\eta(\cdot,\cdot)$ is the maximal efficiency of irreversible transformation of heat into work with power $p$ [5, 6]

$$\eta(p, \overline{\alpha}) = \frac{1}{2}\left[ \frac{p}{\overline{\alpha} T_+} + \eta_k + \sqrt{\left( \frac{p}{\overline{\alpha} T_+} + \eta_k \right)^2 - \frac{4p}{\overline{\alpha} T_+}} \right]. \tag{12}$$

In heat-driven separation systems heat is supplied from the hot reservoir with the temperature $T_+$ to the mixture that is separated. Heat is also removed from the mixture into cold reservoir with the temperature $T$. In absorption-desorption cycle these are the temperatures in desorber and absorber, and in distillation – the temperatures in boiler and condenser. The effective heat transfer coefficient $\alpha$ is expressed in terms of the heat transfer coefficient from the hot reservoir $\alpha_+$ and from the cold one $\alpha_-$ as follows

$$\overline{\alpha} = \frac{\alpha_+ \alpha_-}{\alpha_+ + \alpha_-}, \ \eta_k = 1 - \frac{T}{T_+} . \tag{13}$$

The formulas (9 – 13) are derived from thermodynamic balances (mass, energy and entropy balances) for separation systems. The latter balance includes entropy production, which depends on the mass and heat transfer kinetics. In [4 – 7] the minimal possible entropy production subject to given heat and mass transfer coefficients, given flow rates and mass and energy balances was derived. This result led to finding the minimal extra energy needed. It turned out that the rate of heat-driven separation systems $g_0$ is bounded since increase of the heat flow $q_+$ above some threshold $\overline{q}_+$ reduces the maximal rate of heat-driven separation system. The maximal rate is [5, 8]

$$g_0^{\max} = \frac{-B + \sqrt{B^2 + 4\overline{\alpha} D (T_+^{1/2} - T^{1/2})^2}}{2D} , \tag{14}$$

where $\overline{\alpha}$ is given by (13), and

$$B = \frac{p^0}{g_0}, \quad D = \frac{\Delta p}{g_0^2} . \tag{15}$$

Here $p^0$ and $\Delta p$ are defined by expressions (4) and (9) for incomplete, and by (5) and (10) for complete separations, respectively. Further in the text we will consider the problem of choosing separation sequence for mixture of three-component mixture. The input flow $g_0$ is described by concentrations $x_{10}$, $x_{20}$ and $x_{30}$,

$$\sum_{i=1}^{3} x_{i0} = 1. \tag{16}$$

We assume that separation is carried out in two stages. During the first stage one component is separated out. The residual binary mixture is separated at the second stage. For simplicity we assume that output flows consist of pure substances (Figure 2) and the compositions of the input and output flows are fixed.

This assumption means that we consider zeotropic mixtures only.

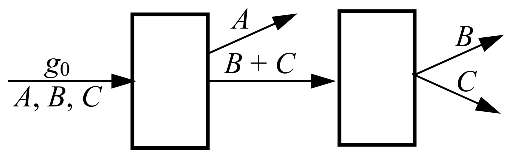


**Figure 2.** Schema of two-stage separation system for three-component mixture.

We would like to find out

1. what component should be separated out first in order to minimise the power needed for separation,
2. what component should be separated first in order to maximise the rate of separation,
3. how to distribute optimally heat and mass exchange surfaces between separation stages in order to minimize power or maximize the rate of separation.

## MECHANICAL SEPARATION SYSTEM

In mechanical separation systems the minimum of the power required corresponds to the minimum of the irreversible losses. Separation is based on the differences between the properties of mixture's components (sizes of molecules, density boiling temperature, etc.).

These differences lead to different interactions between different components of the mixture and membrane or absorber, different rate at which components are transferred from liquid into gaseous phase, etc. We assume that the property used for separation can be measured quantitatively. It is also assumed here that the difference between components' properties used for separation does not depend on the composition of the mixture, which excludes separation of azeotropic mixtures.

We order components in such a way that the difference (in term of this property) between the third and the first component was maximal. Now we can compare two separation sequences only. The first sequence is when the first component is separated first. The second sequence is when the third component is separated first. For the first sequence with the unit rate ($g_0 = 1$) when the first component with concentration $x_{10}$ is separated during first stage, the irreversible losses in accordance with (10) are

$$\Delta p_1 = \frac{x_{10}^2}{\alpha_{11}} + (1 - x_{10})^2 \left[ \frac{1}{\alpha_{11}} + \frac{x_{20}^2 + x_{30}^2}{\alpha_{23}} \right], \tag{17}$$

Here $\alpha_{11}$ and $\alpha_{23}$ are the effective mass transfer coefficients for the first and second stages of separation where the first component is separated out first.

Similarly, when the third component is separated out first we get

$$\Delta p_3 = \frac{x_{30}^2}{\alpha_{13}} + (1 - x_{30})^2 \left[ \frac{1}{\alpha_{13}} + \frac{x_{20}^2 + x_{10}^2}{\alpha_{21}} \right], \tag{18}$$

where $\alpha_{13}$ and $\alpha_{21}$ are the effective mass transfer coefficients for the first stage when third component is separated.

Mass transfer coefficients are proportional to the contact area $S_\nu$ ($\nu = 1, 2$), for apparatus used at the $\nu$ -th stage of separation, and the specific (per unit contact area) mass transfer coefficient $\delta$. This coefficient depends on the properties of the second and first ($\delta_1$) and second and third ($\delta_3$) components. Thus

$$\alpha_{11} = S_1\delta_1,\ \alpha_{13} = S_1\delta_3,\ \alpha_{23} = S_2\delta_3,\ \alpha_{21} = S_2\delta_1. \tag{19}$$

Let us rewrite $\Delta p_2$ and $\Delta p_2$ as follows

$$\Delta p_1 = \frac{K_{11}}{S_1} + \frac{K_{23}}{S_2}, \tag{20}$$

$$\Delta p_2 = \frac{K_{13}}{S_1} + \frac{K_{21}}{S_2}. \tag{21}$$

Here

$$K_{1i}(x_{i0}, \delta_i) = \frac{x_{i0}^2 + (1 - x_{i0})^2}{\delta_i}, \quad i = 1, 3, \tag{22}$$

$$K_{23}(x_{10}, x_{30}, \delta_3) = \frac{(1 - x_{10})^2[x_{30}^2 + (1 - x_{10} - x_{30})^2]}{\delta_3}, \tag{23}$$

$$K_{21}(x_{10}, x_{30}, \delta_1) = \frac{(1 - x_{30})^2[x_{10}^2 + (1 - x_{10} - x_{30})^2]}{\delta_1}. \tag{24}$$

These expressions represent irreversible power losses for one unit contact area and one-unit feed rate of the input mixture.

Suppose that the total contact area is given as $S = S_1 + S_2$ and it is required to distribute it between stages to minimise power needed for separation. The problem of finding the optimal $S_1$ and $S_2$ takes the form

$$[\Delta p_1(S_1, S_2) + \Delta p_2(S_1, S_2)] \Rightarrow \min,$$

subject to $S_1 + S_2 = S$. Since $\Delta p_i$ is convex on $S_1$, $S_2$ the solution of this problem is unique and is determined by the conditions

$$\frac{K_{11}}{S_1^2} = \frac{K_{23}}{S_2^2},$$

for the first sequence and

$$\frac{K_{13}}{S_1^2} = \frac{K_{21}}{S_2^2},$$

for the second one. The optimal distribution of contact area is given by the equalities

$$S_1^* = S\frac{\sqrt{K_{11}}}{\sqrt{K_{11}} + \sqrt{K_{23}}},\ S_2^* = S\frac{\sqrt{K_{23}}}{\sqrt{K_{11}} + \sqrt{K_{23}}}, \tag{25}$$

for the first and

$$S_1^* = S\frac{\sqrt{K_{13}}}{\sqrt{K_{13}} + \sqrt{K_{21}}},\ S_2^* = S\frac{\sqrt{K_{21}}}{\sqrt{K_{13}} + \sqrt{K_{21}}},, \tag{26}$$

for the second separation sequence.

