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Editors

Josip Stepanić, Editor-in-Chief
editor@indecs.znanost.org
Faculty of Mehanical Engineering & Naval Architecture
I. Lučića 5, HR – 10000, Zagreb, Croatia

Josip Kasač, Assistant Editor
josip.kasac@indecs.znanost.org
Faculty of Mehanical Engineering & Naval Architecture
I. Lučića 5, HR – 10000, Zagreb, Croatia

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The winner of the INDECS award for the best published article – INDECSA – for Volume 4 and year 2006, is Dr. Güngor Gündüz. Dr. Gündüz is awarded for his article "Ancient and Current Chaos Theories", INDECS 4(1), 1-18, 2006. That article received the largest number of votes during the process conducted by the Commission for choosing the best article in accordance with the propositions for INDECSA, as stated in the official web site of the Journal. In the name of the INDECS Council and in my personal name let me congratulate Dr. Gündüz for winning the INDECSA.

With publishing of this issue, in which four of the published articles are eligible for INDECSA, the contest for INDECSA 2007 started.

Zagreb, 12 July 2007 Josip Stepanić

Interdisciplinary Description of Complex Systems 5(1), 1-13, 2007

THERMODYNAMIC MEASURE FOR NONEQUILIBRIUM PROCESSES

Katalin Martinás[1,]* and Attila Grandpierre[2]

[1]Department of Atomic Physics, Eötvös Loránd University, Budapest, Hungary

[2]Konkoly Observatory of the Hungarian Academy of Sciences, Budapest, Hungary

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SUMMARY

One of the most fundamental laws of Nature is formulated by the Second Law of Thermodynamics. At present, in its usual formulation the central concept is entropy characterized in terms of equilibrium state variables. We point out that because thermodynamic changes arise when systems are out of equilibrium and because entropy is not a natural state variable characterizing non-equilibrium states, a new formulation of the Second Law is required. In this paper, we introduce a new, more general, but still entropic measure that is suitable in non-equilibrium conditions as well. This new entropic measure has given a name extropy. The introduction of extropy allows us to formulate the Second Law in a more suitable and precise form, and it resolves some conceptual difficulties related to the interpretation of entropy. We point out that extropy has a fundamental significance in physics, in biology, and in our scientific worldview.

KEY WORDS

entropy, thermodynamics, extropy, irreversibility

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*Corresponding author, η: martinas@elte.hu; +36 1 2090 555 Ext 6360;
Atomfizikai tanszek, ELTE, Pazmany Peter setany 1, 1117 Budapest, Hungary.

INTRODUCTION

One of the most fundamental laws of nature is Second Law of Thermodynamics. This law tells that changes during any adiabatic transition of any system proceeds toward the equilibrium, and in this process a certain thermodynamic quantity - called entropy - is never less than that of its initial value [1]. The point is that macroscopic changes do not occur without the presence of non-equilibrium and non-equilibrium processes are not necessarily adiabatic. We attempt here to reformulate the second law with the help of a more general thermodynamic state variable measuring the distance from equilibrium. This distance is not symmetric like geometrical distance just because thermodynamic changes have a preferred direction towards the equilibrium. We call this more general thermodynamic state variable measuring the distance form equilibrium as extropy [2 – 5].

Let us mention some of the characteristic problems related to entropy that we are aware of at present. In the highly popular website "The Second Law of Thermodynamics" (http://www.secondlaw.com – #1 in the Google search list of information under the search term "thermodynamics" and also #1 under "second law"; the site won the Internet Guide Award of The Encyclopedia Britannica) Lambert [6] claims that the idea of the Second Law is "the biggest, most powerful, most general idea in all of science". The concept of entropy, together with the concept of energy, is of central importance for science. Unfortunately, there seems to be wide-ranging confusion regarding this fundamental concept. Recently a new impetus is given to clarify the concept of entropy [7 – 12]. It is pointed out that entropy is not a measure of 'disorder' since 'disorder' is a highly qualitative and not precisely defined concept that sometimes contradicts to the characterization offered by an entropic measure. Instead, Leff [7] and Lambert [6] proposed that entropy is a measure of energy dispersion: "Energy spontaneously tends to flow only from being concentrated in one place to becoming diffused or dispersed and spread out". This formulation is a significant improvement in comparison to the older one regarding entropy as a measure of disorder. Moreover, it could be very much helpful also due to its metaphoric power. The physical content is also valid at least in cases when only one gradient is present. We point out that in more general cases this formulation still requires further improvement. For example, friction is a basic thermodynamic process in which kinetic energy is transformed into heat. Now it is an everyday experience to observe that when someone applies the brakes of a car all of a very sudden, the part of the wheel of the car in contact with the road is suddenly heated to large temperatures. The point is that the process of energy transfer from the car's global kinetic energy to a small region of its wheel is not "energy spreading". More generally, the phenomenon of "spontaneous energy focusing" (www.physics.ucla.edu/Sonoluminescence/page2.html) like sonoluminescence, spark generation, turbulence etc. (e.g. [13 – 17]) seems to indicate that either the concept of spontaneity in Lambert's formulation of the Second Law is problematic in some cases or the validity of Lambert's formulation is not universal. In this paper, we will present a more general and more precise formulation of the Second Law.

ENTROPY IN ISOLATED, CLOSED, AND OPEN SYSTEMS

For *isolated* systems, standard entropy is a good measure of the direction of changes relative to thermodynamic equilibrium. The positive sign of entropy change indicates the direction towards equilibrium state having the highest entropy. For *closed* systems - the difference from the isolated ones is that they are able to exchange energy (heat) with their surroundings – entropy is already not always a good measure of the direction of changes. Equilibrating processes (shortly: e-processes) are the ones occurring in a system that interacts only with its equilibrium environment. In contrast, processes in which the system exchanges matter/energy not only with its environment will be termed as not-necessarily-equilibrating processes or n-processes. For example, the Earth can be regarded as a closed system when we neglect mass transfer with its cosmic environment. The Earth receives a varying amount of solar radiation corresponding to the varying phase of solar activity. Clearly entropy cannot tell the direction of global changes of the Earth in each and every instant.

For *open* systems, i.e. systems that exchange not only energy but matter with their surroundings, entropy is even less good indicator of changes to occur. For example, when we have two systems from the same material, and we know that one having smaller entropy than the other, what do we know about the relation of these systems? One possibility is that the system with smaller entropy is in a larger distance from the thermodynamic equilibrium than the other. Another is that the system with smaller entropy is in the same distance from equilibrium but it has a smaller amount of mass. Entropy in itself does not tell which the case is. The point is that entropy can increase (or decrease) in two types of processes: in e-processes or in n-processes. For example, in the case of biological growth the mass of the given system increases, accompanied by the extensive increase of entropy.

Moreover, not only changes within the system can lead to changes relative to the equilibrium, but also changes in its relations to its environment. Therefore, environmental changes can also induce changes in a system. In order to obtain a general law telling the direction of system's changes, we will need a thermodynamic quantity that is based not only in the thermodynamic parameters of the system, but also on the parameters of its environments. The simplest of all these thermodynamic measures, as we will show here, is the thermodynamic distance from thermodynamic equilibrium as measured in entropic units. Entropic units make it possible that entropy can be a special case of the new, more general thermodynamic variable. How can we obtain such a highly desired variable?

THE INTRODUCTION OF EXTROPY: THE STATE VARIABLE CHARACTERIZING THE ENTROPIC DISTANCE FROM EQUILIBRIUM

We want to characterize the distance of nonequilibrium states from the state of thermodynamic equilibrium (distance from equilibrium, D_e). One immediate idea is to measure the distance of our system from the equilibrium by the temperature difference between it and its environment,

$$D_e = \left|T - T_0\right|.$$

This seems to be a good measure in many cases, and, in everyday life, we measure the degree of nonequilibrium with it in many cases well. This subjective guess proposes the measure, which is larger for larger systems, so we need the distance as the extent of non-equilibrium,

$$D_e = U\left|T - T_0\right|.$$

Nevertheless, the temperature is not a universal measure, since in many cases the system is not in pressure equilibrium with its environment. For pressure the extent of non-equilibrium is

$$V\left|p - p_0\right| = V\Delta p .$$

With the measure $D_e = U|T - T_0|$ we have $U\Delta T$ and $V\Delta p$ in the same units, as it is shown by the First Law of Thermodynamics which states that the work W has the same unit as the internal energy U. It means that if there is a measure for the extent of non-equilibrium, it has the form

$$D_e = K\left[\frac{U}{T}\Delta T + V\Delta p\right],$$

where K is a parameter. This is still not a general measure, since in many cases there is a difference in chemical composition between the system and its environment, and so in the corresponding thermodynamic parameter μ; we have add also $N\Delta\mu$ to D_e. Similarly, in case of electric potential difference can be important; in case of a difference in gravitational potential energy a term $mg\Delta h$ should be added, and so on. In general, the measure of the distance from thermodynamic equilibrium will be

$$D_e = \Delta U + \Delta W + \ldots \ . \tag{1}$$

After a simple transformation of (1), with the choice $K = 1$

$$D_e = U\left|1 - \frac{T_0}{T}\right| + \Delta(Vp) + \Delta(N\mu) + \ldots \ . \tag{2}$$

In this way, we obtain a measure for the distance from thermodynamic equilibrium in energy units. This is a good measure of the distance from thermodynamic equilibrium, if it is

(i) zero in equilibrium,

(ii) positive in nonequilibrium,

(iii) always decreasing in equilibration processes.

We can see that D_e fulfils (i), since in equilibrium $\Delta t = \Delta p = \Delta\mu = \ldots = 0$. In general, it is easy to see that D_e (ii) fulfils, since in nonequilibrium Δt, Δp, $\Delta\mu \ldots > 0$.

From these two requirements, it arises that $D_e > 0$ in nonequilibrium, D_e decreases to zero as we proceed towards equilibrium. The larger the Δt, Δp, Δm etc., the larger is the distance from equilibrium, and the larger is D_e. Regarding (iii), it is important to note that in thermodynamics it is crucial to consider processes in which the various differences (Δt, Δp, Δm etc.) are transformed into each other. For such processes, our measure D_e sums up the contributions of the various differences to the arising distance from equilibrium. Until now, the concept "equilibration" is not characterized quantitatively. If we characterize equilibration by D_e, we find the D_e is not only a suitable measure of the distance from equilibrium, but it is a quantitative measure of equilibration as well. Unfortunately, the absolute value in the formula of D_e makes it difficult to handle.

Now we will show how to obtain thermodynamics from equations (1 – 3). Equation (1) was written on the basis of intuitive notion of the distance from equilibrium. Let us notice that (1) has a remarkable violation of symmetry. All the terms (except the thermal ones) are written in the form extensive×intensive difference. Volume, mole numbers are extensive variables – proportional to the extension of the system), while pressure and chemical potentials are the intensives (they are the same in equilibrium, and their difference is related to the degree of non-equilibrium). On the other hand in the thermal term, energy is extensive, but $1 - T_0/T$ is not a difference of the intensive parameter, it is only a number, not a physical state variable, as all physical variable has a measuring unit, and it is not only a number.

Formally the problem can be solved in two ways - and they lead to different, but concise thermodynamic measures.

a) Instead of energy, an other physical quantity with the dimension energy/temperature is used. It is the entropy. The point we note is that while energy can be measured and determined, it is overly complicated to obtain the exact value of entropy for real systems. With this selection, the measure of distance from equilibrium will be the quantity known as exergy B.

$$B = S(T - T_0) + \Delta Vp + \Delta N\mu + \ldots \ .$$

b) The other way is the modification of the intensives

$$B = S(T - T_0) + \Delta Vp + \Delta N\mu + ... \; \Pi = U\left|\frac{1}{T_0} - \frac{1}{T}\right| + V\left(\frac{p_0}{T_0} - \frac{p}{T}\right) + N\left(\frac{\mu_0}{T_0} - \frac{\mu}{T}\right) + ...$$

For the first sight it may seem clumsier than necessary. But imagine such a physics where instead of Kelvin scale $T^* = 1/T$ is used. The third law would change to the statement - infinite temperature is the lowest, and one never can reach it. Similarly, if in this physics $p^* = p/T$ is used, then the ideal gas equation would be $p^* = R\ N/V$. It would be simpler, naturally other expressions would be more complicated, but it can be done. In that case

$$\Pi = U\left(T^* - T_0^*\right) + V\left(p^* - p_0^*\right) + N\left(\mu^* - \mu_0^*\right) + ... \; .$$

Formally this relation has the same structure as exergy, but it is an entropic measure. Π is called extropy.

An important remark is that in the formula (2) defining D_e the first term $U|1 - T_0/T|$ is the product of U, the internal energy, and $|1 - T_0/T|$, a term without physical meaning because it is dimensionless. We point out that if we write this term into a form $T_0{*}U|1/T_0 - 1/T|$, the last factor $|1/T_0 - 1/T|$ has a physical meaning measuring the distance from equilibrium in terms of $\Delta(1/T)$. If we transform all the other terms correspondingly, this transformation yields an entropic measure, and it has the advantage that entropy does not appear.

$$\Pi = U\left(\frac{1}{T_0} - \frac{1}{T}\right) + V\left(\frac{p_0}{T_0} - \frac{p}{T}\right) + N\left(\frac{\mu_0}{T_0} - \frac{\mu}{T}\right) + ... \; . \tag{3}$$

In the present paper we derive an entropic-like measure for the extent of non-equilibrium. Now we return to Carnot principle, and we show that Clausius made some (allowed) but not needed simplification, that is why he introduced entropy as the only measure for non-equilibrium.

EXTROPY AS A MEASURE OF THE DISTANCE FROM EQUILIBRIUM

We want to consider whether extropy is a monotonous decreasing function of time for equilibration processes. We already had shown that extropy can be written as

$$\Pi = \sum_i Y_i X_i \, .$$

where X_i is for the extensive, $X_1 = U$, $X_2 = N$, …… $X_n = V$, and Y_i for the related intensive $Y_1 = 1/T_0 - 1/T$, …, $Y_n = p_0/T_0 - p/T$, variables. Now we show that the properties of extropy flows from the zeroth and the first law.

The Zeroth Law constraint tells that Π is a homogeneous linear function of U, V, and N. Doubling the system leads to double its extropy.

Proof: Let us make the following transformation: $U \to kU$, $V \to kV$, $N \to kN$, where $k > 0$.

Variables T, p, and μ do not depend on the size of the system,

$$T(U,V,N) = T(kU,kV,kN),$$
$$p(U,V,N) = p(kU,kV,kN),$$
$$\mu(U,V,N) = \mu(kU,kV,kN).$$

That is,

$$Y(kU,kV,kN) = Y(U,V,N),$$

telling that they are homogeneous zeroth order function of U, V, N, in agreement with the Zeroth law of thermodynamics.

The effect of transformation k for Π is

$$\Pi(k) = \sum_i Y_i(k) X_i(k) = \sum_i Y_i X_i(k) = k \sum_i Y_i X_i = k\Pi \; . \tag{4}$$

Now, differentiating both sides by k,

$$\frac{\mathrm{d}\Pi}{\mathrm{d}k} = \sum_{ij} Y_i X_i \,. \tag{5}$$

But considering Π as the function of k, then

$$\frac{\mathrm{d}\Pi}{\mathrm{d}k} = \sum \frac{\partial \Pi}{\partial (kX_i)} \mathrm{d}(kX_i)\mathrm{d}k = \sum \frac{\partial \Pi}{\partial (kX_i)} X_i \,. \tag{6}$$

Comparing (5) and (6) for $k = 1$, we get

$$\frac{\partial \Pi}{\partial X_i} = Y_i \,,$$

so

$$\mathrm{d}\Pi = \sum_i Y_i \mathrm{d}X_i \,. \tag{7}$$

The extropy change of the system is, from eq. (7),

$$\mathrm{d}\Pi = \left(\frac{1}{T_0} - \frac{1}{T}\right)\mathrm{d}U + \ldots$$

Extropy is additive function, so in case of non-equilibrium systems, when the intensives can depend on the spatial coordinates r, then extropy is the sum of the local extropies defined as densities that can vary in space, and so we can work with extropy as a variable depending on variables r, $Y = Y(r)$ and $\rho_i\,(r) = X_i/V$. With this notation,

$$\Pi = \int \sum_i Y_i \rho_i \mathrm{d}V \,,$$

and from eq. (7)

$$\mathrm{d}\Pi = \int \sum_i Y_i \mathrm{d}\rho_i \mathrm{d}V \,, \tag{8}$$

now we have a balance equation for the densities and so we can determine the time dependence of extropy.