Substitution of this optimal area distribution into (20) and (21) yields the following expressions for the irreversible power of separation

$$\Delta p_1 = \frac{g_0^2}{S}\left(\sqrt{K_{11}} + \sqrt{K_{23}}\right)^2, \tag{27}$$

$$\Delta p_2 = \frac{g_0^2}{S}\left(\sqrt{K_{13}} + \sqrt{K_{21}}\right)^2, \tag{28}$$

The condition when this power is lower for the first sequence takes the form

$$\left(\frac{\Delta p_1}{\Delta p_2}\right)^{1/2} = \frac{\sqrt{K_{11}(x_{10},\delta_1)} + \sqrt{K_{23}(x_{10},x_{30},\delta_3)}}{\sqrt{K_{13}(x_{30},\delta_3)} + \sqrt{K_{21}(x_{10},x_{30},\delta_1)}} < 1. \tag{29}$$

If left-hand side of this inequality is higher than 1 then it is energetically more efficient to separate the third component first.

After substitution of (22 – 24) into (29) we get

$$\frac{\sqrt{x_{10}^2 + (1-x_{10}^2)} - (1-x_{30})\sqrt{x_{10}^2 + x_{20}^2}}{\sqrt{x_{30}^2 + (1-x_{30}^2)} - (1-x_{10})\sqrt{x_{30}^2 + x_{20}^2}} < \frac{\delta_1}{\delta_3} = K, \tag{30}$$

where $x_{20} = x_{10} - x_{30}$. The dashed areas in Figure 3 show where first separation sequence is most energy efficient for $K = 0{,}5$, $K = 1$ and $K = 2$.

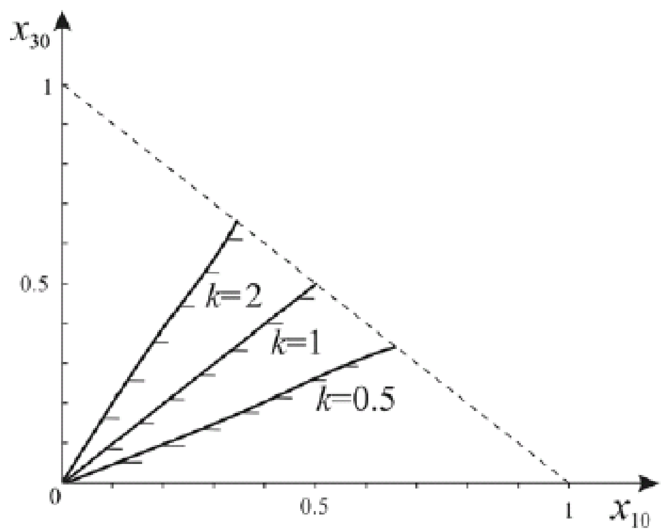


**Figure 3.** Boundaries between two areas where first and second separation sequences are optimal for different *K*. Dashed area corresponds to optimal sequence with separation of first component first.

From the symmetry of the left-hand side of (30) it is clear that the boundary that corresponds to $K = 1$ is a straight line. Calculations show that for different values of $K$ these boundaries are very close (with about 3 % error) to straight lines $x_{10} = x_{30}/K$. It allows us to write down the approximate condition which determines when separation of first component at first stage is optimal as

$$x_{10}\sqrt{\delta_1} \geq x_{30}\sqrt{\delta_3}\ . \tag{31}$$

In particular, if concentrations of components one and two are the same $x_{10} = x_{20} = x_0 < 0{,}5$ then separation of the first component at the first stage is optimal if $\delta_1 > \delta_3$; if $\delta_1$ and $\delta_3$ are close to each other then it is optimal to separate the first component first if $x_{10} > x_{30}$.

In many cases, separation of a multi-component mixture is carried out in stages, when at each stage one component with the highest or lowest value of some property is separated. Since components are numbered in an arbitrary order, we denote the component with the highest value of $R_i = x_{i0}\cdot\delta_i^{1/2}$ as the first one. Distribution of contact surfaces here are found from (25).

For the problem where it is required to separate the middle component, the values of this property for the first and the third components are lower and higher than its value for the middle component.

## HEAT-DRIVEN SEPARATION

When separation sequence is chosen for a heat-driven separation system one needs to take into account that the heat consumed at each stage of separation depend not only on the total power used at the i-th stage $p_i = p_i^0 + \Delta p_i$, (i = 1, 2) but also on the efficiency of the heat into work transformer $\eta_i(p_i, \overline{\alpha}_i)$. When $p_i$ increases then $\eta_i(p_i, \overline{\alpha}_i)$ decrease monotonically from $\eta_K$ at $p_i = 0$ to

$$\eta_i(p_i^{\max}) = 1 - \sqrt{\frac{T}{T_+}}\,, \tag{32}$$

for the maximal possible power for given heat transfer coefficients. In turn, the transformer, which transforms heat into work of separation, cannot generate power that exceeds the maximal power

$$p_i^{\max} = \overline{\alpha}_i (T_{i+}^{1/2} - T_{1/2})^2\,. \tag{33}$$

When $\overline{\alpha}_i$ increases, $p_i$ as well as $\eta_i$ increase (see Figure 4). Note that the expressions (32) and (33) were derived in [9] much earlier than the expression (12), which easily follows from them.

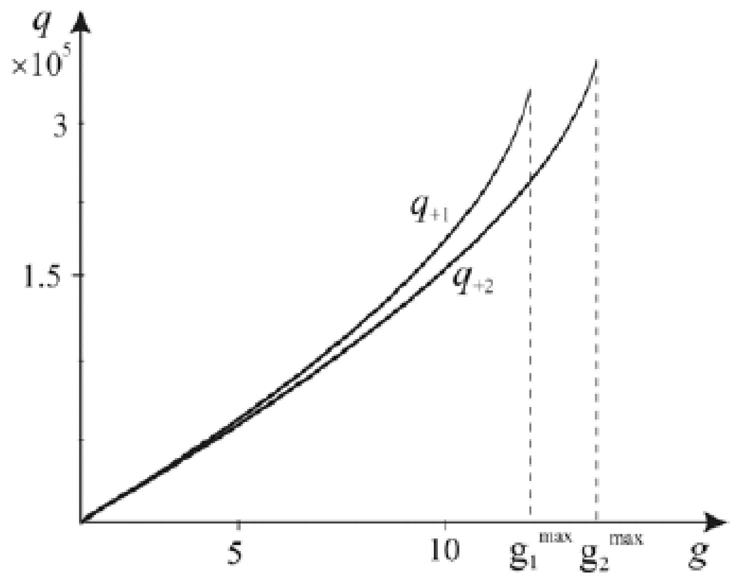


**Figure 4.** Characteristic change of the transformer's efficiency as a function of separation power and the effective heat transfer coefficient.

The maximal rate for a two-staged sequential separation system is determined by the maximal rate of the stage with the lower rate. Since the maximal rate for the i -th stage depends on the heat transfer coefficient $\overline{\alpha}_i$, (i = 1, 2) the maximal rate of two-stage system for the given total area of heat-transfer surfaces

$$\overline{\alpha}_1 + \overline{\alpha}_2 = \overline{\alpha}\,, \tag{34}$$

is achieved when the following equality holds $g_{01}^{\max}(\overline{\alpha}_1) = g_{02}^{\max}(\overline{\alpha}_2)$, i.e.

$$\frac{\sqrt{B_1^2 + 4\overline{\alpha}_1 D_1 (T_{1+}^{1/2} - T^{1/2})^2} - B_1}{D_1} = \frac{\sqrt{B_2^2 + 4\overline{\alpha}_2 D_2 (T_{2+}^{1/2} - T^{1/2})^2} - B_2}{D_2}, \tag{35}$$

where $B_i$ and $D_i$, (i = 1, 2) are defined by (15). Quantities $g_0 B_{ij}$ and $g_0^2 D_{ij}$ (i, j = 1, 2) represent reversible and irreversible losses of energy during the i–th stage of separation, respectively.