TIME DEPENDENCE OF EXTROPY

The time derivative is (it follows from eq. 8)

$$\frac{\mathrm{d}\Pi}{\mathrm{d}t} = \int \sum_i Y_i \frac{\mathrm{d}\rho_i}{\mathrm{d}t} \mathrm{d}V \,. \tag{9}$$

The first law of thermodynamics expresses the conservation of energy. Now the continuity equation for the energy density ρ_i tells that

$$\frac{\partial \rho_i}{\partial t} + \mathrm{div} j_i = 0 \,,$$

and for the mole numbers ρ_j if there is no chemical reaction, in case of chemical reactions

$$\frac{\partial \rho_j}{\partial t} + \mathrm{div} j_j = s_j \,,$$

where s_j is the source/sink of the mole numbers. Inserting these continuity equations into eq. (9), after straightforward manipulations (see Appendix) we obtain for the change of extropy the relation

$$\frac{\mathrm{d}\Pi}{\mathrm{d}t} = \int \sum_i Y_i J_i \mathrm{d}f + \int \sum_i \mathrm{grad}(Y_i) J_i \mathrm{d}V \,,$$

where the first integral is for the surface of the system. We split the flows into to parts: flows to the environment, these are always equilibrating flows, J_{i0}, and J_{ie} flows going to/or coming from other systems being in the environment. We obtain

$$\frac{\mathrm{d}\Pi}{\mathrm{d}t} = \int \sum_i Y_i J_{ie} \mathrm{d}f + \int \sum_i Y_i J_{io} \mathrm{d}f + \int \sum_i \operatorname{grad}(Y_i) J_i \mathrm{d}V$$

Formally,

$$\frac{\mathrm{d}\Pi}{\mathrm{d}t} = J_\Pi - \Sigma ,$$

where

$$J_\Pi = \int \sum_i Y_i J_{ie} \mathrm{d}f .$$

Now

$$\Sigma = -\left(\int \sum_i Y_i J_{\mathrm{io}} \mathrm{d}f + \int \sum_i \operatorname{grad}(Y_i) J_i \mathrm{d}V \right),$$

where J_Π is the extropy carried to the system, and Σ contains the effect all the other processes. It contains the effect of processes within the system and processes between the system and its reservoir.

We will prove the following Thesis: The Carnot –Principle ensures that in real processes Σ is always positive. In a complete cycle the extropy does not change, so

$$\int \frac{\mathrm{d}\Pi}{\mathrm{d}t} \mathrm{d}t = \int J_{ie} \mathrm{d}t - \Sigma(t) \mathrm{d}t = 0 .$$

This equation tells that for a complete cycle

$$\int \Sigma(t) \mathrm{d}t = \int J_{ie} \mathrm{d}t = I_e . \tag{10}$$

Lemma : In a real cycle, the following requirement will be always fulfilled:

$$I_e \geq 0 . \tag{11}$$

Actually, for any part of the cycle, the relevant part of I_e can be negative for a certain period. The point is that, for the complete cycle, I_e in (11) is always positive. It can be zero for the imaginary reversible process, and never can be negative in a complete cycle.

Now we will show the proof of the above formulated Thesis. Classical thermodynamics was built from Carnot on the concept of cycle. First, let us calculate the extropy flow in a cyclic process to the system. Now our system is a heat engine that runs cyclically. It makes contact successively with n reservoirs at temperatures T_i, exchanging from them $\mathrm{d}U_i$ energies, and it has contact with pressure reservoirs with pi pressures, and the relevant volume changes are $\mathrm{d}V_i$.

Then

$$J_{ie} = \sum\nolimits_{cycl} \mathrm{d}U_i \left(\frac{1}{T_0} - \frac{1}{T_i} \right) + \mathrm{d}V_i \left(\frac{p_0}{T_0} - \frac{p_i}{T_i} \right). \tag{12}$$

The Second Law states (see the Appendix) that $Q(1 - T_2/T_1) - W \geq 0$. Now we show that $Q(1 - T_2/T_1) - W$ is just the extropy flow. Then utilising again the First Law, $\mathrm{d}Q = \mathrm{d}U + p\mathrm{d}V$, and the relation telling that the useful work is

$$W = (p_2 - p_1)\mathrm{d}V, \tag{13}$$

where p_2 is the pressure of the environment, we obtain

$$U\left(\frac{1}{T_2} - \frac{1}{T_1} \right) + \mathrm{d}V \left(\frac{p_2}{T_2} - \frac{p_1}{T_1} \right) \geq 0 . \tag{14}$$

A simple generalization is then obvious; we may consider a more complicated heat engine that runs cyclically, making contact successively with n reservoirs at temperatures T_i. Then the first law is $W = \Sigma_i W_i$, and Carnot's principle becomes

$$\sum_{cycl}\left[\mathrm{d}Q_i\left(1-\frac{T_0}{T_i}\right)-\mathrm{d}W_i\right]\geq 0\,, \tag{15}$$

Utilizing the First Law and substituting (13) into (15), it yields

$$\sum_{cycl}\left[\mathrm{d}Q_i\left(1-\frac{T_0}{T_1}\right)-\mathrm{d}W_i\right]=\sum_{cycl}\mathrm{d}U_i\left(\frac{1}{T_0}-\frac{1}{T_i}\right)+\mathrm{d}V_i\left(\frac{p_0}{T_0}-\frac{p_i}{T_i}\right)\geq 0 \tag{16}$$

It is easy to see that

$$\sum_{cycl}\mathrm{d}U_i\left(\frac{1}{T_0}-\frac{1}{T_i}\right)+\mathrm{d}V_i\left(\frac{p_0}{T_0}-\frac{p_i}{T_i}\right)=\int J_{ie}\mathrm{d}t = I_e\,,$$

is the total extropy flow to the system during the cycle that must be positive for every cyclic process, and so we proved our Lemma telling that $I_e > 0$ is always valid for real processes.

Our Lemma implies that in real circumstances a process is always present which consumes the extropy. As it is valid for any type (imaginary or real) cycle, it implies that

$$\Sigma(t)\geq 0\,. \tag{16}$$

The result we obtained tells that in any real cycle extropy Π is consumed.

We utilized the Carnot principle for showing that Π has a special property: in real processes it is always decreasing. Therefore we proved that extropy fulfils our requirement (iii). Only extropy inflow can increase the amount of extropy and to realize a complete cycle in which every state variable regains its original value. The production of extropy from nothing is not allowed.

Simple manipulations yield, that

$$\Pi=\sum_i\int_{Y_i}^{Y_{io}}\left(Y_{io}-Y_i\right)\mathrm{d}X_{\mathrm{i}}$$

or introducing the entropy matrix:

$$g_{ik} = -\frac{\partial^2 S}{\partial X_i \partial X_k}$$

and $s_{ik} = g_{ik}^{-1}$, we can write $dX_i = - s_{ik} dY_k$, so

$$\Pi=\sum_i\int_{Y_i}^{Y_{io}}\left(Y_{io}-Y_i\right)s_{ik}\,\mathrm{d}Y_{\mathrm{k}}$$

Extropy is zero in the equilibrium state with the environment. Expression $\Pi = 0$ means that the system is not distinguishable from its environment. There is no way to get energy from it. There is no order.

Extropy is a function of the parameters of the reservoir, and the parameters of the system being this reservoir.

We note that Π is a non-equilibrium potential function measuring the thermodynamic distance from thermodynamic equilibrium in entropic units. It is a thermodynamic distance, and it is not symmetric, at variance with geometric distance.

The geometric distance d between two point A and B d(A,B) has the properties:

1. $d(A, B) \geq 0$
2. $d(A, A) = 0$
3. symmetry: $d(A, B) = d(B, A)$
4. triangle inequality $d(A, B) + d(A, C) \leq d(A, C)$

In contrast, extropy as a thermodynamic distance is asymmetric, since it is measured from the equilibrium state E to the actual state A or B. Its properties are

$$d(E, A) \geq 0,$$
$$d(E, E) = 0.$$

Instead of the triangle inequality it is subadditive

$$d(E, A) + d(E, B) \leq d(E, A + B)$$

where A + B is the symbol of the unified A and B systems. Subadditivity follows from the superaddivity of entropy.

Now let us estimate the thermodynamic distance Π from the equilibrium state in the case of the human body. In a simplest approach, we can keep the dominant term in eq. (16), and so we can approximate the value of extropy of the human body as

$$\Pi \approx \mathrm{d}N\left(\frac{\mu}{T} - \frac{\mu_0}{T_0}\right). \tag{25}$$

Equation (25) is similar to the Gibbs potential $G = \mu N$. In chemistry, the chemical potential is not calculated relatively to the environment of the system. Nevertheless, if someone regards G as measured relative to the environment (e.g. [18]), and redefine the chemical potential in a way that $\mu_0 = 0$, than a simple relation will be found between the Gibbs potential and the extropy:

$$\Pi \approx \frac{G}{T}. \tag{26}$$

Now a simple method to determine the value of extropy is available through estimating G. To determine the Gibbs free energy, we can use the formula $G = H - TS$. In human organisms, the processes are isothermal at $T \sim 310$ K. The fuel content of a 70 kg person is given as triacylglycerols (fat), 15,6 kg; proteins, 9,5.kg; carbohydrates, 0,5 kg. The combustion heat of fat is 38,9 kJ/g, therefore the chemical entropy of fat of the human body is 606,8 MJ. The combustion heat of protein and carbohydrate is 17,2 kJ/g, all together 172 MJ. The enthalpy present in the chemical bonds of the 70 kg human body H = 778,8 MJ. The entropy of glucose is 212,13 $\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$ = 1,18 $\mathrm{J \cdot K^{-1} \cdot g^{-1}}$ and that of liquid water 69,94 $\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$ = 3,88 $\mathrm{J \cdot K^{-1} \cdot g^{-1}}$. Approximating the entropy content of living matter with that of glucose, the 9,5 kg protein will give 11,21 $\mathrm{kJ \cdot K^{-1}}$, the 15,6 kg fat 18,4 $\mathrm{kJ \cdot K^{-1}}$, and the 0,5 kg carbohydrates 0,59 $\mathrm{kJ \cdot K^{-1}}$, all together 30,20 $\mathrm{kJ \cdot K^{-1}}$. The 44,4 kg water has an entropy of 172,25 $\mathrm{kJ \cdot K^{-1}}$. In this way, the estimated entropy of the material of the 70 kg human body is found to be $S_h \approx 202{,}4$ $\mathrm{kJ \cdot K^{-1}}$. With $T \approx 310$ K, $T_S \approx 62{,}6$ MJ, and so $G \approx 716{,}2$ MJ. Therefore extropy, or thermodynamic distance from equilibrium, will be $\Pi \approx G/T \approx 2{,}31$ $\mathrm{MJ \cdot K^{-1}}$, an order of magnitude higher than entropy, $S_h \approx 0{,}20$ $\mathrm{MJ \cdot K^{-1}}$. In this way, we obtained that

$$\Pi(\text{human organism}) \approx 2{,}31\ \mathrm{MJ \cdot K^{-1}} >> S(\text{human organism}) \approx 0{,}20\ \mathrm{MJ \cdot K^{-1}} \tag{27}$$

The fact that the extropy of the human organism is much larger than its entropy corresponds to the general experience of the ultimate easy to move our fingers, hands or foots.

FORMULATION OF THE SECOND LAW WITH THE HELP OF EXTROPY

The second law of thermodynamics, in a concise form, states that any thermodynamically *isolated* system tends to equilibrate with its environment:

"Isolated systems seek the dead level" (I)

But this formulation seems to be clearly restricted. Attempting to obtain a formulation of the Second Law as a general law of Nature, it is highly desirable to expand its range of application. Now it is easy to expand the range of this law to more general type of systems by regarding the system plus its equilibrium environment *together* as representing an isolated system.

The Second Law of Thermodynamics can be formulated in a precise form telling:

> "In cases when the considered material system A is governed by its equilibrating interactions with its equilibrium environment B, α_i, the thermodynamic parameters of A, will tend toward β_i, the thermodynamic parameters of its environment B." (II)

We can observe that (II) does not apply in cases when friction, sparkling or other spontaneous energy focusing processes are present. For example, in the case of a braking car the temperature of the car can increase, heat goes from a colder to a hotter place, and so the thermodynamic parameter of the system T_A does not tend toward the temperature of the environment T_B during the process of braking. At the same time, we may observe that the kinetic energy decreases drastically in the process of braking; therefore, if a thermodynamic parameter could be construed that involves somehow the sum of thermal, kinetic, chemical, electric etc. processes, in a way that this new thermodynamic parameter measures the distance from the equilibrium, this new parameter could work well characterizing the direction of changes.

A simple formulation of the Second Law is available formulated with extropy:

> In cases when the considered material system A is governed by its equilibrating interactions with its equilibrium environment B, the decrease of extropy of the system $\Pi_A \leq 0$ determines the direction of changes relative to thermodynamic equilibrium. (III)

Certainly, our formulation (III) surpasses previous formulations of the Second Law like (I) and (II) in the sense that it can be applied to a wider set of phenomena, and so it may be regarded as a better and as a more general formulation of the Second Law.

The biosphere – or the biological organisms are embedded in the environment, but they interact with each other (and with the Sun). In the present (extropic approach) first we characterize the systems with the extropy – that is they are in the equilibrium environment, and after we describe their interaction. As in the interactions material and energy may go from one system to the other, extropy flow also appears.

Living system can maintain themselves, because they acquire extropy from their environment.

THE DIFFERENCES AND SIMILARITIES OF EXTROPY WITH ITS PREDECESSORS: ENTROPY, GIBBS FREE ENERGY, NEGENTROPY AND EXERGY

Entropy is measured from zero degree (0 K) instead from the equilibrium with the actual environment, and this circumstance leads to hardly tractable complications. One point is that entropy in general cannot be simply calculated, since heat capacity variations with temperature influences its value when obtaining by extrapolations from zero degree, and variations of heat capacities around zero degree involve complicated quantum processes.

Therefore, in general, it is simply not possible to determine the exact value of entropy theoretically. Moreover, it is not easy to obtain empirical determination of such complex materials that exist in our environment and in biological organisms. Another point is that entropy does not have any convenient interpretation.

Unfortunately, there are many problems with the interpretation of entropy, and therefore it is not always easy to obtain quantitative insights on the relation of these two quantities of overall importance.

Let us take two examples. We have two piles of apples, and you have to tell which pile you prefer: the pile with higher or lower entropy. If you select the pile with lower entropy, in the hope that the apples of this pile will be not rotten, and this is why their entropy is lower, you can receive less number of apples, but they can be all rotten. Now if you select the pile of higher entropy, it can contain more apples; but you cannot know in advance if they are rotten or not. Therefore, entropy is not a good measure in itself for a selection of apples if you are hungry. The situation became confused because the apples can change their entropy content, depending on their state. We can assume that a fresh apple has smaller entropy than the old one. On the other hand two apples have twice as much entropy as one apple has.

Another quantity closely related to extropy is the Gibbs free energy. Haynie [18: pp.85-86] defines the free energy through the change of the chemical potential $\mu_A - \mu_A^0$ relative to the standard state corresponding to T = 298,16 K and p = 1 bar. At the same time, Haynie claims [18: p.74] that the Gibbs free energy measures the maximum amount of work that can be done by a process going from a non-equilibrium state to an equilibrium state (at constant temperature and pressure). We point out that there is a hidden awkwardness or ambiguity in the different uses of the term Gibbs energy. The introduction of the concept of extropy sheds light to this awkwardness and, at the same time, it resolves the problem lying in the background. Work can be made only relatively to the environment. The same compressed gas can make different amount of work in different environment. Therefore, if one wants to interpret G as the maximum amount of work that can be done by a process going from a non-equilibrium state to an equilibrium state (at constant temperature and pressure), one has to define the zero point of chemical potential to the environment. Extropy is a concept that rules out such awkwardness, it is a precise and exact thermodynamic state potential, and it can be applied generally.

Another closely related concept to the concept of extropy is Brillouin's negentropy $N = S_0 - S$, where S_0 is the entropy of the system in the corresponding equilibrium and S is its entropy content in its actual state. In many cases, the concept of negentropy is very useful in describing and understanding nonequilibrium system's behaviour. But from a thermodynamic viewpoint negentropy does not have the property frequently attributed to it, namely, measuring the distance from the equilibrium, since equilibrium is always referring to the actual environment, and mass and energy exchange with this environment always introduces conditions not taken into account by the system's parameters only. For example, the Brillouin negentropy N does not change when the same system is in different environment, like room once in a summertime, once in a wintertime environment; N(wintertime) = N(summertime). Certainly, the same room with the same degree of temperature 20 C is farther from thermodynamic equilibrium in winter (T = 0 °C) than in summer (T = 15 °C). Extropy performs better in this respect, too, since it is equivalent with

$$\Pi \approx \left(S_{syst} + S_{env}\right)_0 - \left(S_{syst} + S_{env}\right). \tag{28}$$

This equations shows that for an isolated system extropy is equal with Brillouin's negentropy, but in the case of closed and open systems the two entropic measure is different; Brillouin's negentropy does not measure the distance from equilibrium while extropy does.

SUMMARY AND CONCLUSIONS

We found that extropy as an indicator of direction of changes performs better, since it is simpler, more general and elegant than entropy. Simplicity and universality are one of the most fundamental aspects of scientific theories. The Second Law of thermodynamics is one of the most fundamental laws of Nature. Therefore, extropy has an even more fundamental role in physics than entropy.