The conditions (34) and (35) determine the optimal distribution of heat-transfer surface $\overline{\alpha}_1$ and $\overline{\alpha}_2$ between two stages of heat-driven separation system. In turn, at each stage of separation for given $\overline{\alpha}_i$ the distribution of heat-transfer surface between heating and cooling is determined by the condition of minimum of the total cost of heat-exchangers subject to (13). If these cost costs are equal, then $\overline{\alpha}_{i+} = \overline{\alpha}_{i-} = 2\overline{\alpha}_i$.

Since $\eta(p, \overline{\alpha})$ decreases monotonically when $p$ increases, reduction of power (for example, as a result of optimal redistribution of mass exchange surface $S$), reduces the amount of heat $q_+$. That is why we assume that this surface $S$ is distributed between the stages in such a way that combined irreversible power losses at these stages is minimal. This condition leads to expressions (25) or (26) for $S_i^*$.

Suppose that during first stage of separation the first component is separated. The concentration of the first component in the input flow with rate $g_0$ is $x_{10}$. The residual binary mixture is then separated into pure components at the second stage of separation. Then for the first stage the power is $p_1 = p_1^0 + \Delta p_{11}$, where

$$p_1^0 = RTg_0 \left[ (1 - x_{10})(x_{20} \ln x_{20} + x_{30} \ln x_{30}) - \sum_{i=1}^{3} x_{i0} \ln x_{i0} \right] = g_0 B_{11}, \tag{36}$$

$$\Delta p_{11} = \frac{g_0^2}{S_1^*} K_{11} = \frac{g_0^2}{S_1^*} \frac{x_{10}^2 + (1 - x_{10})^2}{\delta_1} = D_{11} g_0^2. \tag{37}$$

Here $S_1{}^*$ corresponds to the expression (25). The subscripts of $B_{ij}$ and $D_{ij}$ in (36) and (37) denote the separation sequence and separation stage number, respectively.

Similarly, for the second stage we have

$$p_2^0 = -RTg_0 (1 - x_{10})(x_{20} \ln x_{20} + x_{30} \ln x_{30}) = g_0 B_{12}, \tag{38}$$

$$\Delta p_{12} = \frac{g_0^2 K_{23}}{S_2^*} = \frac{g_0^2 (1 - x_{10})^2}{S_2^*} \frac{x_{20}^2 + x_{10}^2}{\delta_3} = g_0^2 D_{12}. \tag{39}$$

We now introduce auxiliary notations for limiting power of heat into work of separation for the single-unit effective heat transfer coefficient $r_1 = (T_{+1}^{1/2} - T^{1/2})^2$, $r_2 = (T_{+3}^{1/2} - T^{1/2})^2$. Then for the first separation sequence from power balance it follows that

$$g_{01} B_{11} + D_{11} g_{01}^2 = \alpha_1 r_1, \tag{40}$$

for the first stage of separation. Similarly for the second stage of separation we get

$$g_{01} B_{12} + D_{12} g_{01}^2 = \alpha_2 r_2. \tag{41}$$

Since the maximal rate of the two-stage sequence of separations is determined by the minimal rate of its stages, we assume that the heat transfer surface is distributed in such a way that the maximal rates for both stages are equal. We denote this rate for the first separation sequence as $g_{01}$. Let us express $\alpha_1$ and $\alpha_2$ from (40) and (41). After adding them together we obtain the following expression for $g_{01}$,

$$\overline{\alpha} = \frac{g_{01}B_{11} + D_{11}g_{01}^2}{r_1} + \frac{g_{01}B_{12} + D_{12}g_{01}^2}{r_2} = F_1(g_{01}). \tag{42}$$

Similarly we get for the second separation sequence

$$\overline{\alpha} = \frac{g_{02}B_{21} + D_{21}g_{02}^2}{r_1} + \frac{g_{02}B_{22} + D_{22}g_{02}^2}{r_2} = F_2(g_{02}). \tag{43}$$

Solution of these equations with respect to $g_{0i}$ gives the maximum rate for the i–th separation sequence

$$g_{0i}^{\max} = \frac{-(B_{i1}r_2 + B_{i2}r_1) + \sqrt{(B_{i1}r_2 + B_{i2}r_1)^2 + 4(D_{i1}r_2 + D_{i2}r_1)\overline{\alpha}r_1r_2}}{2(D_{i1}r_2 + D_{i2}r_1)}, \ i = 1, 2. \tag{44}$$

The closed-form expressions for $B_{ij}$ and $D_{ij}$ are the following:

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = RT \begin{pmatrix} (1-x_{10})\sum_{i=2,3} x_{i0}\ln x_{i0} - \sum_{i=1}^{3} x_{i0}\ln x_{i0} & (x_{10}-1)\sum_{i=2,3} x_{i0}\ln x_{i0} \\ (1-x_{30})\sum_{i=1,2} x_{i0}\ln x_{i0} - \sum_{i=1}^{3} x_{i0}\ln x_{i0} & (x_{30}-1)\sum_{i=1,2} x_{i0}\ln x_{i0} \end{pmatrix}, \tag{45}$$

$$\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{K_{11}}{S_1^*} & \dfrac{K_{23}}{S_2^*} \\ \dfrac{K_{13}}{S_{12}^*} & \dfrac{K_{21}}{S_{22}^*} \end{pmatrix} = \frac{1}{S}\begin{pmatrix} K_{11} + \sqrt{K_{11}K_{23}} & K_{23} + \sqrt{K_{11}K_{23}} \\ K_{13} + \sqrt{K_{13}K_{21}} & K_{21} + \sqrt{K_{13}K_{21}} \end{pmatrix}. \tag{46}$$

## SEPARATION SEQUENCE FOR THREE-COMPONENT MIXTURE IN HEAT-DRIVEN SEPARATION SYSTEMS

The choice of separation sequence with maximal rate depends on the value of $\overline{\alpha}$ for which the maximal rates of two separation stages are equal and the curves $F_1(g_{01})$ and $F_2(g_{02})$ intersect. The point of intersection $g_0$ is determined by the condition

$$g_0 = \frac{\dfrac{B_{11} - B_{22}}{r_1} + \dfrac{B_{12} - B_{21}}{r_2}}{\dfrac{D_{11} - D_{22}}{r_1} + \dfrac{D_{12} - D_{21}}{r_2}}, \tag{47}$$

Positive $g_0$ exists if numerator and denominator in (47) have the same signs. Otherwise, the optimal sequence is determined by the sign of numerator in (47): it is positive then the second sequence is better, while if it is negative the first sequence has higher maximal rate. If curves intersect then this rule holds for $g_{0i} < g_0$. In case of $g_{0i} > g_0$ the opposite rule holds.

It is clear that for $r_1 = r_2$ maximal rate always corresponds to the minimal irreversible power loss for separation.

## EXAMPLE

We consider separation of three-component mixture with initial concentrations $x_{10} = 0{,}1$; $x_{20} = 0{,}6$ and $x_{30} = 0{,}3$. The specific (per surface unit) mass transfer coefficients are $\delta_1 = 0{,}2$ and $\delta_2 = 0{,}1$.

From (31) it follows that if mechanical separation is used then the third component should be separated first (0,3 > 0,141).

Let us consider heat-driven separation and assume that the temperature of the input mixture is $T = 300$ K, the temperature of the hot reservoir $T_+$ are $T_{+1} = 400$ K for separation of the first component and $T_{+3} = 350$ K for separation of the third one. The heat transfer coefficient is $\overline{\alpha} = 20\ 000$. The total heat-exchange area for both stages is $S = 10$ m$^2$.

### Step 1

We calculate the optimal distribution of heat-exchange area between stages. From (22 – 24) $K_{11}$ = 4,1; $K_{13}$ = 5,8; $K_{23}$ = 3,645 and $K_{21}$ = 0,907. The optimal distribution of area for the first separation sequence are $S_1$ = 5,15; $S_2 = S - S_1$ = 4,85 and $S_1$ = 7,17; $S_2 = S - S_1$ = 2,83.

### Step 2

For the first stage of the first separation sequence from (36) and (37) we get $B_1$ =739,6; $D_1$ = 0,8 and for the second stage we obtain $B_2$ = 1662,5; $D_2$ = 0,75 see (38) and(39). For the second separation sequence we get $B_{21}$ =1300,3; $B_{22}$ = 1336,5; $D_{21}$ = 0,81 and $D_{22}$ =0,32.

### Step 3

See (44). For the first separation sequence (separation of the first component at first stage) we get $g_0$ = 20,48. For the second separation sequence we obtain $g_0$ = 22,9. Therefore if third component is separated first then we can separate larger flow (with the same composition) than we can if we separated the first component first.

## CONCLUSIONS

In this paper we show how to select the most efficient separation sequence for three-component mixtures in a two-staged separation system. We also obtained the optimal distribution of contact surfaces for mass and heat transfer between separation stages. The minimal power required for mechanical separation at given production rate is derived. For heat-driven separation we obtain the maximal flow rate of the input mixture.

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# REDOSLIJED OPTIMALNOG RAZDVAJANJA U TRO-KOMPONENTNIM SUSTAVIMA

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## SAŽETAK

U članku se određuje optimalni slijed razdvajanja za trokomponentnu smjesu u dvostupanjskom procesu. Dva su rješenja dobivena. Prvo rješenje minimizira korištenu energiju, pri stalnom toku ulazne smjese, putem izdvajanja optimalnog slijeda razdvajanja i optimalnim dijeljenjem kontaktne plohe između prvog i drugog stupnja. Pokazano je kako je ulazni tok toplinom upravljanog dvostupanjskog procesa razdvajanja omeđena iznosom (najveće moguće brzine toplinom upravljanog razdvajanja) koji ovisi o slijedu razdvajanja. Izvedeni su izrazi za navedenu ovisnost.

## KLJUČNE RIJEČI

razdvajanje, trokomponentni sustav, zeotropska smjesa

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# SOCIAL FREE ENERGY OF A PARETO-LIKE RESOURCE DISTRIBUTION

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## SUMMARY

For an organisation with a Pareto-like distribution of the relevant resources we determine the social free energy and related social quantities using thermodynamical formalism. Macroscopic dynamics of the organisation is linked with the changes in the attributed thermodynamical quantities through changes in resource distribution function. It is argued that quantities of thermodynamical origin form the optimised set of organisation's state indicators, which is reliable expression of micro-dynamics.

## KEY WORDS

resource distribution function, Pareto distribution, social free energy

## CLASSIFICATION

JEL: C15

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## INTRODUCTION

Deepening the understanding of diverse human systems, their subsystems and smaller parts, is essential for a number of reasons, e.g. in order to enable reliable predictions about near-future states, improved allocation of scarce resources, improved planning of policies, increasing possibility for improvement of human living and working conditions, etc. The listed topics are important for different kinds of human systems, ranging from a family or a small group of persons to national and international systems. Within such systems there are institutions, processes and organisations the precise value of which is, in the socio-economic context, important both for them and for the environment. Here we particularly consider an organisation, not the whole system that it is a small part of. We assume that the organisation's resources are documented, e.g. book-kept. That may be due to internal procedures, relevant legislature, taking part in stock-markets, the transparency needed for external funding, etc. A straightforward consequence of documenting is that distribution of organisation's resources is known in different times. That enables their diverse statistical treatments, enlarging thereby the possibility to gain additional insight in resource structure and dynamics, i.e. to unveil possible, additional pieces of information.

Our aim is to develop a quantity which figures as an indicator of the state of the social system, and which may be intuitively and clearly determined from documented microscopic quantities.

A particular, interdisciplinary way of interpreting functioning of an organisation is possible through meta-theoretical description in which two system-oriented branches of theoretical physics, the thermodynamics and statistical physics, are exploited. That long-lasting approach has been recently revived by the idea that a part of socio-economic activity is expressible and measurable by way of social free energy. The social free energy is a measure of resources which can be transferred for a given purpose within a social system without changing its structure [1]. Formally, it was recently introduced meta-theoretically as the analogue of the physical free energy in a number of independent works: as the combination of innovation and conformity of a collective, profit, common benefit, availability, or free value of the canonical portfolio (see [1] and references therein). Substantially, it was recognised as a quantity with intrinsically social interpretation whose meta-theoretical origin enables its quantification and relation to other quantifiable functions [2]. In particular, that approach was introduced, developed in more detail, and accompanied by the modelling which in itself serves as a precursor to further application of the notion of social free energy on realistic cases [3].

In this article we start with the assumed form of a distribution of resources for a general organisation. In particular we take the distribution as one-parameter, Pareto-like distribution. Using it, we determine thermodynamical quantities. In that way we develop the notion of social free energy on the one hand, and contribute to searching for the existence of invariants in different organisations. While some results strictly depend on the form of the distribution chosen, others - including the most general ones - are independent of that choice. In particular, the power tail, the characteristic of Pareto distribution, was encountered in a large number of diverse systems [4] – ranging from original work on distribution of income, via distribution of outflows, via distribution of movies' popularity to distribution of flaws during quality control [5 – 7]. It is not clear whether in these examples there is some unifying underlying mechanism for generating Pareto distribution of observed resources, or not [6]. Nevertheless, this uncertainty about microscopic origin does not prevent the development of macroscopic approach in which underlying mechanisms are suppressed. Before proceeding, let us mention that the Pareto-like form chosen enables the development of compact expressions for relevant quantities.

In the second section of the article, we describe the distribution and argue about its adequacy. In the third section we determine thermodynamical quantities, that we analyse in the fourth section. Conclusions and projections of future work are given in the fifth section. The simplified case of the presented model is treated separately in the appendix.

## MODEL

The model distribution function of the resources of a value $u$ is the following function

$$p(u,a) = \frac{N(a)}{1+u^{a}}, \tag{1}$$

as shown in Fig. 1, with the norm $N(a)$ equal to

$$N(a) = \frac{\sin(\pi/a)}{\pi/a}. \tag{2}$$

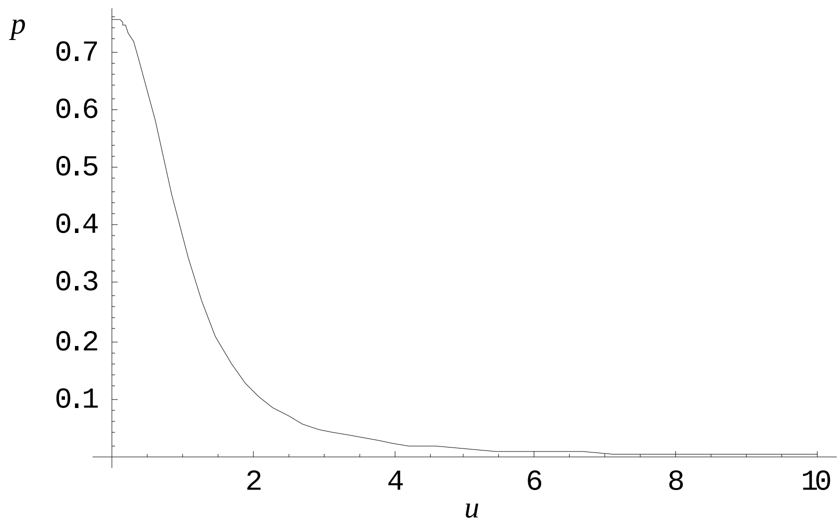


**Fig. 1.** Resource distribution function (1) for $a$ = 2,5.

Resources $u$ in (1) are considered to be non-dimensional quantities, which means that they are measured in units of some referent resource. We assume here that it is a constant during considered processes, and hence we leave it unspecified.