The fact that extropy as a driver of processes is based on differences presents an unexpected and fundamental challenge for us physicists accustomed to the preconceptions based on Newtonian physics. In modern physics, it is generally regarded that the basic drivers of physical processes are forces. The four fundamental forces of physics, gravitation and electromagnetism, weak and strong nuclear forces are forces corresponding to the properties of the objects themselves. In contrast, our finding is that the basic driver of thermodynamic processes is not a physical factor corresponding to the properties of the objects themselves, but to the relation between the objects and their environments. Modern physics regards as Aristotelian the view that the factor driving natural processes depends on the relation between the objects and their environments. The force beyond thermodynamic changes is not of a Newtonian type, because it is based on differences and not the material properties of the objects themselves. We learned that this thermodynamic force is originated from the fact that the system is not in thermodynamic equilibrium. This thermodynamic force is not symmetric, not fulfilling Newton's third law. It seems that we have to change our basic preconceptions regarding the nature of physical world and learn to be accustomed to a new worldview based on the extropic aspects of thermodynamic.

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TERMODINAMIČKA MJERA ZA NERAVNOTEŽNE PROCESE

K. Martinás[1] i A. Grandpierre[2]

[1]Odsjek za atomsku fiziku – Sveučilište Eötvös Loránd
Budimpešta, Madžarska

[2]Konkolijev opservatorij – Madžarska akademija znanosti
Budimpešta, Madžarska

SAŽETAK

Jedan od temeljnih zakona prirode iskazan je kao drugi zakon termodinamike. U današnje vrijeme, u njegovom uobičajenom iskazu središnji pojam je entropija određena putem varijabli ravnotežnog stanja. Ističemo da je, zbog toga što termodinamičke promjene nastupaju kad je sustav van ravnoteže i zbog toga što entropija nije prirodna varijabla za opis neravnotežnih stanja, potreban novi iskaz drugog zakona termodinamike. U ovom radu uvodimo novu, općenitiju mjeru, i dalje entropijsku, prikladnu za neravnotežen uvjete – ekstropiju. Uvođenje ekstropije omogućava nam iskzivanje drugog zakona termodinamike u prikladnijem i preciznijem obliku te razrješava neke konceptualne nejasnoće povezane s interpretacijom entropije. Ističemo fundamentalno značenje ekstropije u fizici, biologiji i znanstvenom pogledu na svijet.

KLJUČNE RIJEČI

entropija, termodinamika, ekstropija, ireverzibilnost

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SOME REMARKS ON THE EFFECTS OF PRODUCTIVITY ON GROWTH

Zsolt Gilányi

Department of Microeconomics, Corvinus University of Budapest
Budapest, Hungary

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SUMMARY

Standard economic models predict that a *ceteris paribus* increase in the overall productivity results in an increased production if the economy departs from an equilibrium state. We show that this result is valid under specific conditions In other cases, even if the initial conditions of an economic system are so that the economy converges into (or starts from) an equilibrium state, the increase of overall productivity generally results, in the long run, in the collapse of the economy due to the unbalanced change in the money holdings. Hence, in general, an appropriate expansive monetary policy should accompany the increase of productivity.

KEY WORDS

economic growth and aggregate productivity, multisector growth, monetary growth

CLASSIFICATION

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Corresponding author, η: zsolt.gilanyi@uni-corvinus.hu; +36 30/4300342;
1093 Budapest, Fővám tér 8. Hungary

INTRODUCTION

Our purpose here is to examine the conditions under which the standard economics' result on the effect of a change in overall productivity on production holds. This standard result is intuitive: a ceteris paribus increase in overall productivity increases production. (see e.g. [1]).

To be able to predict the effect of any shocks or economic policy, one needs an appropriate representation of our economies.

Standard economic analysis [2] makes this representation by dividing time into periods defined by the actions that one can execute in the period. These actions are production, consumption and exchange. The notion of short or long run is used in several senses. It can refer either to the number of exogenous variables during the optimization problem of economic agents, or (in a sequential model) to the number of periods during which a given (equilibrium) state is reached.

A representation of the complex system of our economies naturally requires shortcuts and simplifications. Economic theory is intended to explain the choice of key variables and connections (causalities) between them. As a result, in standard economic models, the general hypotheses/results are: the neutrality of money in production as well as in consumption [3], and the positive effect of the increase of overall productivity on production.

Ayres and Martinás [4 – 8] showed that a re-foundation of the assumptions on the behaviour of economic agents questions the habitual results; the teachings of standard economic analysis being just a special case. Moreover, this reformulation permits to handle other theoretical findings, which are non-conform to the standard formulation[1]. As in Ayres and Martinás' work, agents do not seek the traditional profit and utility maximum, the neutrality of money does not necessarily hold any more in production; as a result an increase in productivity changes the commodity/money ratio and usually the economy is driven away from a stable equilibrium state and collapses after a while. It follows that an appropriate monetary policy accompanying changes in production is needed to save the standard results.

The paper is organised as follows. First, we consider a simplified version of the Ayres-Martinás model, next we examine the conditions under which the Ayres-Martinás model has the same properties as the standard economic models, and finally we discuss the effects of productivity change on production in the Ayres-Martinás model.

A SIMPLIFIED AYRES-MARTINÁS MODEL

Let us consider a simplified model of Ayres-Martinás (for a detailed discussion see [5]).

We construct two macro-agents, called sector 1 and sector 2. A sector comprises not only firms but also consumers drawing their revenues from the sector. We suppose that sectors are disjunctive. We omit banks as usual in standard models and consider money exogenously[2].

Time is divided into periods, which in turn are divided into sub-periods. In the first sub-periods sectors produce, in the second sub-period they exchange and in the third they consume. Hence, the stocks x of a sector changes as follows:

$$\text{sector 1: } x^1{}_t = x^1{}_{t-1} + x^1{}_{p,t} + x^1{}_{e,t} - C^1{}_t,$$

$$\text{sector 2: } x^2{}_t = x^2{}_{t-1} + x^2{}_{p,t} + x^2{}_{e,t} - C^2{}_t,$$

where $x^2{}_t$ is the stock x held by sector 2 at the end of period t, $x^2{}_{p,t}$ is the change of stock (flow) x of sector 2 in the period t due to the production process.

We suppose that there are four stocks: commodity y_1 produced exclusively by sector 1, commodity y_2, produced exclusively by sector 2, labour L and money M. There is no depreciation. For the sake of simplicity, we consider a linear production technology, i.e.: $x_p = a_l$, where $a = (a_1, a_2, a_L)$ is the input-output vector; by assumption for sector 1 $a_1 > 0$, $a_2 \leq 0$ and $a_L < 0$; for sector 2 $a_2 > 0$, $a_1 \leq 0$ and $a_L < 0$; l is the level of production.

Sectors, being economic agents, seek to get richer for economic decisions, hence we suppose that

- they can evaluate wealth. This evaluation is given by the following wealth function, identical for both sectors,

$$Z(y_1, y_2, L, M),$$

- they avoid avoidable losses for economic decisions in time:

$$\mathrm{d}Z/\mathrm{d}t \geq 0.$$

For production decisions from 2, if stocks are not binding, with linear approximation we obtain:

$$x_p = a_l = a \cdot K \cdot v^T \cdot a,$$

where v^T is the transposed row vector of values $v^T = (y_1/M, y_2/M, L/M)$, K is a parameter showing the intensity of reaction of the sector, i.e. a unit value added induces K times increase in the level of production.

For exchange decisions from 2, if stocks are not binding, with linear approximation we obtain:

$$x_e = N(v - p)$$

where N is a parameter showing the intensity of reaction of the sector in response to the surplus realisable on a commodity and p is the price offered.

For the sake of comparability with the standard model, let us suppose that exchange takes place always at equilibrium prices. We suppose that $N_1 = N_2 = N$. in that case, $p = (v_1 + v_2)/2$. By assumption, there is no exchange of labour. Consumption is given exogenously: c % of the commodities is consumed. The quantity of labour is fixed. We measure overall production as usual by the real GDP: first period prices multiplied by the quantities produced.

THE STANDARD ECONOMIC MODEL AS SPECIAL CASE OF THE AYRES-MARTINÁS MODEL

Standard economic models are equilibrium models. This means that:

- exchange takes place always at equilibrium prices, i.e. prices at which planned demand (plans being made before exchanges) in a period equals planned supply. Hence, the reallocation of stocks by exchange is independent from the exchange mechanism. In other terms, there are no exchange mechanisms but just price adjustment mechanisms in these models [9]. As a result, money is neutral in exchange,
- (planned) excess demand is always equal to zero in the current period referred often to as short run equilibrium. In multi-period models agents make expectations on future prices. The situation where planned excess demand is zero in all periods is called long run equilibrium. The suite of short run equilibriums is called stationary equilibrium.

One can easily check that in general in the Ayres-Martinás dynamic model there is no equilibrium as in standard models even if (as we have also chosen the specification of the Ayres-Martinás model)

1. *the evaluation function* (utility function in the standard model and the wealth function in the Ayres-Martinás model) *is well behaved* (strictly quasi-concave, strictly increasing twice differentiable function); and the production technology inhibits the Arrow-Debreu

[10, 11] model's assumptions (production set determined by the technology is nonempty, closed, convex, irreversible, possibility of inactivity, no output without using some inputs),

2. *sectors are symmetrical*, that is to say:
   - they have the same production technology $a_1^{\ 1} = a_2^{\ 2}$, $a_2^{\ 1} = a_1^{\ 2}$ and $a_L^{\ 1} = a_L^{\ 2}$,
   - they have the same reaction parameters K and N,
   - they consume the same proportion c,
   - they have the same stocks in symmetry i.e.: $y_1^{\ 1} = y_2^{\ 2}$, $y_2^{\ 1} = y_1^{\ 2}$, L and M are the same,
3. *exchange takes place only at equilibrium prices* i.e. while prices evolve in a sub-period stocks do not change.

A typical path of the economy with the above assumptions is as shown in Fig. 1.

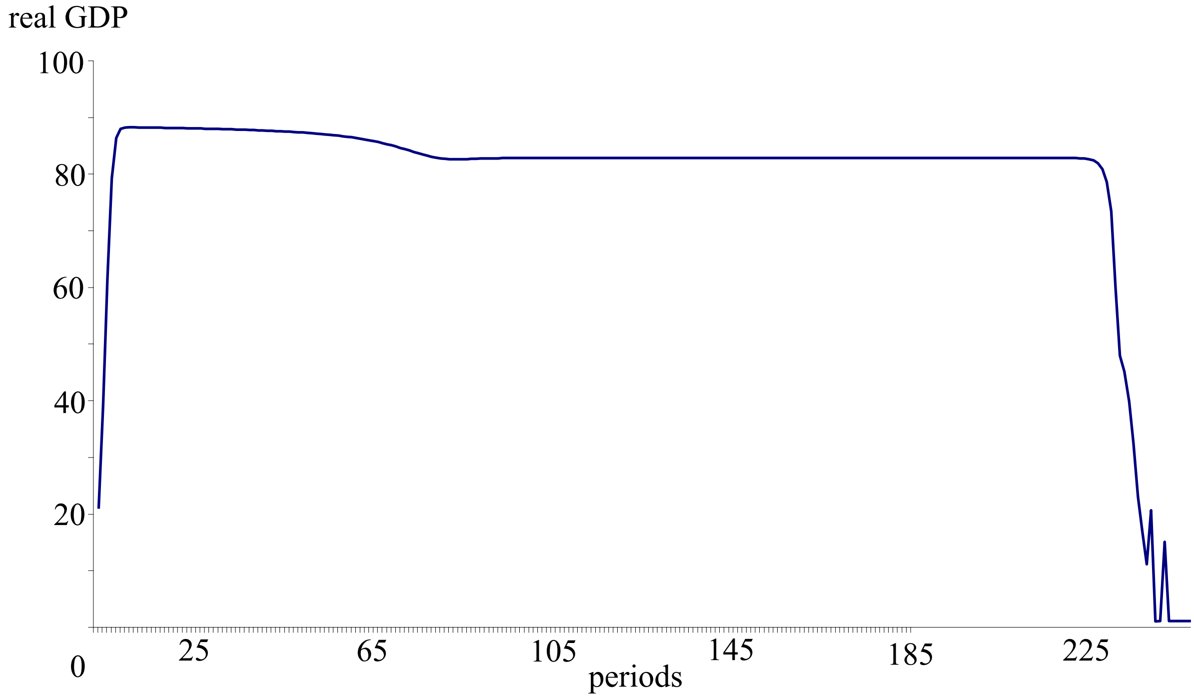

Fig. 1. Typical economic path.

That is to say, after a while, economy collapses because of the non-appropriate (non proportional) change in stocks. An appropriate change (in our case increase) of money supply permits to avoid the collapse of the economy. As an illustration, we show in the figure below an appropriate (i.e.: permits production in the long run) and an excessive increase in the money stocks, Fig. 2.

That figure points that the elimination of money balance effects in exchange by the assumption of exchange at equilibrium prices is not sufficient to assure, in general, stationary equilibrium even with the above enumerated additional simplifications. Neutrality of money (i.e.: no effect on exchange and/or production decisions) should be assured also for production decisions to obtain the standard economic findings.

This is the case of standard profit maximising behaviour. Formulated in the Ayres-Martinás framework for sector 1:

$$\max_{y_1^v y_2^v, L^v} y_1^v, y_2^v L^v M$$

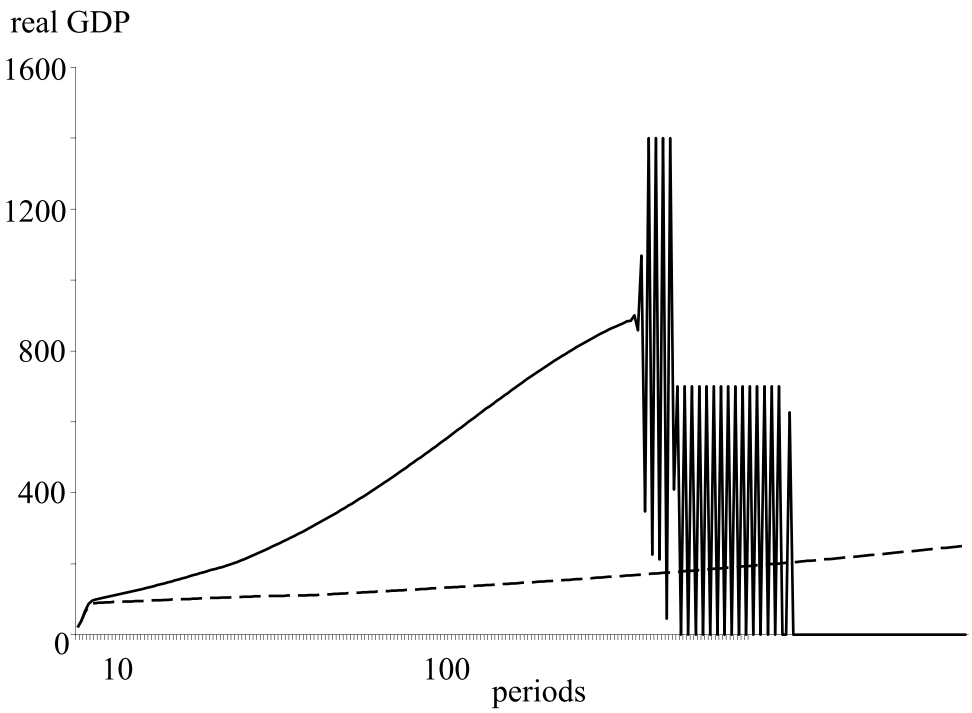

Fig. 2. Expansive monetary policy. Solid (dashed) line denotes high (low) expansion of M.

$$s.t.:\begin{pmatrix} y_1 \\ y_2 \\ L \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_L \end{pmatrix} l$$

$$y^{v}{}_{j} = y^{i}{}_{j} + yj, \quad j = 1, 2,$$
$$L^{v} = L_{i} + L.$$

where $y^{v}{}_{j}$ is the stock of sector 1 at the end of the production period in product j, $y^{i}{}_{j}$ is the endowment of sector 1 at the beginning of the production sub-period and y_j is the change of stock j during the production process (l is the level of production already defined).

Without loss of generality we suppose that $a_1 = 1$. Hence, we can transcribe the above problem in the following manner:

$$\max_{y_1^v y_2^v, L^v} y_1^v, y_2^v L^v M$$

$$s.t.: y_j = \min\left\{\frac{y_2}{a_2}; \frac{L}{a_L}\right\}$$

$$y^{v}{}_{j} = y^{i}{}_{j} + y_{j}, \quad j = 1, 2,$$
$$L^{v} = L^{i} + L.$$

It is straightforward that the in optimum production does not depend on the quantity of money. In that case, the standard result on the existence of stationary equilibrium holds.

PRODUCTIVITY AND GROWTH: SOME RESULTS

Standard economic model predicts that a ceteris paribus increase in the overall productivity increases production in the short and in the long run. As we have seen this result is true in every case when stocks change proportionally (or do not change). This can be assured for all cases either

- by assuming that consumption is always so that leaves stocks at the level of the beginning of the period; or
- by assuming that money is neutral. This is assured for example if we assume that firms maximise profits for one period (given their expectations on future).

If we omit the above hypotheses, we cannot assure, in general, the existence of stationary equilibrium and hence production collapses after a time. That is to say, in the short run it is always true that an increase in overall productivity increases production. But in the long run production can decrease because of disproportional variations.