The distribution (1) requires $a > 2$. For large enough $u$, (1) becomes

$$N(a) \cdot u^{-a}, \tag{3}$$

which is Pareto distribution function with the exponent of the power law distribution equal to $a$ (and Pareto distribution shape parameter $k$ equal to $k = a - 1$) [4]. In realistic cases and in performed simulations for Pareto distribution, values of $a$ range from 1,5 to 3. Generally, Pareto distribution is valid only in a certain, finite interval of resource values. In cases when it is used for income or outflows of individuals, or human groups, it includes the inegalitariness of the system it is attributed to [8]. The inegalitariness lessens for larger values of $a$. We consider as the measure of inegalitariness the portion of people whose resources are smaller than some constant, otherwise unspecified number. Independently of the value of that number, the portion is smaller for larger $a$. In addition, both originally and nowadays still prevalently Pareto distribution is related to income and outflow, i.e. transfer of resources distributions. Generally, one may argue that for owned, held resources it is not universally applicable. For the purpose of simplicity, we consider that Pareto distribution is relevant also in cases of representing distribution of book-kept resources, at least for certain classes of organisations. Nevertheless, the formalism described is straightforwardly applicable to other distributions, a topic that we will address in detail in future.

We describe the region $u \approx 0$ using a modified Pareto distribution (1), which denotes realistic, finite number of zero-value resources. Generally, distributions which in the large resource tail follow Pareto form, approach a plateau in the low resource part. The particular form of that plateau, introduced through the constant (taken to be equal to 1 in denominator in (1)) is, thus a representation of a realistic feature. It is expected that in realistic cases the region of low resource values will be rather insignificant for interpretations of the all-organisation characteristics, hence the precise form of the plateau is not crucial for the distribution.

Form (1) is a particular form of a whole family of possible forms representing organisations' distribution of resources. In that sense, the formalism to follow means more than just elaboration of facts for a particular choice of resource distribution, as it represents a general procedure for calculating the aggregated quantities for a general resource distribution function. For a detailed description of physically-interpreted microeconomic behaviour see [9].

In (1) a reference is an idealised situation of quasi-equilibrium organisation's operations, without turmoil in environment and other sources of significant fluctuations. Closely related to (1) is the resource distribution of an organisation which has just undergone a turmoil and which is, somewhat generalised, represented as a sudden and drastic change of a portion of resources in the following way:

$$p(u;\ \{u\},a) = \frac{N'(\{u\},a)}{1+u^a}\theta(u_0 - u)\left[\theta(u_< - u) + \theta(u - u_>)\right], \tag{4}$$

where {u} stands for $u_{<,\,>,\,0}$ (with $u_< < u_> < u_0$) while $N'$ is a straightforwardly obtained norm. Expression (4) is nevertheless a somewhat cumbersome expression. In (4) the sudden lack of resources of a given interval of amounts is assumed. It is reasonable to consider a sudden enhancement of resources in a finite interval of values by substituting the bracket with $[1 + \theta(u - u_<) + \theta(u_> - u)]$.

## THERMODYNAMICS

The total energy $U$ of the system with resources described by (1) is given with

$$U(a) = \int_0^\infty up(u,a)du = \frac{1}{2\cos(\pi/a)} \quad . \tag{5}$$

The entropy $S(a)$ is determined using the statistical mechanics formula

$$S(a) = -\int_0^\infty p(u,a)\ln p(u,a)\cdot du\ , \tag{6}$$

$$S(a) = \ln\left|\frac{\pi/a}{\sin(\pi/a)}\right| - C - \Psi(1 - 1/a), \tag{7}$$

in which $\Psi(\cdot)$ is digamma function, and $C = 0{,}577$ is the Euler constant.

Temperature $T(a)$ is determined using

$$T\mathrm{d}S = \mathrm{d}U, \tag{8}$$

that gives

$$T(a) = \frac{\frac{\pi}{2}\frac{\sin(\pi/a)}{\cos^2(\pi/a)}}{\psi'(1-1/a) + a - \pi\mathrm{ctg}(\pi/a)}, \tag{9}$$

while free energy is obtained using

$$F(a) = U(a) - T(a)S(a). \tag{10}$$

The closed expression for (10) is omitted here because of its non-tractability. Dependence of entropy (7) and temperature (9) on $a$ is shown in Figures 2 and 3, respectively, while the dependence of free energy on $a$ is shown in Figure 4.

(10) may be interpreted from a formal and a substantial viewpoint. Formally, (10) is applicable in case of a quasi-equilibrium processes, because the expression (8) is appropriate in quasi-stationary dynamics of organisation's resources, with relatively small part of resources transferred in the relevant time unit. Because of that, flow of resources is relatively small, fluctuation in resource distribution as well, and consequently there is quasi-equilibrium situation for which (10) is relevant expression. Substantially, (10) means that individual processes in the organisation, which include resource transfer, are changed in a time interval which is relatively small compared to the time interval of a characteristic change in organisation's

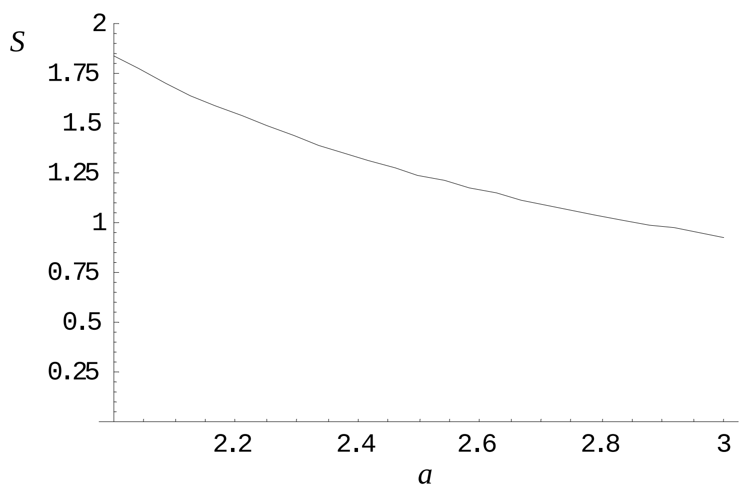


**Fig. 2.** Entropy as a function of $a$.

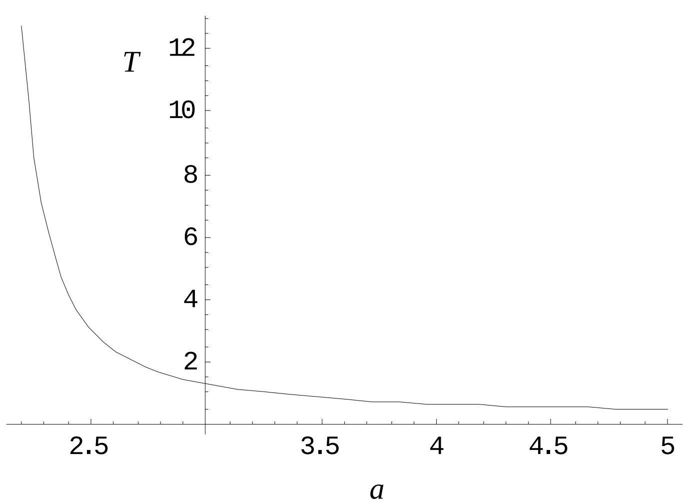


**Fig. 3.** Temperature as a function of $a$.

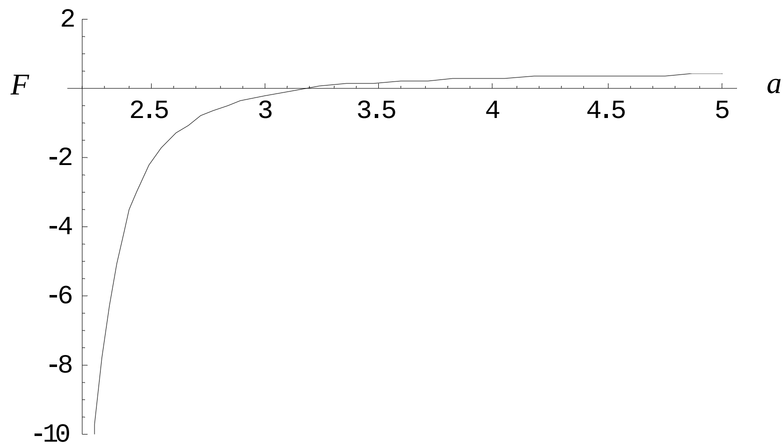


**Fig. 4.** Free energy given by (10).

environment. In that case, no matter what the environment state is, the organisation's resource distribution function has the assumed form (1). The change in the environment is thus projected onto the change of the parameter *a* of the resource distribution function.