However, if the parameters (production response intensity K, exchange response intensity N, technology a, initial stocks) of the considered model are so that there exists stationary equilibrium we can say a little bit more. In that case, still referring to the symmetric case, a “small” increase in productivity gives the traditional result. The stationary equilibrium, depending on the concrete values of the parameters, is more or less sensitive to these parameters. However, in any case when the symmetric behaviour of the sectors is broken (K, N is different) the initial stationary equilibrium brakes down with an overall productivity increase resulting in the collapse of production.

A straightforward solution to avoid the collapse of production is to consider not a *ceteris paribus* change in the overall productivity, but a simultaneous change in the money supply. For, in the Ayres-Martinás model, production depends on the evaluation of agents, which in turn depends on the money holdings. Hence, the variation of money holdings also effects production.

CONCLUSIONS

Following the proposition of standard economics a ceteris paribus increase in overall productivity increases production. We have examined this proposition in the light of Ayres-Martinás works. These authors have re-founded the usual behavioural assumptions on economic agents; their theory incorporating as a special case the standard economic theory.

We have found that this standard proposition holds for the short run, but for the long run, in general, without an appropriate monetary policy accompanying the productivity increase, production decreases in the long run even if the economy departs from a stationary equilibrium state. The reason for this is the following: all economic decisions depend on how agents evaluate their stocks. As money holdings play in that evaluation and, in general, non-proportional changes occur, some agents loose all their money holdings and cannot take part any more in the production process.

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REMARKS

[1]For example, balance sheet approaches [6, 12 – 14].

[2]Opposed to the balance sheet approach following which this cannot be done.

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NAPOMENE O UČINKU PRODUKTIVNOSTI NA RAST

Z. Gilányi

Odjel za mikroekonomiju, Korvinovo sveučilište u Budimpešti
Budimpešta, Madžarska

SAŽETAK

Standardni ekonomski modeli predvidađaju kako, uz sve ostale uvjete nepromijenjene, porast opće produktivnosti rezultira povećanom proizvodnjom ako se ekonomski sustav udaljuje iz ravnotežnog stanja. U radu je pokazano kako taj rezultat vrijedi uz određene uvjete. U ostalim slučajevima, čak i kad su početni uvjeti ekonomskog sustava takvi da ekonomija teži (ili započinje u) ravnotežnom stanju, porast opće produktivnosti dugoročno rezultira kolapsom ekonomije zbog neuravnotežene promjene u čuvanju novca. Zbog toga u općem slučaju porast produktivnosti mora biti praćen približno ekspanzivnom monetarnom politikom.

KLJUČNE RIJEČI

ekonomski rast i agregirana produktivnost, višesektorski rast, monetarni rast

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ECONOMIC GROWTH WITH LEARNING BY PRODUCING, LEARNING BY EDUCATION AND LEARNING BY CONSUMING

Wei-Bin Zhang

College of Asia Pacific Management, Ritsumeikan Asia Pacific University
Oita Prefecture, Japan

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SUMMARY

This paper proposes a dynamic economic model with wealth accumulation and human capital accumulation. The economic system consists of one production sector and one education sector. We take account of three ways of improving human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect competition. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examine effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system.

KEY WORDS

learning by producing, learning by consuming, learning by education, economic growth, education production

CLASSIFICATION

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Corresponding author, η: wbz1@apu.ac.jp; +81 977 73 9787, +81 977 78 1020;
Ritsumeikan Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-ken 874-8577, Japan

INTRODUCTION

Dynamic interdependence between economic growth and human capital is currently a main topic in economic theory. This study attempts to provide another contribution to the literature by examining interdependence between savings and research within a new approach to consumers' behavior with endogenous saving). This study attempts to provide another contribution to the literature by examining interdependence between savings and education within a new approach to consumers' behavior with endogenous saving.

Our model is built upon the three main growth models – Solow's one-sector growth model [1, 2], Arrow's learning by doing model [3], and the Uzawa-Lucas's growth model with education [4] – in the growth literature. The main mechanisms of economic growth in these three models are integrated into a single framework. One of the first seminal attempts to render technical progress endogenous in growth models was made by Arrow in 1962. He emphasized one aspect of knowledge accumulation - learning by doing [3]. Uzawa [4] introduced a sector specifying in creating knowledge into growth theory. The knowledge sector utilizes labor and the existing stock of knowledge to produce new knowledge, which enhances productivity of the production sector. In 1981 Schultz emphasized the incentive effects of policy on investment in human capital [5]. There are many other studies on endogenous technical progresses. But on the whole theoretical economists had been relatively silent on the topic from the end of the 70s until the publication of Romer's 1986 paper. The literature on endogenous knowledge and economic growth have increasingly expanded since Romer re-examined issues of endogenous technological change and economic growth in his 1986's paper, e.g., [6 - 9]. Since then various other issues related to innovation, diffusion of technology and behavior of economic agents under various institutions have been discussed in the literature. There are also many other models emphasizing different aspects, such as education, trade, R&D policies, entrepreneurship, division of labor, learning through trading, brain drain, economic geography, of dynamic interactions among economic structure, development and knowledge. This study is to model interaction between physical capital and human capital accumulation by taking account of Arrow's learning by doing, Uzawa-Lucas's learning through education, and Zhang's learning by consuming.

Our purpose is to combine the economic mechanisms in the three key growth models - Solow's growth model, Arrow's learning by doing model, the Uzawa-Lucas education model into a single comprehensive framework. The synthesis of the three growth models within a single framework is still analytically tractable because we propose an alternative approach to consumers' behavior. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and human capital accumulation. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect economic competition. Section 3 examines dynamic properties of the model. We simulate the model to demonstrate effects of changes in some parameters on the economic system. Section 5 concludes the study.

BASIC MODEL

The economy has one production sector and one education sector. Most aspects of the production sector are similar to the standard one-sector growth model, see [10 - 12]. It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use inputs such as labor with varied levels of human capital, different kinds of capital, knowledge and natural resources to produce material goods or services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We assume a homogenous and fixed population N_0. The labor force is distributed between the two sectors. We select commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that wage rates are identical among all professions. We introduce

$F_i(t)$ – output level of the production sector at time t,

$K(t)$ – level of capital stocks of the economy,

$H(t)$ – level of human capital of the population,

$N_i(t)$ and $K_i(t)$ – labor force and capital stocks employed by the production sector, respectivelly,

$N_e(t)$ and $K_e(t)$ – labor force and capital stocks employed by the education sector, respectivelly,

$T(t)$ and $T_e(t)$ – work time and study time, respectivelly,

$p(t)$ – price of education (service) per unit of time, and

$w(t)$ and $r(t)$ – wage rate and rate of interest, respectivelly.

Total capital stock $K(t)$ is allocated between the two sectors. As full employment of labor and capital is assumed, we have

$$K_i(t) + K_e(t) = K(t), \; N_i(t) + N_e(t) = N(t)$$

in which $N(t) \equiv T(t) \cdot N_0$, where $N(t)$ is the total work time of the population. We may rewrite previous relations as follows

$$n_i(t) k_i(t) + n_e(t) k_e(t) = k(t), \; n_i(t) + n_e(t) = 1 \tag{1}$$

in which

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad n_j(t) \equiv \frac{N_j(t)}{N(t)}, \quad k(t) \equiv \frac{K(t)}{N(t)}, \; \mathrm{j} = \mathrm{i, e}. \tag{1}$$

THE PRODUCTION SECTOR

We assume that production is to combine 'qualified labor force' $H^m(t) \cdot N_i(t)$ and physical capital $K_i(t)$. We use the conventional production function to describe a relationship between inputs and output. The function $F_i(t)$ defines the flow of production at time t. The production process is described by

$$F_i(t) = A_i K_i^{\alpha_i}(t) \left[H^m(t) N_i(t)\right]^{\beta_i}, \quad A_i, \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1.$$

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rate are determined by markets. Hence, for any individual firm $r(t)$ and $w(t)$ are given at each point of time. The production sector chooses the two variables $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)} = \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)} = \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}, \tag{2}$$

where δ_k is depreciation rate of physical capital.

ACCUMULATION OF HUMAN CAPITAL AND THE EDUCATION SECTOR

We assume that there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow first introduced learning by doing into growth theory [3]; Uzawa took account of trade offs between investment in education and capital accumulation [4], and Zhang introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory [13, 14]. We propose that human capital dynamics is given by

$$\dot{H} = \frac{\upsilon_{\mathrm{e}} F_{\mathrm{e}}^{a_{\mathrm{e}}} \left(H^{\mathrm{m}} T_{\mathrm{e}} N_0\right)^{b_{\mathrm{e}}}}{H^{\pi_{\mathrm{e}}} N_0} + \frac{\upsilon_{\mathrm{i}} F_{\mathrm{i}}^{a_{\mathrm{i}}}}{H^{\pi_{\mathrm{i}}} N_0} + \frac{\upsilon_{\mathrm{h}} C^{a_{\mathrm{h}}}}{H^{\pi_{\mathrm{h}}} N_0} - \delta_{\mathrm{h}} H \,, \tag{3}$$

where δ_k (>0) is the depreciation rate of human capital, υ_e, υ_i, υ_h, a_e, b_e, a_i and a_h, are non-negative parameters. The signs of the parameters π_e, π_i and π_h are not specified as they can be either negative or positive.

The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term

$$\frac{\upsilon_{\mathrm{e}} F_{\mathrm{e}}^{a_{\mathrm{e}}} \left(H^{\mathrm{m}} T_{\mathrm{e}} N_0\right)^{b_{\mathrm{e}}}}{H^{\pi_{\mathrm{e}}} N_0},$$

describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_e, and in the (qualified) total study time, $H^m \cdot T_e \cdot N_0$. The population N_0 in the denominator measures the contribution in terms of per capita. The term H^{π_e} indicates that as the level of human capital of the population increases, it may be more difficult (in the case of π_e being large) or easier (in the case of π_e being small) to accumulate more human capital via formal education. The term N_0 in the denominator term measures the contribution in terms of per capita. We take account of learning by producing effects in human capital accumulation by the term $\upsilon_i \cdot F_i^{a_i}/H^{\pi_i}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_i and is dependent on the level of human capital. The term H^{π_i} takes account of returns to scale effects in human capital accumulation. The case of $\pi_e > (<)\ 0$ implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital. We take account of learning by consuming by the term $\upsilon_h \cdot C^{a_h}/(H^{\pi_h} N_0)$. This term can be interpreted similarly as the term for learning by producing.

It should be noted that in the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (see [12])

$$\dot{H}(t) = H(t)^{\eta} G[T_{\mathrm{e}}(t)] \,,$$

where the function $G(\cdot)$ is increasing as the effort rises with $G(0) = 0$. In the case of $\eta < 1$, there is diminishing return to the human capital accumulation. This formation is due to Lucas [7]. As $\dot{H}/H < H^{\eta-1}G(1)$, we conclude that the growth rate of human capital must eventually tend to zero no matter how much effort is devoted to accumulating human capital. Uzawa's model may be considered a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. It seems reasonable to consider diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all – as though each additional percentage increment were harder to gain than the preceding one. Solow adapts the Uzawa formation to the following form

$$\dot{H}(t) = H(t) \cdot \kappa \cdot T_{\mathrm{e}}(t) \,.$$

This is a special case of the previous equation. The new formation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes. This results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $G_H(t) = \kappa$ (human capital grows at its maximum rate as results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. As remarked by Solow, the above formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital (i.e., $\dot{H}(t)$), and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education. Moreover, we treat teaching also as a significant factor in human capital accumulation. Efforts in teaching are neglected in Uzawa-Lucas model.

We assume that the education sector is also characterized of perfect competition. Here, we neglect any government's financial support for education. Indeed, it is important to introduce government's intervention in education. Students are supposed to pay the education fee $p(t)$ per unit of time. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w(t) \cdot N_e(t) + r(t) \cdot K_e(t)$. The total education service is measured by the total education time received by the population, $T_e \cdot N_0$. The production function of the education sector is assumed to be a function of $K_e(t)$ and $N_e(t)$. We specify the production function of the education sector as follows

$$F_e(t) = A_e K_e^{\alpha_e}\left(H^m N_e\right)^{\beta_e}, \quad \alpha_e, \beta_e > 0, \quad \alpha_e + \beta_e = 1, \tag{4}$$

where A_e, α_e and β_e are positive parameters. The education sector maximizes the following profit

$$\pi(t) = p(t) A_e K_e^{\alpha_e}\left(H^m N_e\right)^{\beta_e} - \left(r(t) + \delta_k\right) K_e(t) - w(t) N_e(t).$$

For given $p(t)$, $H(t)$, $r(t)$ and $w(t)$ the education sector chooses $K_e(t)$ and $N_e(t)$ to maximize the profit. The optimal solution is given by

$$r + \delta_k = \frac{\alpha_e p F_e}{K_e} = \alpha_e A_e p H^{m\beta_e} k_e^{-\beta_e}, \quad w(t) = \frac{\beta_e p F_e}{N_e} = \beta_e A_e p H^{m\beta_e} k_e^{\alpha_e}. \tag{5}$$

The demand for labor force for given price of education, wage rate and level of human capital is given by

$$N_e = K_e\left(\frac{\beta_e A_e p H^{m\beta_e}}{w}\right)^{1/\alpha_e}.$$

We see that the demand for labor force from the education sector increases in the price and level of human capital and decreases in the wage rate.

CONSUMER BEHAVIORS

Consumers make decisions on choice of consumption levels of services and commodities as well as on how much to save. Different from the optimal growth theory in which utility defined over future consumption streams is used, we assume that we can find preference structure of consumers over consumption and saving at the current state. The preference over current and future consumption is reflected in the consumer's preference structure over current consumption and saving. We denote *per capita* wealth by $\bar{k}(t)$, where $\bar{k}(t) \equiv k(t)/N_0$. By the definitions, we have $\bar{k}(t) = T(t) \cdot k(t)$. *Per capita* current income from the interest payment $r(t) \cdot \bar{k}(t)$ and the wage payment $T(t) \cdot w(t)$ is given by

$$y(t) = r(t)\bar{k}(t) + T(t)w(t).$$

We call $y(t)$ the current income in the sense that it comes from consumers' daily toils (payment for human capital) and consumers' current earnings from ownership of wealth. The current income is equal to the total output. The sum of money that consumers are using for consuming, saving, and education are not necessarily equal to the temporary income because consumers can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + \bar{k}(t) = [1 + r(t)]\bar{k}(t) + T(t)w(t). \tag{6}$$

The disposable income is used for saving, consumption, and education. At each point of time, a consumer would distribute the total available budget among saving $s(t)$, consumption of goods $c(t)$, and education $p(t)\cdot T_e(t)$. The budget constraint is given by

$$c(t) + s(t) + p(t)T_e(t) = \hat{y}(t) = (1 + r(t))\bar{k}(t) + T(t)w(t). \tag{7}$$

The consumer is faced with the following time constraint

$$T(t) + T_e(t) = T_0,$$

where T_0 is the total available time for work and study. Substituting this function into the budget constraint (7) yields

$$c(t) + s(t) + (p(t) + w(t))T_e(t) = \bar{y}(t) \equiv (1 + r(t))\bar{k}(t) + T_0 w(t). \tag{8}$$

In our model, at each point of time, consumers have three variables, the level of consumption, the level of saving, and the education time, to decide. We assume that consumers' utility function is a function of level of goods $c(t)$ and level of saving $s(t)$ and education service $T_e(t)$ as follows

$$U(t) = U(c(t), s(t), T_e(t)).$$

The utility function can be considered as a function of $c(t)$, $s(t)$ and $T_e(t)$. For simplicity of analysis, we specify the utility function as follows

$$U(t) = c^{\xi}(t)s^{\lambda}(t)T_e^{\eta}(t), \quad \xi, \lambda, \eta > 0; \quad \xi + \lambda + \eta = 1, \tag{9}$$

where ξ is called the propensity to consume, λ the propensity to own wealth, and η the propensity to obtain education. This utility function is applied to different economic problems [13, 15]. A detailed explanation of the approach and its applications to different problems of economic dynamics are provided in [16].

For the representative consumer, wage rate $w(t)$ and rate of interest $r(t)$ are given in markets and wealth $\bar{k}(t)$ is predetermined before decision. Maximizing $U(t)$ in (9) subject to the budget constraint (8) yields

$$c = \xi\bar{y}, \quad s = \lambda\bar{y}, \quad (p + w)T_e = \eta\bar{y}. \tag{10}$$

The demand for education is given by $T_e = \eta\bar{y}/(p + w)$. The demand for education decreases in the price of education and the wage rate and increases in $\bar{y}$. An increase in the propensity to get educated increases the education time when the other conditions are fixed. In this dynamic system, as any factor is related to all the other factors over time, it is difficult to see how one factor affects any other variable over time in the dynamic system. We will demonstrate complicated interactions by simulation.