Substantially, (12) enables the determination of the quantity of resources which are extractable from the organisation when its state changes from some initial *i*¸ to some final *f*:

$$\Delta F = F_{\mathrm{f}} - F_{\mathrm{i}}. \tag{11}$$

That quantity then gives the resources which are not bound to the organisation's dynamics but are exploitable for other purposes, one of which is regularly preventing or lowering the destructive influence of environment dynamics. Thus *F* measures the system level of adaptation. However, adaptation is not separated from other, intrinsically system quantities, like entropy. The higher level of adaptation brings about lower entropy, the interplay of which is discussed for general resource distribution elsewhere [1]. If $\Delta F > 0$ in (11), the system level of adaptation rises, and vice versa. Regarding changes in entropy, if $\Delta S > 0$, the system is considered to evolve spontaneously.

## RESULTS AND DISCUSSION

Energy, entropy (Fig. 2) and temperature (Fig. 3) are lower for larger *a*, in accordance with the fact that for larger *a*, the portion of low-value resources is larger.

For $a < 3{,}187$ the social free energy is negative. We do not consider in detail the implications of *F* changing the sign, as we further in the section concentrate on the region $1{,}5 < a < 3$. With rising *a* the number of important resources of significant value decreases. In other words, system is more adapted with smaller number of valuable resources if we consider larger value of *F* as a sign of better adaptation.

Let us consider possible small fluctuations in distribution of resources, thus in values of *a*. Small changes in value of *a* are less important for the value of *F* in case of relatively large *a*. In that sense, larger *a* means that fluctuations are suppressed. This is because small fluctuations mean fluctuations in quantity of low-value resources, which are relatively large in number in case of a sufficiently large *a*. In order to preserve the functional form of the distribution function (1), the value of large-value resources should be somewhat changed, but without crucial changes in their quantity that would bring about a significant change in *a*. In cases of a relatively small value of *a*, its small fluctuations bring about relatively large change in *F*, because there is relatively large portion of large-value resources which can contribute to fluctuations.

Social free energy as a measure of adaptation means that changes in alignment with environment for relatively small values of *a* are reachable using intra-system redistribution of resources. To the contrary, changes in alignment (i.e. better adaptation) for relatively large values of *a* are brought about through changes in the number of small-valued resources. Thus, for large values of *a* there is structured response to environmental influences, and the corresponding systems can be considered as having some hierarchy because different resources function differently in response to environmental influences. Furthermore, this implies that large-valued resources contribute more to structure (or entropy) and intensity of dynamics than to free resources, i.e. the more-valued the resource is, the stronger it is bound to the system. This is valid in case of relatively small environmental influences, which is considered to be a regular characteristic of environment dynamics.

## CONCLUSIONS

The process of determining macroscopic quantities from a given microscopic quantity – here a resource distribution function – is presented. In particular, for a realistic, Pareto-like resource distribution function, the social free energy is determined in characteristic situations. For the distribution chosen, the social free energy is constantly negative. Changing its value closer to zero is related to the model with better adaptation, the possibility of intensely localised-in-time dynamics and with rising proportion of low-value resources.

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# SOCIJALNA SLOBODNA ENERGIJA PARETOVE RASPODJELE RESURSA

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## SAŽETAK

Za organizaciju kojoj je raspodjela resursa opisana funkcijom sličnom Paretovom određujemo socijalnu slobodnu energiju i druge veličine primjenom termodinamičkog formalizma. Makroskopska dinamika organizacije povezana je s promjenama u pridruženim termodinamičkim veličinama putem promjena u funkciji raspodjele resursa. Diskutiramo kako veličine termodinamičkog porijekla tvore optimalni skup indikatora stanja organizacije, kao pouzdani iskaz mikrodinamike.

## KLJUČNE RIJEČI

funkcija raspodjele resursa, Paretova raspodjela, socijalna slobodna energija

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# COMPLEX SYSTEMS BUILT BY SIMPLE ELEMENTS

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## SUMMARY

There are a lot of systems, which behave complexly, around us. We cannot predict their behaviour. Unpredictability is almost a character of complexity but how can we tackle the phenomenon of it. The formal mathematical descriptions of them are more and more complex and only several times solvable. Is the making a system of non-linear equations the only way to handle and descript systems like them? Using simple elements we can build models which show complex behaviour. Simple rule-systems can be a model of a complex system. For example algorithms can be appropriate for this task. We can implement these models for the language of computers, as well, and running simulations. Can we observe or perceive emergent characters? What is the measure of emergent phenomena? These are the questions to which I am searching the answers. The algorithms can give us a better way to understand the complex world.

## KEY WORDS

complex systems, emergence

## CLASSIFICATION

PACS: 89.75.-k

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## INTRODUCTION

The main point is that the methods of traditional economics determine the limits of inquiring. The economics relies on mathematics, but the systems of axioms draw the boundaries. Today the systems working around us are unpredictable. So how can we build models which have complex behaviour? First we must find a theory and then some tools which can help us.

## THE THEORY OF COMPLEX SYSTEMS

The theory of complex systems says that there are constituents and connections between them. They build subsystems and systems. The constituent is the basic of a system. It doesn't matter how we can describe these elements, but in a system there are some similarities among the constituents. The connections among them make the system complex. The connections make loops and feedbacks so the effects propagating on them will widespread all over the system. The behaviour of a system is the trajectory on which it runs. The trajectory is the series of the successive states which is changing with the movements of the system. The movements is the changing states itself. But a system described in the former mentioned manner will have some similar characteristics.

The constituents are only similar but not the same. So the subject of examination is a set of heterogeneous elements or more exactly the subject is their aggregate behaviour in time. The time brings the dynamic viewpoint into the inquiring. But we have to preserve the unpredictability of the systems, we should use probabilistic logic, so all members of the heterogeneous population usually don't do the same thing in same manner. But its consequence is the path-dependent behaviour and unpredictable system-path. The system will be sensitive to the initial conditions, to the initial state. There will emerge a few of possible state-loop or optimal blot in the state-space of the complex system. It is a possibility the being of more than one optimal point or state. And it depends on the intensiveness of interaction between the system and the environment or the adaptation process of the system which one of them will be reached by the system. The system can reach its optimum only in long run, but it isn't sure that we can recognise it.

Characteristically neither economic system is chaotic. There is always a certain trend among the sequence of economic data. These systems are somewhere among the order and chaos. The economic evolution balances among these 2 extreme states. So the equilibrium is a wide concept. Assuming the complexity of the system and at the same time the equilibrium does exist. It is true that the system can move toward the order or chaos. The process of self organising is spontaneous. It's the way of evolving of order and the phenomenon of synergy. Kauffman states that the position along the axis of the system is connected with this process. The less connection means the higher order, like evolving of oligopolies [1]. The basis of dynamical processes is the complexity of the economic systems which is the optimum itself of the dynamical system.