We now find dynamics of capital accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda\bar{y}(t) - \bar{k}(t). \tag{11}$$

For the education sector, the demand and supply balances at any point of time

$$T_e N_0 = F_e(t). \tag{12}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t) + S(t) - K(t) + \delta_k K(t) = F_i(t) \tag{13}$$

where $C(t)$ is the total consumption, $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation. We have

$$C(t) = c(t)N_0, \quad S(t) = s(t)N_0 .$$

It is straightforward to show that this equation can be derived from the other equations in the system. We have thus built the dynamic model. We now examine dynamics of the model.

DYNAMICS AND ITS PROPERTIES

This section examines dynamics of the model. First, we show that the dynamics can be expressed by the two-dimensional differential equations system with $k_i(t)$ and $H(t)$ as the variables.

LEMMA

The dynamics of the economic system is governed by the two-dimensional differential equations

$$\dot{k}_i(t) = \tilde{\Omega}_i(k_i,\ H),$$
$$\dot{H}(t) = \tilde{\Omega}_h(k_i,\ H), \tag{14}$$

where the functions $\tilde{\Omega}_i(k_i,\ H)$ and $\tilde{\Omega}_h(k_i,\ H)$ are functions of $k_i(t)$ and $H(t)$ defined in (A10) and (A13) in the Appendix. Moreover, all the other variables can be determined as functions of $k_i(t)$ and $H(t)$ at any point of time by the following procedure: $\bar{k}(t) = \varphi_0(k_i,\ H)\varphi(k_i,\ H)$ (where φ_0 and φ are defined respectively in (22) and (21)) → $k_h(t) = \alpha k_i(t)$ → $T(t)$ and $\bar{y}(t)$ by (A9) → $k(t)$ by (A8) → $p(t)$ by (A2) → $n_i(t)$ and $n_h(t)$ by (A3) → $r(t)$ and $w(t)$ by (2) → $c(t)$, $T_e(t)$, and $s(t)$ by (10) → $N(t) = N_0 T(t)$ → $N_j(t) = n_j(t)N(t)$ (j = i, e), → $K(t) = k(t)N(t)$ → $K_j(t) = k_j(t)N_j(t)$ → $F_j(K_j(t),\ N_j(t))$.

The differential equations system (14) contains two variables $k_i(t)$ and $H(t)$. Although we can analyze its dynamic properties as we have explicitly expressed the dynamics, we omit analyzing the model as the expressions are too complicated. Instead, we simulate the model to illustrate behavior of the system. In the remainder of this study, we specify the depreciation rates by δ_k = 0,05; δ_h = 0,04 and let T_0 = 1. The requirement T_0 = 1 will not affect our analysis. We specify the other parameters as follows

$$\begin{aligned}
&\alpha_i = 0{,}3;\ \ \alpha_e = 0{,}34;\ \ \lambda = 0{,}8;\ \ \eta = 0{,}008;\ \ N_0 = 50\ 000;\ \ m = 0{,}6;\\
&v_e = 0{,}8;\ \ v_i = 2{,}5;\ \ v_h = 0{,}7;\ \ \pi_e = -0{,}2;\ \ \pi_i = 0{,}7;\ \ \pi_h = 0{,}1;\\
&a_e = 0{,}3;\ \ b_e = 0{,}5;\ \ a_i = 0{,}4;\ \ a_h = 0{,}1;\ \ A_i = A_e = 0{,}9.
\end{aligned} \tag{15}$$

The propensity to save λ is 0,8 and the propensity to consume education is 0,008. The propensity to consume goods ξ = 1 – 0,8 – 0,008 = 0,192. The technological parameters of the two sectors are specified at $A_i = A_e$ = 0,9. The specification m = 0,6 implies that there is a diminishing effect in turning human capital to labor force. The conditions π_e = – 0,2; π_i = 0,7 and π_h = 0,1 mean respectively that the learning by education exhibits increasing effects in human capital; the learning by producing exhibits (strong) decreasing effects in human capital; and the learning by consuming exhibits (weak) increasing effects in human capital.

By (14), an equilibrium point of the dynamic system is given by

$$\tilde{\Omega}_i(k_i,\ H) = 0,$$

$$\widetilde{\Omega}_h(k_i,\ H) = 0\,. \tag{16}$$

Simulation demonstrates that the above equations have the following unique equilibrium solution

$$k_i = 7{,}9643,\ H = 1{,}1716\,.$$

The equilibrium values of the other variables are given by the procedure in Lemma. We list them as follows

$$r = 0{,}0175;\quad w = 1{,}255;\quad p = 0{,}920;\quad H = 1{,}172;\quad N = 32138{,}7$$

$$\begin{pmatrix} f_i \\ f_e \end{pmatrix} = \begin{pmatrix} 1{,}793 \\ 1{,}790 \end{pmatrix},\ \begin{pmatrix} n_i \\ n_e \end{pmatrix} = \begin{pmatrix} 0{,}731 \\ 0{,}269 \end{pmatrix},\ \begin{pmatrix} k_i \\ k_e \end{pmatrix} = \begin{pmatrix} 7{,}964 \\ 9{,}573 \end{pmatrix},\ \begin{pmatrix} F_i \\ F_e \end{pmatrix} = \begin{pmatrix} 42109{,}7 \\ 16778{,}1 \end{pmatrix},\ \begin{pmatrix} K_i \\ K_e \end{pmatrix} = \begin{pmatrix} 187\,092 \\ 82\,781{,}4 \end{pmatrix},$$

$$k = 8{,}397;\quad T = 0{,}643;\quad T_e = 0{,}357;\quad c = 1{,}295.$$

The consumer spends about 35,7 % of the total available time for study. The relative importance of the education sector is given by the following variables

$$\frac{pF_e}{F_i + pF_e} = 0{,}268;\quad \frac{K_e}{K_i + K_e} = 0{,}307;\quad \frac{N_e}{N_i + N_e} = 0{,}269.$$

The relative share of the education sector is about 26,8 % percent of the national product. The share seems to be large if one considers any real economy. As we are mainly concerned with effects of changes in some parameters, it seems that these unrealistic shares will not affect our main conclusions about comparative dynamic analyses.

We are now concerned with motion of the system. We specify initial conditions for the differential equations (14) as follows

$k_i(0) = 7{,}32$ and $H(0) = 0{,}7$.

As shown in Figure 1, only one variable monotonously changes – the level of the human capital monotonously increases from the initial state to the equilibrium value. The economic development experiences a kind of *J*-curve process. It first experiences declination in per capita levels of consumption and wealth. After a few years these variables start to increase. The wage slightly declines and soon begins to increase. During the simulation period, the price of education increases and then starts to decline. It should be remarked that the price of education changes only slightly during the whole period. The education time also experiences a *J*-curve change during the study period. It first declines as the price increases and the real wage rate declines. The rate of interest increases and then starts to decline.

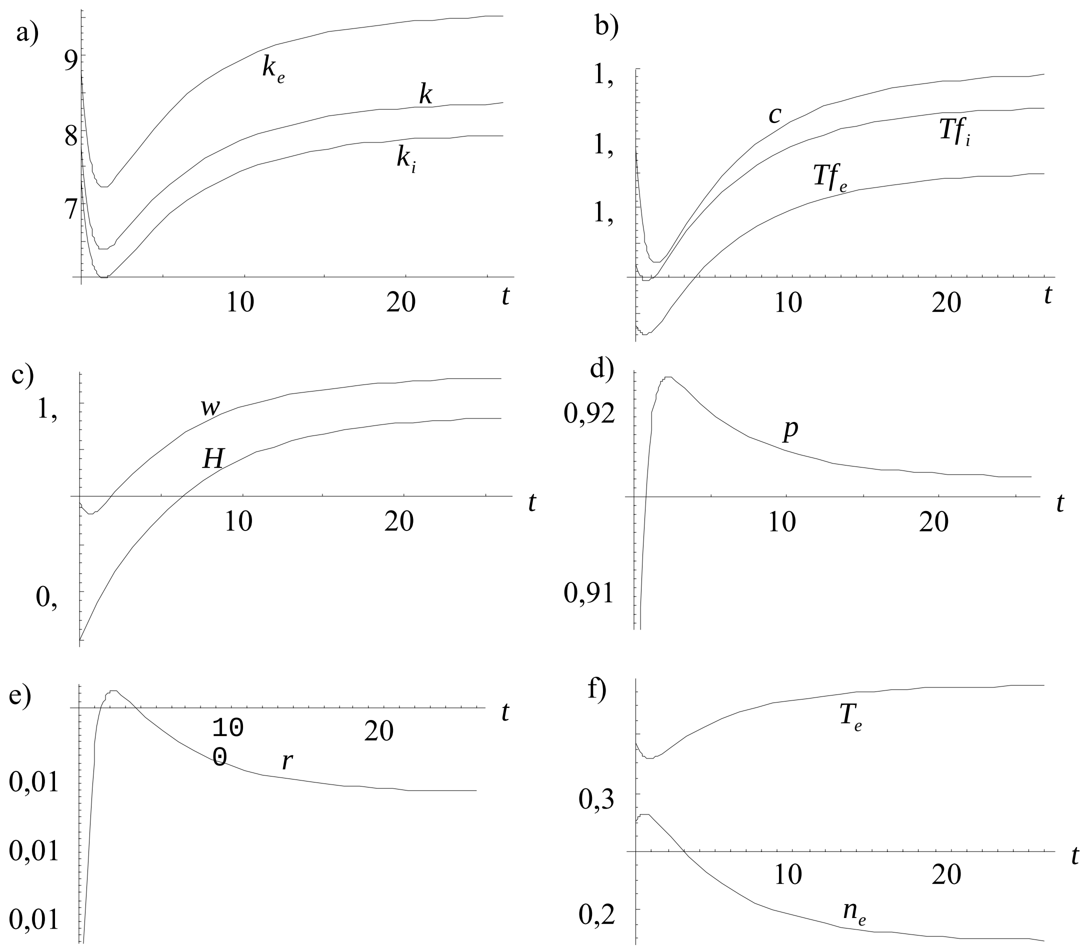

Figure 1. The motion of the economic system. Graph a) shows the capital intensities for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (H), d) the price of education p, e) the rate of interest r; and f) the study time T_e and the sectorial share of labor force n_e. Values of parameters are as in (15).

COMPARATIVE DYNAMIC ANALYSIS IN SOME PARAMETERS BY SIMULATION

We now examine impact of changes on dynamic processes of the system. First, we examine the case that all the parameters, except the education efficiency parameter, A_e, are the same as in (15). We increase the education efficiency parameter from $A_e = 0{,}9$ to $A_e = 1{,}2$. The simulation results are demonstrated in Figure 2. The solid lines in Figure 2 are the same as in Figure 1, representing the values of the corresponding variables when $A_e = 0{,}9$; the dashed lines in Figure 2 represent the new values of the variables when $A_e = 1{,}2$. We see that as the education sector improves its productivity, the price of education will be reduced and the study time is increased. The level of human capital increases and the wage rate is increased. The per capita levels of consumption and wealth are increased. The share of the labor force of the education sector in the total labor force declines as the productivity of the education sector is improved. The per capita level of the education sector's output is increased and the per capita level of the production sector's output is reduced.

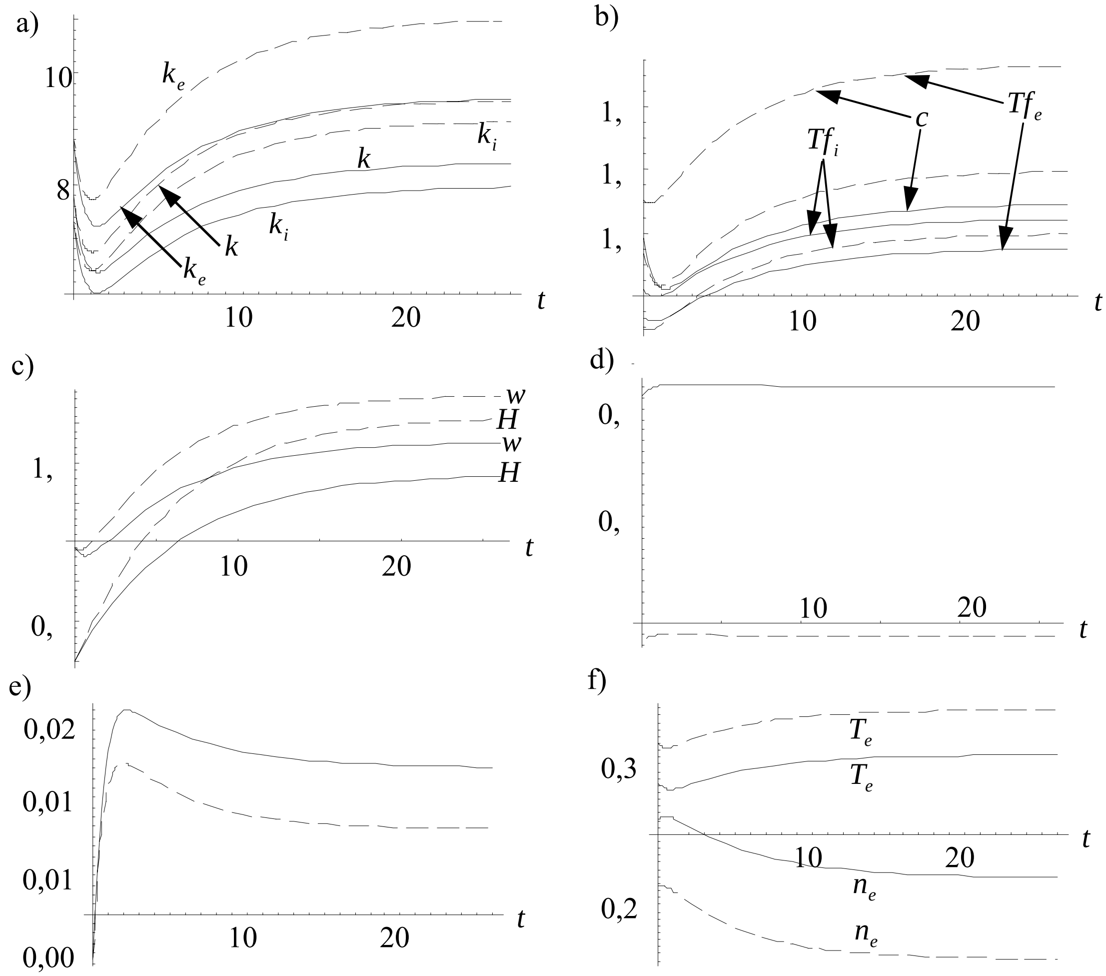

Figure 2. For A_e equal 0,9 (solid lines) and 1,2 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (H), d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

We now increase the production sector's productivity from $A_i = 0{,}9$ to $A_i = 1{,}2$. The simulation results are demonstrated in Figure 3. The solid lines in Figure 3 are the same as in Figure 1, representing the values of the corresponding variables when $A_i = 0{,}9$; the dashed lines in Figure 3 represent the new values of the variables when $A_i = 1{,}2$. We see that as the production sector improves its productivity, both the price of education is increased and the study time is increased. The effects are different from the effects due to increases in the education sector's productivity. The level of human capital increases and the wage rate is increased. The per capita levels of consumption and wealth are increased. The share of the labor force of the education sector in the total labor force declines as the productivity of the production sector is improved. The per capita levels of the education sector's output and the production sector's output are increased.

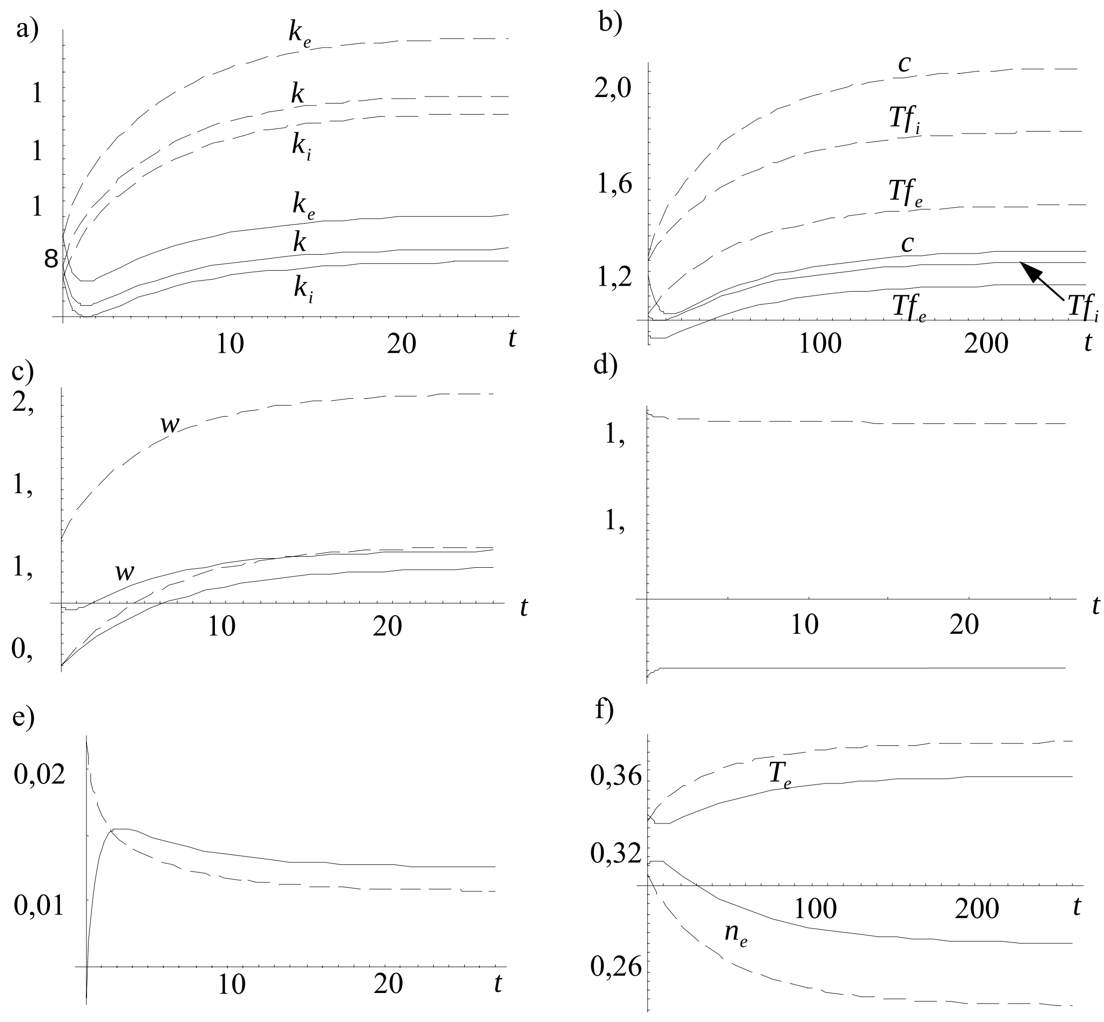

Figure 3. For A_i equal 0,9 (solid lines) and 1,2 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital (curves without letters), d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

It is important to examine effects of changes in the household's preference for education. We allow the propensity to receive education to increase from η = 0,008 to η = 0,014. The simulation results are demonstrated in Figure 4. The solid lines in Figure 4 are the same as in Figure 1, representing the values of the corresponding variables when η = 0,008; the dashed lines in Figure 4 represent the new values of the variables when η = 0,014. We see that as the household's propensity to receive education increases, the per capita level of consumption declines first and then increases. The level is only slightly increased. The per capita level of the education sector increases and that of the production sector declines as the household's preference for education is increased. The per capita level of wealth increases. The wage rate and level of human capital are increased. As the propensity to receive education is increased, the study time increases and the price level falls down.

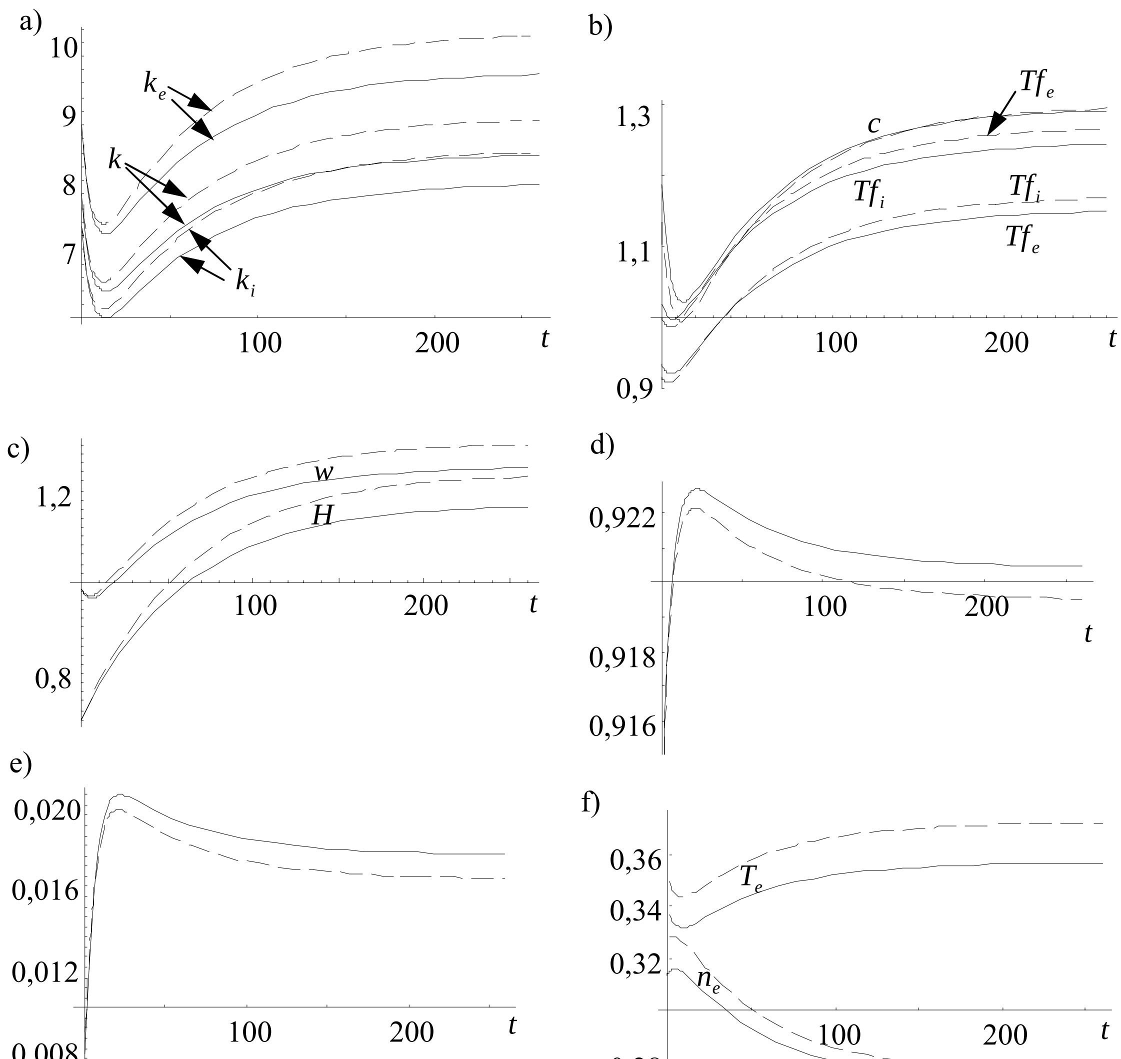

Figure 4. For η equal 0,008 (solid lines) and 0,014 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c, with solid and dashed curve almost overlapped) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

The effects of change in the population have negative effects on the living conditions, as demonstrated in Figure 5. In Figure 5, we increase the population from $N_0 = 50\ 000$ to $N_0 = 60\ 000$. We see that as the population is increased, the per capita level of consumption declines. The per capita levels of the two sectors are reduced. The per capita level of wealth declines. The wage rate and level of human capital are reduced. The study time falls and the price level rises.

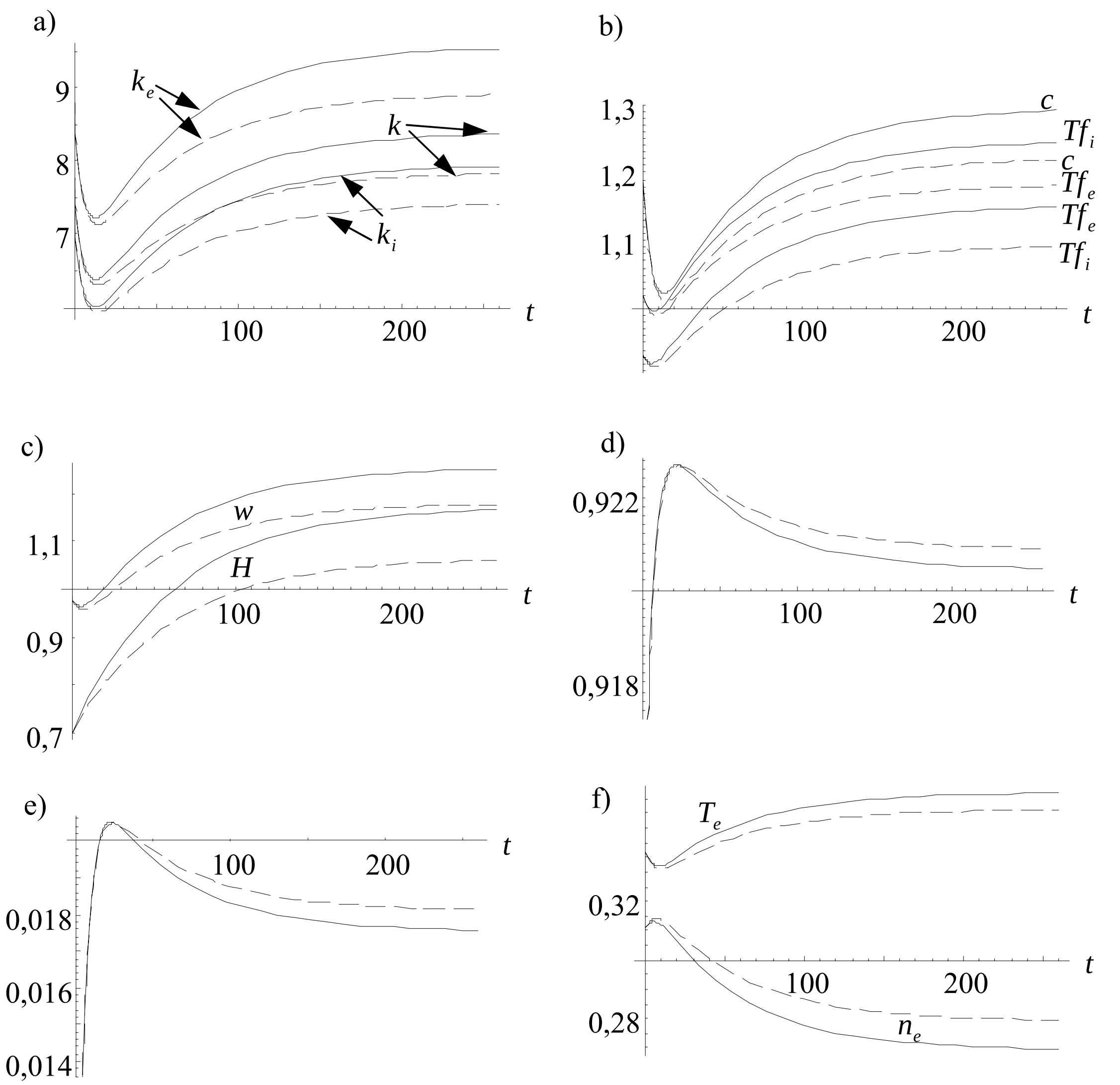

Figure 5. For N_0 equal 50 000 (solid lines) and 60 000 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c, with solid and dashed curve almost overlapped) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

In Figure 6, we show the effects of change in the efficiency of learning by consuming. We increase the efficiency parameter from $\nu_h = 0{,}7$ to $\nu_h = 10$. We see that as the efficiency of learning by consuming, the economic conditions are improved and the level of human capital is improved.

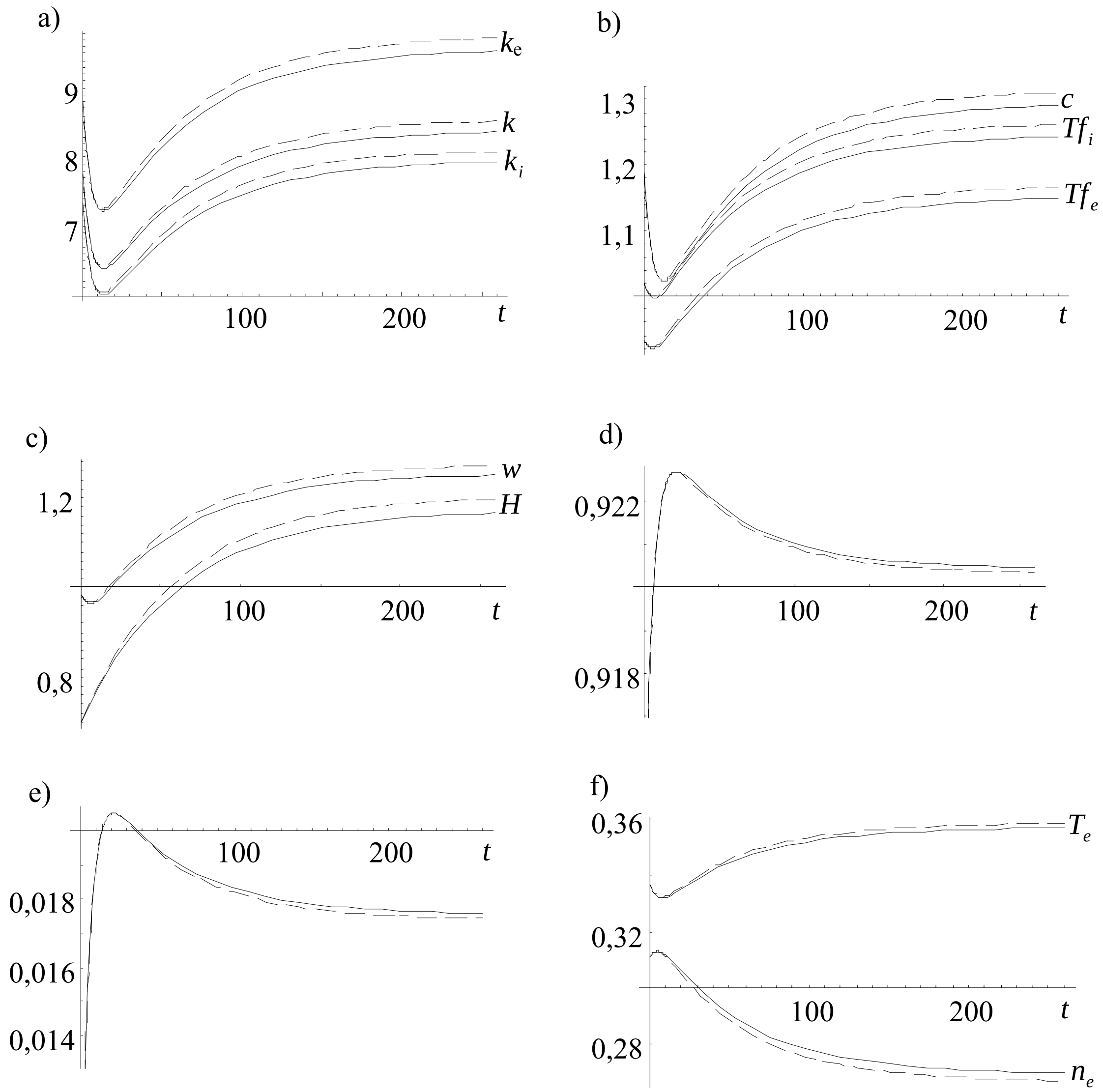

Figure 6. For ν_h equal to 0,7 (solid lines) and 10 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

In Figure 7, we show the effects of change in the efficiency of education in improving human capital. We increase the efficiency parameter from $\nu_e = 0,8$ to $\nu_e = 1$. We see that as the efficiency of learning by consuming, the economic conditions are improved and the level of human capital is improved.

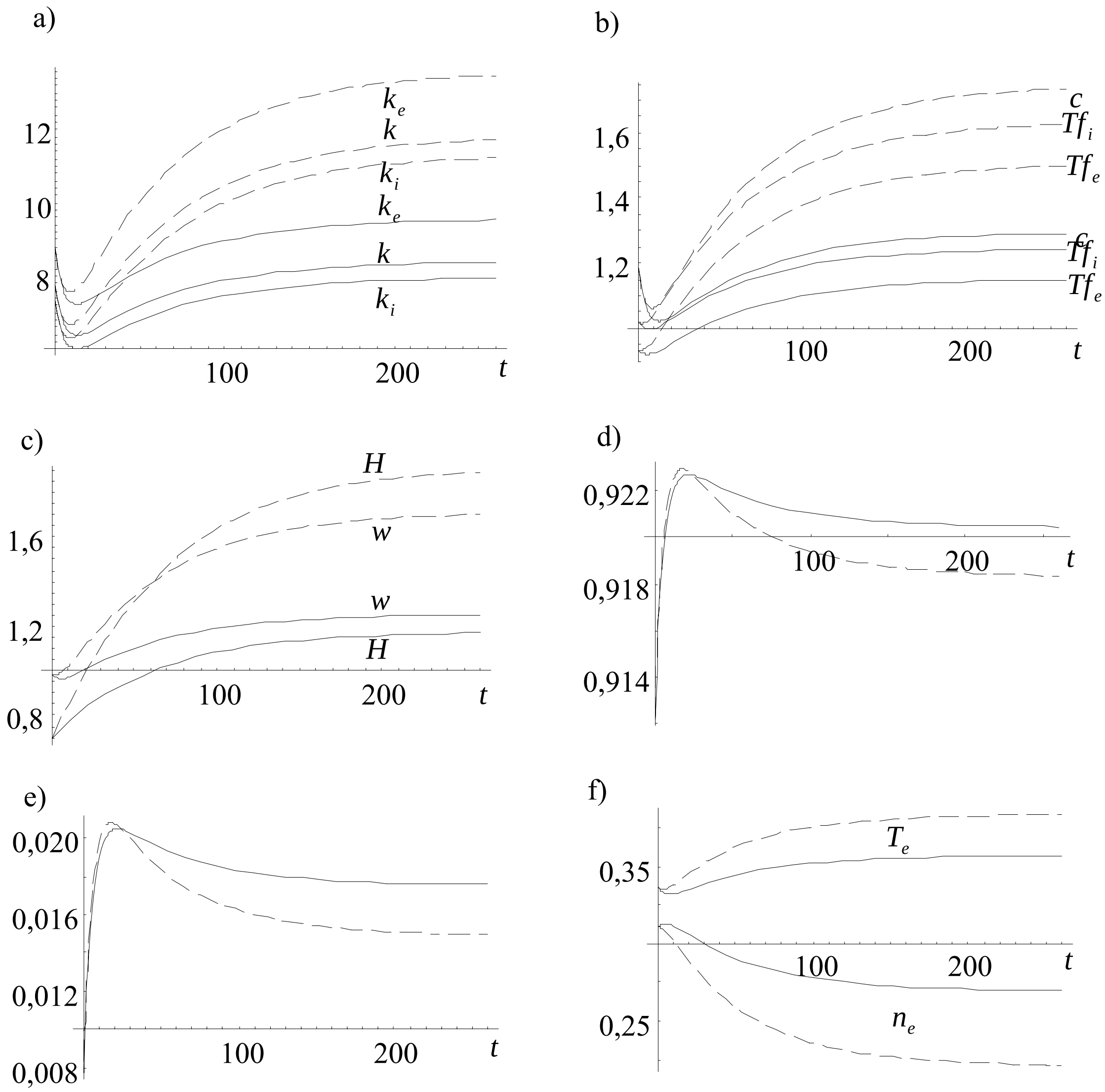

Figure 7. For ν_e equal to 0,8 (solid lines) and 1 (dashed lines) the graphs show a) the capital intensities and wealth for production (k_i) and education (k_e) and per capita wealth ($\bar{k}$), b) the per capita consumption (c) and production (Tf_i), c) the wage rate (w) and the level of human capital, d) the price of education, e) the rate of interest and f) the study time T_e and the sectorial share of labor force n_e.