You can observe some general processes in operating complex systems:

- Structural deepening: Specializing among the system. Some part of the system make a specialization into subsystems. Such as the organs in a body. Subsystems make the whole system more complicated.
- Heterogeneity: The constituents of the system can be characterised in only similar manner. They are not the same, but only similar.

- Supra-criticality: The possible connections or interactions among the constituents of the system can bring a huge number of possible changes.
- Sub-criticality: There should be some limit for the number of possible changes. The limited number of basic constituents makes some limit on the possible interaction and connections.
- Optimality and adaptability: A system has to work in a changing environment. The environment and the systems in it interact on each other. So the systems need to adapt to actual environment and to its possible changes. The aim of the systems is the surviving, but it leads to the question of optimality and adaptability. The systems should find the optimal ability of adaptability.
- Isolation: Isolation can make spread the changes among the system, because it isn't sure that a new change won't extinct.
- Criticality and turbulence: The connections among the constituents of the system build backward and forward feedings so the effects of a change in the systems spread away in the system in an unpredictable manner. A slight change in the system can make revolutionary changes among the system. These phase-transitions-like phenomenon in the system seem to happen accidentally.

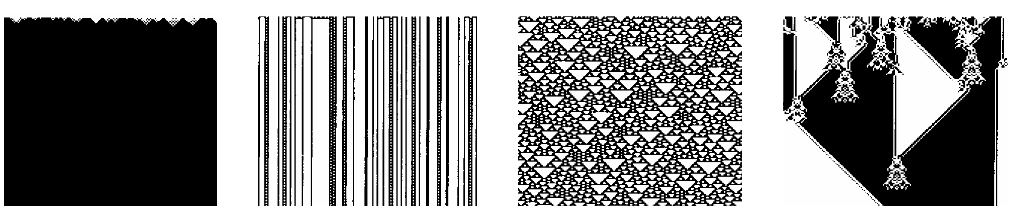
Can we build system which shows these processes?

## THE MODELLING AND DESCRIPTION OF A COMPLEX SYSTEM

The traditional economics was built on the basis of mathematics. Maths describes the complex problems with differential equations. As characteristics are becoming more complex, the number of equations raises. So finding the solution is getting harder and harder. Even the rearranging of equations to more simple or understandable forms can be impossible. So it is not sure that you can find the optimum. If the equations are connected along positive or negative feedbacks the movements of the system in the state-space are irreversible and even it cannot assure the stability, convergence and one-way effect–mechanism of system. But its result is a chaotic system. It is impossible to differentiate – without any axiom – the deterministic and stochastic characteristics. So the economic theories are simple and built on strict axioms.

## MODELLING WITH A SIMPLE TOOL

Let us concentrate on the cellular automatons. You could meet them in seventies or eighties of the XX. century like life-games. They consist of only some simple rules which show you the rules of the changing cells which can have some colours, but regularly there were only black and white cells. You can determine the initial state and you can run the system and watch the life on the screen. You can produce interesting patterns running automatons like these [2: p.231].



1. class 2. class 3. class 4. class

**Figure 1.** The 4 classes of cellular automatons.

The fourth class is the most interesting for us. It shows the chaotic and ordered behaviour at the same time. Becauese you can see isolated patterns which can interact with some of the others. Why is it so important for us? With a cellular automaton we can produce patterns which are similar to the empirical observations. As the next picture shows [2: p.402.]. Is it not similar to a leaf?

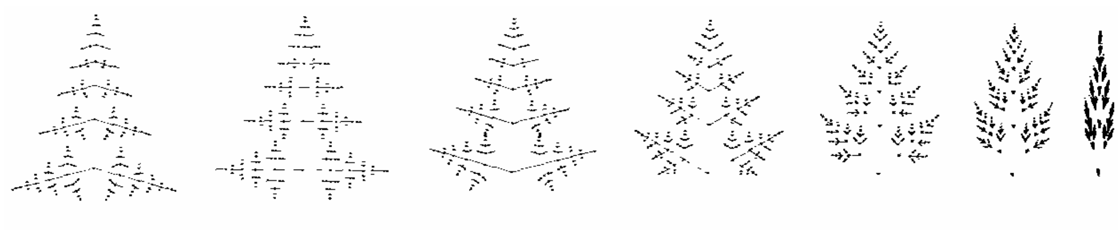


**Figure 2.** The behaviour of a cellular automaton with substituting rules.

A few of cellular automatons are universal. The universal automaton can be a model of every other models. It means that you cannot mention an electrical or mathematical or other model which you cannot substitute with a cellular one.

It is true that this substitution does not necessarily bring a quicker or more understandable model-behaviour, but the main point is that a model based on simple rules can be universal. Even the ability of predictability can be lost with using simple models. A lot of inquirers said among the history of theories that it's not necessary the ability of predictability of a theory. It is a conclusion of István Magas that unpredictability does not make a theory unscientific.

As in the case of cellular automatons, the unpredictability is a characteristic of the complex systems. You can determine precisely the state of a complex system at a moment, but that is obscure what will happen in the next moment. This character comes from the complexity of the system.

There were some attempts for using cellular automatons for building models. As Fig. 3 shows, some simple rule can result in a form which has a point-distribution similar to data of exchange rates in a market.

**Figure 3.** Rules and behaviour of a cellular automaton and the distribution of black and white points.

Another model was built to simulate the elections in a virtual country. Every point in this model had an initial interest – for which party he/she will elect – but the neighbours interact as the time passes. Every point will choose own opinion if most of his neighbours has other. The results were that not the given initial state of the system is the most important for the possible elections but the distribution of the opinions among the system.

It is a good question that what is the appropriate transformation of the behaviour of a cellular automatons, which will show the appropriate results. How we can formulate appropriate questions? Or we can only make those questions which will give the appropriate answers which are shown by the behaviour of our models?

The examples showed me that we can build a model using cellular automatons than we can run simulations and after these we can formulate our questions for which these results would be similar.

But if we can use simple models to build a model which can behave complexly, why we have to use cellular automatons, or how much a model should be well formulised mathematically. According to my opinion it does not necessary that every characteristic of a complex system should be formulised. I want to use simple algorithms to build economic models. Then these models became available for computer simulations, tests and observation.

If the constituents and/or interactions – moreover the influencing connections – are characterised by algorithms, you can attribute the model with every character, which you can recognize in the theory of complex systems. The disadvantage of this versus the traditional theory is that a model like this is characterized deterministic and stochastic attributions. But in behalf of this you have to sacrifice the concept of unambiguously definable optimum or equilibrium.

You can observe certain interrelations and the patterns of model behaving during the subsequent computer simulations. Therefore you can foreshadow the trends and probable direction of possible dynamic equilibrium.

## THE Z-FUNCTION

The utility is an economic concept for evaluating the usage or the commodities themselves for the actors in the economy. But usage of the utility concept is not coherent in the economic theories. It is not true that the economic actor will behave in the same manner if he or she poor or wealthy. So the utility depends on the owned sets of commodities. Even the money is a commodity which is not neutral for the economic actors and their decisions.

If the price of oil is raising then the world demand will not be diminished as we can expect. It is a fact which we can perceive in the real world, as well. So we should introduce a new concept. K. Martinás advise the concept of Z-function. Z is a function which can evaluate all parts of the sets of commodities which an economic actor has. Z can be the wealth itself.

There is a problem about this function. Because it is hard to say when will the fortune of an actor raise. If there is a shortage of fuel then actors can stockpile of it, or the same amount of fuel will have higher value. So if there are two distinct state when the same actor has almost the same sets of goods, the difference is only in amount of one of owned commodities, then we can surely say that this state has a higher or lower Z-value. But if the actor has not got the same sets of goods then we cannot compare the Z-values of these states. How can we introduce new goods or commodities in an economy with Z-function?