CONCLUDING REMARKS

This paper proposes a dynamic economic model with wealth accumulation and human capital accumulation. The economic system consists of one production sector and one education sector. We took account of three ways of improving human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect competition. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system. We may extend the model in some directions. For instance, we may introduce some kind of government intervention in education into the model. It is also desirable to treat leisure time as an endogenous variable.

APPENDIX: THE TWO-DIMENSIONAL DIFFERENTIAL EQUATIONS

This section examines dynamics of the model. First, we show that the dynamics can be expressed by a two-dimensional differential equations system. From (2) and (3), we obtain

$$\frac{K_e(t)}{N_e(t)} = \alpha \frac{K_i(t)}{N_i(t)}, \quad \text{i.e.} \quad k_e(t) = \alpha k_i(t), \tag{A1}$$

where $\alpha \equiv \alpha_e \beta_i / \alpha_i \beta_e$ ($\neq 1$, assumed). From (A1), (2) and (4), we obtain

$$p(t) = \frac{\alpha_i A_i}{\alpha_e A_e} \alpha^{\beta_e} H^{-m\beta} k_i^{\beta}, \tag{A2}$$

where $\beta \equiv \beta_e - \beta_i$. From (A1) and (1), we solve the labor distribution as functions of $k_i(t)$ and $k(t)$

$$n_i = \frac{\alpha k_i - k}{(\alpha - 1)k_i}, \quad n_e = \frac{k - k_i}{(\alpha - 1)k_i}. \tag{A3}$$

Dividing (13) by N_0, we have

$$c + s - \delta \bar{k} = A_i T n_i H^{\beta_i m} k_i^{\alpha_i},$$

where $\delta \equiv 1 - \delta_k$. Substituting $c = \xi \bar{y}$ and $s = \lambda \bar{y}$ into the above equation yields

$$\bar{y} = \left\{ \frac{\alpha A_i H^{\beta_i m} k_i^{\alpha_i}}{\alpha - 1} + \delta k - \frac{A_i H^{\beta_i m} k}{(\alpha - 1) k_i^{\beta_i}} \right\} \frac{T}{\xi + \lambda}. \tag{A4}$$

where we use the equation for n_i in (A3) and $\bar{k} = Tk$. Insert (2) and $\bar{k} = Tk$ into the definition of $\bar{y}$ in (8)

$$\bar{y} = \left(\delta + \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i}\right) kT + T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}. \tag{A5}$$

From (A4) and (A5), we solve

$$\left(\frac{\alpha A_i H^{\beta_i m} k_i}{\alpha - 1} + \delta \eta k k_i^{\beta_i} - \frac{A_i H^{\beta_i m} k}{(\alpha - 1)} + (\xi + \lambda) \alpha_i A_i H^{m\beta_i} k \right) T = (\xi + \lambda) T_0 \beta_i A_i H^{m\beta_i} k_i. \tag{A6}$$

From (12) and (4), we have

$$T_e = A_e T n_e H^{\beta_e m} k_e^{\alpha_e}.$$

Insert $T + T_e = T_0$ and n_e in (A3) into the above equation

$$T = \left(1 + \frac{\alpha^{\alpha_e} A_e H^{\beta_e m} (k - k_i)}{(\alpha - 1) k_i^{\beta_e}} \right)^{-1} T_0. \tag{A7}$$

Substituting (A7) into (A6) yields

$$k = \varphi(k_i, H) \equiv \left(\frac{\alpha_0 \beta_i - \alpha - A k_i^{\alpha_e} H^{\beta_e m}}{(\alpha - 1)\delta \eta k_i^{\beta_i} / A_i H^{\beta_i m} - A k_i^{\alpha_e} H^{\beta_e m} - 1 + \alpha_0 \alpha_i} \right) k_i, \tag{A8}$$

where $\alpha_0 \equiv (\alpha - 1)(\xi + \lambda)$ and $A \equiv \alpha^{\alpha_e} \beta_i A_e (\xi + \lambda)$. By (A8), we can express $k(t)$ as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A7) and (A5), we can also express $T(t)$ and $\bar{y}(t)$ as functions of $k_i(t)$ and $H(t)$ as follows

$$T = \varphi_0(k_i, H) \equiv \left(1 + \frac{\alpha^{\alpha_e} A_e H^{\beta_e m}(\varphi(k_i, H) - k_i)}{(\alpha - 1)k_i^{\beta_e}}\right)^{-1} T_0,$$

$$\bar{y} = \Lambda(k_i, H) \equiv \left(\delta + \alpha_i A_i H^{m\beta_i} k_i^{-\beta_i}\right)\varphi(k_i, H)\varphi_0(k_i, H) + T_0 \beta_i A_i H^{m\beta_i} k_i^{\alpha_i}. \tag{A9}$$

These functions show that $T(t)$, $\bar{y}(t)$, $N(t)$ (with $N(t) = T(t)N_0$), and $K(t)$ (with $K(t) = k(t)N(t)$) can be treated as functions of $k_i(t)$ and $H(t)$ at any point of time. By (A3) and $N_j(t) = n_j(t)\,N(t)$, we see that the labor distribution, $n_j(t)$ and $N_j(t)$ (j = i, s), are functions of $k_j(t)$ and $H(t)$. It is straightforward to see that $F_j(t)$ and $C(t)$ can be expressed as functions of $k_j(t)$ and $H(t)$ at any point of time.

We now express dynamics of the system in terms of $k_j(t)$ and $H(t)$. First, substituting the functions $T = T_0 - T_e$, $F_j(t)$ and $C(t) = \xi N_0\,\bar{y}\,(t)$ into (3), we obtain

$$\dot{H}(t) = \tilde{\Omega}_h(k_i, H) \equiv \Omega_e(k_i, H) + \Omega_i(k_i, H) + \Omega_h(k_i, H) - \delta_h H, \tag{A10}$$

where

$$\Omega_e(k_i, H) \equiv \frac{\upsilon_e N_0^{a_e + b_e - 1} A_e^{a_e} \alpha^{a_e \alpha_e}}{(\alpha - 1)^{a_e}} (\varphi(k_i, H) - k_i)^{a_e} (T_0 - \varphi_0(k_i, H))^{b_e} \varphi_0^{a_e}(k_i, H) k_i^{-a_e \beta_e} H^{m b_e + a_e \beta_e m - \pi_e},$$

$$\Omega_i(k_i, H) \equiv \frac{\upsilon_i A_i^{a_i} N_0^{a_i - 1}}{(\alpha - 1)^{a_i}} (\alpha k_i - \varphi(k_i, H))^{a_i} \varphi_0^{a_i}(k_i, H) k_i^{-a_i \beta_i} H^{m a_i \beta_i - \pi_i},$$

$$\Omega_h(k_i, H) \equiv \upsilon_h \xi^{a_h} N_0^{a_h - 1} \Lambda^{a_h}(k_i, H) H^{-\pi_h}.$$

The one-dimensional differential equation expresses change in $H(t)$ as a function of $k_i(t)$ and $H(t)$.

We now show that change in $k_i(t)$ can also be expressed as a differential equation in terms of $k_i(t)$ and $H(t)$. First, substitute $\bar{y} = \Lambda(k_i, H)$ and $\bar{k} = Tk = \varphi_0(k_i, H)\varphi(k_i, H)$ into (11)

$$\dot{\bar{k}}(t) = \lambda\Lambda(k_i, H) - \varphi_0(k_i, H)\varphi(k_i, H). \tag{A11}$$

Taking derivatives of $\bar{k} = Tk = \varphi_0(k_i, H)\varphi(k_i, H)$ with respect to time, we have

$$\dot{\bar{k}} = \left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)\dot{k}_i + \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h. \tag{A12}$$

where we use (A10). Substituting (A12) into (A11) yields

$$\dot{k}_i = \tilde{\Omega}_i(k_i, H) \equiv \left[\lambda\Lambda - \varphi_0\varphi - \left(\frac{\partial \varphi_0}{\partial H}\varphi + \frac{\partial \varphi}{\partial H}\varphi_0\right)\tilde{\Omega}_h\right]\left(\frac{\partial \varphi_0}{\partial k_i}\varphi + \frac{\partial \varphi}{\partial k_i}\varphi_0\right)^{-1}. \tag{A13}$$

The one-dimensional differential equation (A13) expresses change in $k(t)$ as a function of $k_i(t)$ and $H(t)$. The two differential equations (A10) and (A13) contain two variables $k_i(t)$ and $H(t)$. We thus proved Lemma.

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EKONOMSKI RAST S UČENJEM IZ PROIZVODNJE, UČENJEM IZ OBRAZOVANJA I UČENJEM IZ POTROŠNJE

W.-B. Zhang

Azijsko pacifičko sveučilište Ritsumeikan
prefektura Oita, Japan

SAŽETAK

U radu je predložen ekonomski model s akumulacijom bogatstva i ljudskih resursa. Ekonomski sustav sastoji se od jednog proizvodnog i jednog obrazovnog sektora. Ujedno se uzimaju tri načina unaprijeđenja ljudskih resursa: učenje iz proizvodnje, učenje iz obrazovanja i učenje iz potrošnje. Model opisuje dinamičku povezanost između akumulacije bogatstva, ljudskih resursa i podjele posla u slučaju idealne kompeticije. Simulacijom modela demonstrirana je egzistencija ravnotežnih točaka i gibanje dinamičkog sustava. Također je ispitan učinak promjene mogućnosti obrazovanja, učinkovitosti učenja i učinkovitosti obrazovanja na dinamiku sustava.

KLJUČNE RIJEČI

učenje iz proizvodnje, učenje iz potrošnje, učenje iz obrazovanja, ekonomski rast, produkcija u obrazovanju

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ENERGY SYSTEM PLANNING ANALYSIS USING THE INTEGRATED ENERGY AND MACROECONOMY MODEL

Helena Božić

Institute Hrvoje Požar
Zagreb, Croatia

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SUMMARY

In the past, the energy planners through setting the desired level of economic growth simply used this figure as a base to which additional increases were made, dependent on changing population and supply conditions. Planning proceeded from the national, macroeconomic position, to the aggregate, sectoral and finally project levels. Such process was a virtual one-way linkage from economic growth rate to the energy sector; it is viewed in isolation from the reminder of the economy. Integration of energy system optimization model MARKAL and the macroeconomic growth model MACRO makes possible the analysis of two-way linkage between energy system and the economy. This paper presents review of relation between energy system and economy, including the basics of technology and economy oriented models and their integration in one model with applications.

KEY WORDS

model, energy system, economy, environment

CLASSIFICATION

JEL: C61, Q43, Q51

Corresponding author, η: hbozic@eihp.hr; 385 1 6326 106;
Institute for Energy *Hrvoje Požar*, Savska cesta 163, HR - 10 000 Zagreb, Croatia

INTRODUCTION

Models are usually developed to address specific questions and a therefore only suitable for the purpose and objectives they were design for. Besides many ways of classifying energy models, analytical admission distinguish engineering and economic approach. Because of strong relation between the energy system and economy, the interaction between this two modelling approaches is necessary.

RELATION BETWEEN ENERGY SYSTEM AND ECONOMY

Energy alone is not sufficient for creating the conditions for economic growth, but it is certainly necessary. Most studies of the relationship between energy use and economic development have focused on how the latter affects the former. Economic growth always leads to increased energy use, at least in the early stages of economic development. Empirical analysis demostrates the importance of energy in driving economic development [1].

The neoclassical production function attributes economic growth to increases in the size of the labour force and to the amount of capital available, as well as to increase in total factor productivity. By explicitly incorporating an energy variable in the production function, it is possible to estimate the contribution energy made to the growth of gross domestic product in several countries that grew veray rapidly in the 1980s and 1990s (the United States was included in the sample for comparison).

In every country studied, except China, the combination of capital, labour and energy contributed more to economic growth than did productivity increases. Energy contributed significantly to economic growth in all countries and was the leading driver of growth in Brazil, Turkey and Korea. Its contribution was smaller in India, China and the United States. This results suggest that energy plays a bigger role in countries at an intermediate stage of economic development, because industrial production often makes a large contribution to economic growth at this stage. The results also reflect government policies and the resource endowment of individual countries. Brazil and Mexico, where energy played the leading role in economic growth, have both industrialised rapidly. In Indonesia, the relatively low importance of energy probably reflects the country`s policy of importing sophisticated manufacturing technology via foreign direct investment.

The complementary relationship between energy use and economic growth is intuitively obvious. Less obvious is the extent to which constraints on the availability of energy and its affordability can affect economic development. Numerous studies have demostrated that energy, capital and labour can be substituted for one another to some degree. An increase in energy-input costs can be compensated by investing more in energy-efficient technology, shifting to less-intensive production or using more labour, where it is in surplus supply.

TECHNOLOGY AND ECONOMY ORIENTED MODELS

Technological models often neglect market-related decentralise behaviour of agents and cover only the energy-environment systems. Detailed technological models like MARKAL and EFOM usually characterised as bottom-up approaches, adopt a system-wide optimisation of energy system costs. They have been often criticized as lacking explicit representations of markets, related policy instruments and individual behaviour of agents.

Table 1. Contribution of factors of production and productivity to GDP growth in selected countries, 1980-2001.

	Average annual GDP growth (%)	**Contribution of factors of production and productivity to GDP growth (% of GDP growth)**			
		Energy	**Labour**	**Capital**	**Total factor productivity**
Brazil	2.4	77	20	11	-8
China	9.6	13	7	26	54
India	5.6	15	22	19	43
Indonesia	5.1	19	34	12	35
Korea	7.2	50	11	16	23
Mexico	2.2	30	60	6	4
Turkey	3.7	71	17	15	-3
USA	3.2	11	24	18	47

MARKAL is a dynamic linear programming model that optimizes a technology-rich network representation of an energy system. The model was developed at the Brookhaven National Laboratory (BNL) in a collaborative effort under the auspices of the International Energy Agency (IEA). In MARKAL model the entire system is represented as s Reference Energy System (RES), showing all possible flows of energy from resource extraction through energy transformation and end-use devices to demand for useful energy services. Each link in the RES is characterized by a set of technical coefficients (capacity, efficiency), environmental emission coefficients and economic coefficients (capital costs, date of commercialization). MARKAL finds the “best“ RES for each period by selecting the set of options that minimizes the cost of the total system over the entire planning horizon [2].

Other example of technological models are market-oriented energy-environment system models as GEMS, GEMINI, ENPEP, NEMS and PRIMES (for EU). This models are often characterised as partial equilibrium models because they cover only the energy system and not the rest of economy [3].

PRIMES is a modelling system that simulates a market equilibrium solution for energy supply and demand in the EU member-states. PRIMES being a detailed energy system model, is able to provide evidence about the feasibility of quantified emission reduction objctives in medium to long term horizon. It can also evaluate a wide range of policy instruments, including command and control policy, including chnages (that affect the optimality of technology choices) within the industry structure, competition regimes and decentralisation of decision making. The dynamics of technology penetration can be simulated and influenced through a number of market and non market factors.

Macro-economy oriented models may well represent market-orientation, through the economic equilibrium paradigm, but often neglect the technological change aspects and the energy system details.