It is an aim to determine an appropriate Z-function which can give the driving-forces for decision-making processes in the model.

## A GROUP OF ALGORITHMS AS AN ECONOMIC MODEL

An economic actor can operate in several kinds of actions. He acts according to certain patterns in every action, and after a while, it became routine actions. Among decision making, first of all, he collects information (each actor behaves differently in one way) but every member works up the information and after the balancing process he decides, executes and during the controlling process he makes up conclusions in according to his own decision making procedure. Therefore he alters the processes or algorithms himself. I think that is why you can build up a model using algorithms for decision making procedure.

Imagine a market, where the economic actors can sell and buy some kind of commodity or good. In order to easily implement it into computers the model handle the time discretely. One trading day is one moment in time. The time is relative since Einstein. The time really is a series of actions or occurrences. Every day every economic actor can alter his own decision making knowledge base and what kind of procedure he can use during the decision making, simply how high probability is ordered to each decision making strategy from the strategy set. Because of the wide range of economic literature easily accessible, every potential trader has a chance to know the different available methods which are used by expert traders. One actor can make only one assignment in one specific time. This assignment can be selling, buying or holding the positions.

Economic actors, who operate on the market, can be grouped in several different ways. Usually in a society the wealth is in inverse ratio to the number of definite society layer, therefore the number of rich people is small. Involving this assumption in the model there should be a few wealthy agent (who take a risk with enormous money) and many poorer agents. The activity on the hall of the market is inverse ratio to the number in the group of agents. Therefore a few risk-taking agents behave actively, while the smaller ones can trade fewer times.

Every economic actor has its own knowledge base from decision making strategies and he makes his decisions using this base. His decisions focus attention on when, how, and how much sell or buy goods, otherwise simply holds his positions. The knowledge base is influenced by the information come from the group of acquaintance (those people whom he know were successful or not), by the general movement of prices and indexes and by the measure of the average prices. Naturally the enumeration is incomplete, but the economic literature is not able to give satisfactory list.

The knowledge base itself is a strategy- and probability-set (ordered to each strategy). Each economic actor decides itself, the activity in altering the knowledge base and how you want to enter and exist in the market. It is connected with its own wealth (higher wealth, higher activity), with the relation of risk (higher risk taking, higher activity), with the actual market price (diminishing prices, higher activity).

Each economic actor is a constituent or individual of the model. The critical points of algorithms used by them are the decision making procedures or methods which are the knowledge base itself.

Within the economic actors the connection network means ordered randomizing web, which is alterable later. The links in the web can evolve and disappear randomly or it can be influenced by the neighbouring actors. For example, it has relatively higher probability for my friend’s friend to become my friend. The success of the economic actor is his wealth, which are the cash-funds and the sum of actual price of the owned goods. The success of his decision is measured by the net-yield which gained from the decisions. The strategy which brings net-yield, gains a higher probability in the knowledge-base. And it has a diminishing effect on the friends’ knowledge-base. This process is similar to the selection procedure of the biological evolution. This ensures the fine-spun tuning of the system and slow adaptation and successive approximation towards the optimum.

Beside of this each economic actor knowledge base or wealth can randomly alter. This phenomenon is similar to the mutation procedure of the biological evolution which is coarse-grained tuning, so smaller or bigger jumps in every direction in the state-space of the model. It is a problem how we can realize the patterns of the behaviour. Which patterns should be watched or how can be characterized the states of the system. For example is the level of concentration of decision making strategy set of an actor important? Or the connections among the actors in the system are more important? We can draw a picture in which the points can be the actors and the lines among them are the connections and the distance among the points or the colours of the points can be tackled as the wealth or strategy concentrating of points. Has it some meaning? How we can it translate for the events in the real world?

We should accept that we cannot determine as exact results as the mathematics can. The behaviours of these systems can show many kind of those processes which we can expect from a system which behaves complexly. But it is not sure that we will use these results for predicting a lot of complex phenomenon in the economic world. There is not any model which can be used as a universal model for economy. We can determine the steps of model-building processes and the main methods for pattern-recognition.

My aim is that during the simulation of computer model I realize the unpredictability of the model, but using of the algorithms you can build up complex models, which behaviour analogue more to the real life situations.

## SUMMARY

You can hardly describe mathematically the model of the complex system. So my conclusion is that you have to use simple methods for describing and building complex systems.

The theory of complex systems has some consequences which are about the behaviour of a complex system. We can observe these characters in our everyday systems. And there is a fact that we can build models of a cellular automaton which can modelling every kind of other system. So simple process(es) can lead to modelling systems which have complex behaviour. The cellular automatons are too abstract to describe anything about the real systems. We have to use other simple tools to make simple models. Well, it is a possible threatening that we should sacrifice some character which is important scientifically, for example the predictability, but we can gain some notion about emergent characters, too.

The algorithms can be the appropriate tool. It has the advantage of easily coding, implementing and simulating on computers. Simulating these models you have to seek for trends and patterns, which will be emergent characters.

So the aim of my research is to prove that algorithms are good tools for building complexly behaving models. We can build models in according to the empirical statistics. The question is that the results of running model whether can be similar to the real life.

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# KOMPLEKSNI SUSTAVI IZGRAĐENI OD JEDNOSTAVNIH ELEMENATA

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## SAŽETAK

U našoj je okolini velik broj sustava, kompleksnog ponašanja. Njihova stanja ne možemo predvidjeti. Nepredvidljivost je gotovo svojstvo kompleksnosti, ali kako pristupiti toj pojavi? Formalni opis tih sustava vrlo je složen i razriješiv samo u nekoliko slučajeva. Da li je postavljanje sustava nelinearnih jednadžbi jedini način za bratanje i opis takvih sustava ? Koristeći jednostavne elemente možemo izgraditi modele koji pokazuju kompleksno ponašanje. Jednostavni pravilima određeni sustav može biti kompleksni sustav. Npr., algoritmi mogu biti prikladni za takav pristup. Postavljene modele možemo uklopiti u računalne jezike i simulacije. Možemo li opaziti, ili osjetiti svojstva izviranja? Koja je mjera izvirujućih pojava? To su pitanja na koja tražim odgovore. Algoritmi nam omogućuju bolji način razumijevanja kompleksnog svijeta.

## KLJUČNE RIJEČI

kompleksni sustavi, izviranje

## MANUSCRIPT PREPARATION GUIDELINES

Manuscript sent should contain these elements in the following order: title, name(s) and surname(s) of author(s), affiliation(s), summary, key words, classification, manuscript text, references. Sections acknowledgments and remarks are optional. If present, position them right before the references.

**SUMMARY** Concisely and clearly written, approx. 250 words.

**KEY WORDS** Not more than 5 key words, as accurate and precise as possible.

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Include figures and tables in the preferred position in text. Alternatively, put them in different locations, but state where a particular figure or table should be included. Enumerate them separately using Arabic numerals, strictly following the order they are introduced in the text. Reference figures and tables completely, e.g., "as is shown on Figure 1, *y* depends on x …", or in shortened form using parentheses, e.g., "the *y* dependence on *x* shows (Fig. 1) that…".

Enumerate formulas consecutively using Arabic numerals. In text, refer to a formula by noting its number in parentheses, e.g. formula (1). Use regular font to write names of functions, particular symbols and indices (i.e. sin and not *sin*, differential as d not as *d*, imaginary unit as i and not as *i*, base of natural logarithms as e and not as *e*, $x_\mathrm{n}$ and not $x_n$). Use italics for symbols introduced, e.g. $f(x)$. Use brackets and parentheses, e.g. {[( )]}. Use bold letters for vectors and regular GoudyHandtooled BT font (for MS Windows) or similar font for matrices. Put 3pt of space above and below the formulas.

Symbols, abbreviations and other notation that requires explanation should be described in the text, close to the place of first use. Avoid separate lists for that purpose.

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