An example of such stand-alone model is GEM-E3 model for EU member states. GEM-E3 being a macroeconomic equilibrium model, is suitable for global characterisation of policies and the exploration of the interactions between the economy, the energy system and the environment. A general equilibrium model like GEM-E3 is, by design, appropriate to evaluate distributional effects over sectors and countries.

INTEGRATION OF THE MODELS

Both PRIMES and GEM-E3 models were conceived specifically for the study of climate change strategies but also for other purposes. They do not consider the very long term analyses. MARKAL-MACRO model is an example of model suitable for energy-economy-ecology long-term analyses.

The MARKAL-MACRO model is an integration of MARKAL and MACRO, a single-sector, macroeconomic growth model. Current MARKAL and MARKAL-MACRO users consists of most countries in OECD, and in many economies in transition and developing countries.

By combining MARKAL (a “bottom-up“ technological model) and MACRO (a “top-down“ neoclassic macroeconomic model) in a single-modeling framework (Figure 1), MARKAL-MACRO is able to capture the interplay between the energy system, the economy and the environment [4 – 6].

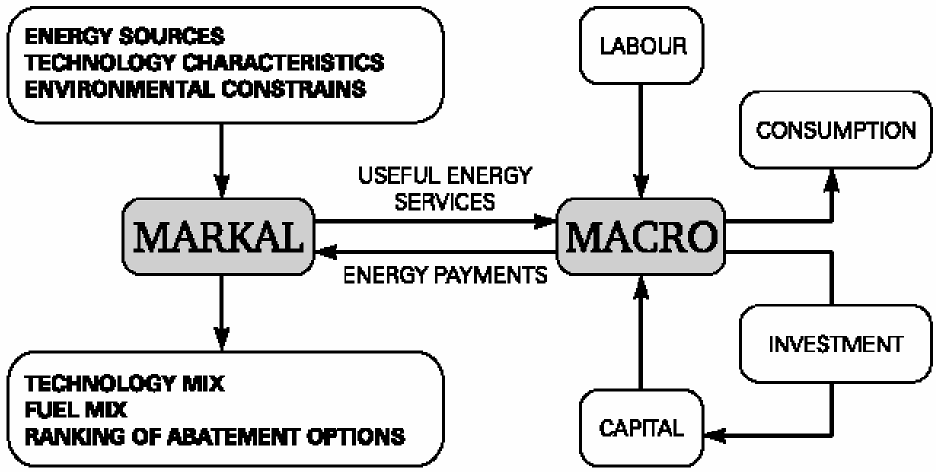

Figure 1. An overview of model MARKAL-MACRO.

As shown in Figure 1, there are two types of linkage between MARKAL and MACRO models. There are physical flows of energy between MARKAL and MACRO and energy cost payments from MACRO to MARKAL. The physical flow of energy is defined as useful energy demands which are exogenous to the stand-alone MARKAL version, but endogenous to the linked model. The costs of energy supply appear in the objective function of MARKAL, but enter into MACRO through the period-by-period constraints governing the allocation of the economy`s aggregate output between consumption, investment and energy cost payments.

The linkage between MARKAL and MACRO is based upon one key idea – the concept of an economy-wide production function. The principal advantage is that this enables to make a direct link between a physical process analysis and a standard long-term macroeconomic growth model.

The inputs to the production function consists of capital, labor and useful energy demands. Capital, labor and energy may each substituted for the other, but there are diminishing returns ot the substitution process [7]. To avoid the econometric estimation of many parameters, the production function is a nested constant elasticity of substitution (CES) form as presented with [8]:

$$Q = A\left[\delta\, K^{-\rho} + (1-\delta)\, L^{-\rho}\right]^{-1/\rho}, \tag{1}$$

where K and L stand for production factors kapital and labour, respectively. Variable A is efficiency factor, δ is parameter of distribution and ρ parameter of substitution.

At the top level, there is a capital-labor aggregate that may be substituted for an energy aggregate. At the bottom level, there is a unitary elasticity of substitution between capital and labor and the energy aggregate as shown in Figure 2.

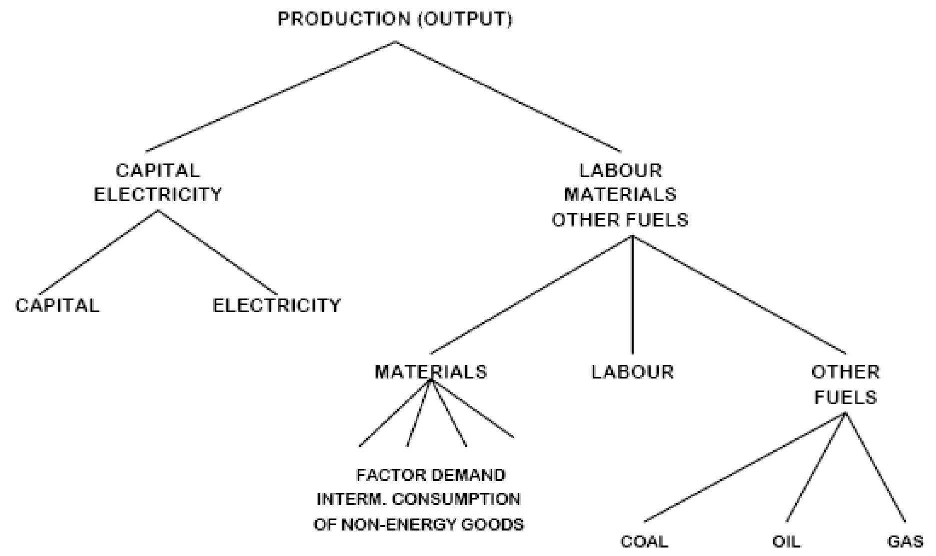

Figure 2. Structure of CES production function.

In the standrad version of MARKAL, it is dynamic linear model where markets are simulated by minimising an objective function incorporating the costs of energy technologies and resources. Model MACRO takes an aggregated view of long-term economic growth. The basic input factors of production are capital, labor and individual forms of energy. The economy`s outputs are used for investment, consumption and interindustry payments for the cost of energy. MACRO is solved by nonlinear optimization. It uses the criterion of maximum discounted utility of consumption to select among alternative time paths of energy costs, macroeconomic consumption and investment.

Different methods and modeling tools for energy system studies are often grounded in one of two disciplines: energy economics and system engineering. Energy-economic studies focus on links between energy demand and economic development. Their modeling tools are designed to account for economic feedback on energy demand from changes in energy prices, but include little detail on technological change. In contrast, engineering studies of energy demand focus on the efficiency of energy-using equipment. However, economic feedback is in general not included in the analysis tools. In studying the possibilities and costs of mitigating emissions of greenhouse gases, energy-economic and systems engineering studies have given different results.

RESULTS OF INTEGRATED MODEL APPLICATION

The example of MARKAL-MACRO application is national energy analysis for Sweden, for investigation of consequences of reducing energy-related carbon dioxide emissions for the Swedish energy system and economy. Different reduction level is studied on the basis of different international climate protocols. MARKAL-MACRO has been considered a valuable tool for this type of analysis, since the societal cost in term of reduction in GDP can be estimated na dthe marginal cost of reducing CO2 can be calculated [9].

MARKAL-MACRO has been used to evaluate different environmental tax and subsidy schemes. The results clearly show that the change in Swedish CO2 taxes and subsidy programs between 1990 and 1996 should lead to substantial reductions in CO2 emissions as presented in Figure 3. Tha tax change will lead to a strongly increased use of biomass, especially in the district heating sector. The tax scheme of 1996 includes energy tax, CO2 taxes for all energy use except for electricity production, sulphur tax and subsidies for biomass based cogeneration, wind power and solar heat. Obviously, the compounded change in energy taxation strongly curbs the emissions until the year 2020. However from 2020, at which time it is assumed that most of the nuclear power will be phased out, it is not enough to keep total emissions down.

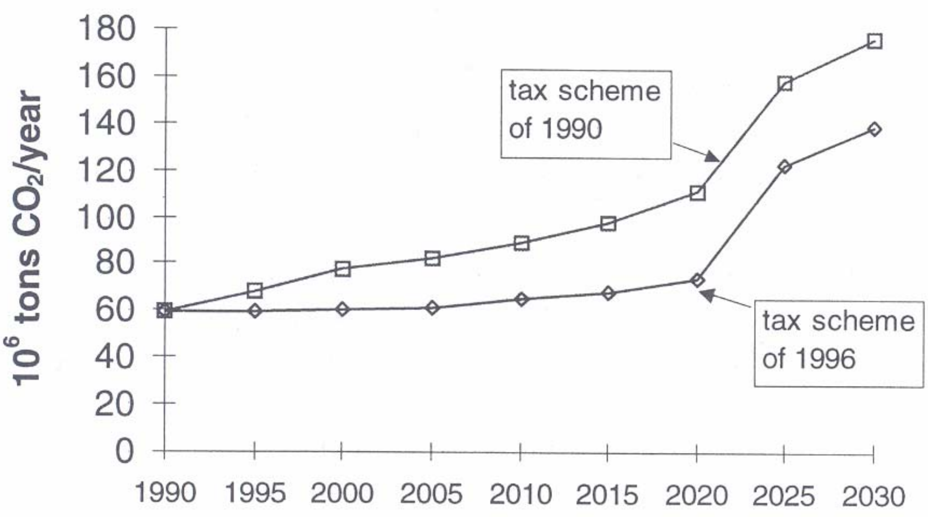

Figure 3. The total CO2 emissions from the Swedish energy system with the energy tax and subsidy scheme of 1990 versus the tax scheme of 1996.

MARKAL-MACRO has been used to study three different energy-environment issues of interest for Swedish energy policy. They illustrate the importance of the energy-economy interface and the value of using a linked energy-economy model. This issues are: the possibilities and cost of restricting carbon dioxide emissions; the possibilities and cost of phasing out the Swedish nuclear power and the value of carbon-free resources, such a biomass and end use energy efficiency improvements. The results have focused partly on the societal value of technologies or groups of technologies or resources and partly on the interplay between the technical energy system and the economy.

The value of an energy resource or a technology depends on the system of which it is a part and the external demands on this system. This statement is well illustrated by the MARKAL-MACRO results, which show that the value of biomass resources and end use energy efficiency improvements depends strongly on whether nuclear power is phased out and on the existence of CO2 restrictions. The value of a resource or technology is estimated by comparing the resulting development of GDP with and without the availability of the resource.

A certain CO2 reductions is achieved through a combination of technological changes and economic feedback effects. Firstly, technological changes within the energy system, such as fuel switching and efficiency improvements, lead to a reduction in CO2 emissions per unit of useful energy. These changes are modelled in the MARKAL model. The increasing energy prices lead to a reduction in the demand for useful energy, further reducing total CO2 emissions. This economic feedback results in a decrease in general economic activity (GDP) and partly in decreasing energy use per GDP. Both effects are taken into account through the link to the macroeconomic production function (MACRO). The allocation of CO2 reductions in the case of stabilizing emissions on the 1990 level is shown in Figure 4. The total reduction can be divided between different causes of reduction: reduced GDP growth, reduced energy use per GDP and technological changes within the energy system. A further division is then made between different sectors within the energy system.

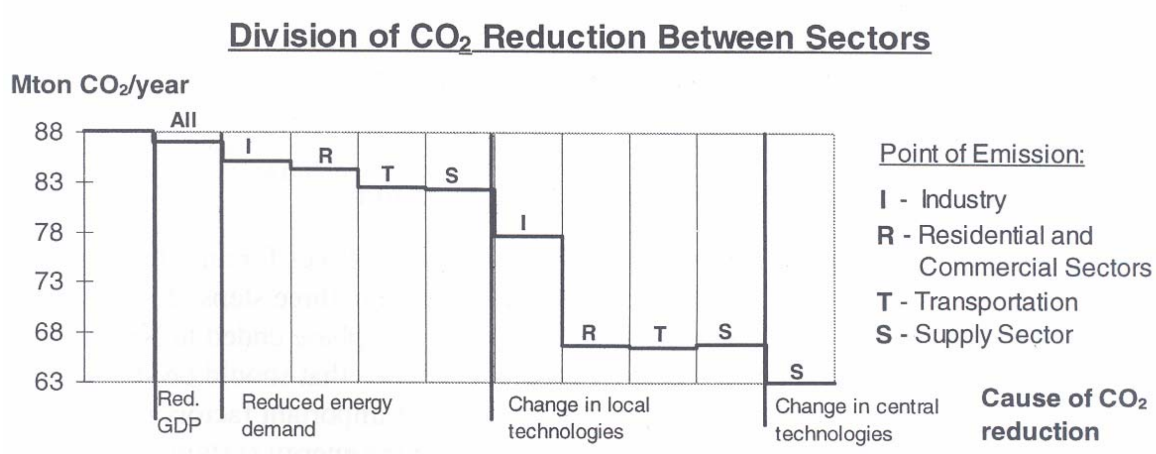

Figure 4. The CO2 reduction in the case of stabilization of emissions at the 1990 level compared to the business-as-usual case.

Another example of MARKAL-MACRO application is a project for the Clean Development Mechanism in Taiwan [10]. Under the Kyoto protocol a Clean Development Mechanism (CDM) was established which authorizes emission trading with a developing country as a means by which an industrialized country can meet its oblifgation to reduce greenhouse gases.

Before a CDM project can start, the participants – the host country and the investor in an industrialized country – must set and define baselines and criteria for quantification of emission reduction. Equally important is an agreement on how the accrued emission credits should be shared between the host and the investors.

This issues can be addressed within a single modeling framework; the MARKAL-MACRO model was used to evaluate the costs and benefits of what a CDM project might be, together with its impact on economic development.

The project to be evaluated was the transfer of technologies that are being promoted by the USEPA Energy Star Buildings program (in US this is market-based program in which commercial building owners and operators agree to make a series of energy-efficient improvements).

The case study started with the MARKAL-MACRO model of Taiwan and a database that already includes many conservation measures and efficient technologies currently available in the market. The database also included some advanced technologies that are likely to enter the market in the near future.

MARKAL-MACRO calculates the marginal cost of providing the new technology (compact fluorescent tubes, building tune-up, fan systems and heating and cooling system upgrades etc.) as the incremental change in national welfare when the technology is provided.

Total energy system cost are lower when the Energy Star Buildings measures are introduced. These costs include energy resource and fuel costs, investment in supply and demand technologies and operating and maintenance costs. Restricting future carbon dioxide emissions makes a pronounced reduction in these energy costs. This is the result of the substantial reduction in the use of fossil energy, especially coal.

The introduction of the Energy Star program has a positive impact on the growth in gross domestic product (GDP). Since this incremental growth stems from reduced fossil energy, it can fairly be considered an increase in the direction of sustainable development.

CONCLUSION

The energy-economy modelling together with the issues related to CO2 emission reduction implies more complexity because global energy, economy and environmental systems are affected, longer time horizon analyses with technology changes are necessary and market-related decentralised behaviour of agents need to represent. Most of the available empirical models cannot fulfill this requirements; technological models often neglect market-related decentralised behaviour of agents and cover only the energy-environment system. The capability of macro-economy oriented models are opposite to technical models. Because this situation limits the insight we can have on new issues, combination of such models became the best solution for complex analyses.

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ANALIZA PLANIRANJA ENERGETSKOG SUSTAVA INTEGRIRANIM ENERGETSKIM I MAKROEKONOMSKIM MODELIMA

H. Božić

Energetski institut Hrvoje Požar
Zagreb, Hrvatska

SAŽETAK

U prošlosti, planiranje vezano uz energiju odvijalo se postavljanjem željene razine ekonomskog rasta i njegove jednostavne uporabe kao temelja daljnjih porasta ovisnih o poulaciji i dovodima. Planiranje se kretalo od nacionalne, makroekonomske razine do agregata, sektora i naposljetku projektne razine. Takav je proces virtualna jednosmjerna poveznica ekonomske stope rasta i energetskog sektora, izoliran od ostale ekonomije. Integracija modela MARKAL za optimiranje sustava i modela makroekonomskog rasta MACRO omogućava analizu dvosmjerne veze između ekonomije i energetskog sustava. U ovom radu razmatra se stanje relacije između energetskog sustava i ekonomije, uključujući osnove tehnologije, modele usmjerene na ekonomiju, njihovu integraciju u jedan model i primjene.

KLJUČNE RIJEČI

model, energetski sustav, ekonomija, okoliš

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CYCLICAL DEVELOPMENT OF EMPIRES - AN APPLICATION OF KRUGMAN'S THEORY

Viktor Eszterhai

c/o Eötvös Loránd University
Budapest, Hungary

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SUMMARY

The study of empires' development has prided itself on great traditions in historical sciences. However, understanding the development courses meets difficulties if the geographical environment is disregarded. In my opinion, with the help of studying the geographical environment and also Krugman's economic geographical theory, a more complex understanding of empires' history can be obtained.

KEY WORDS

empire, centre, periphery, trading periphery, cyclical development

CLASSIFICATION

JEL: A10

Corresponding author, η: igorio@freemail.hu; +36303362006;
ELTE, Pazmany Peter setany 1, 1117 Budapest, Hungary.

INTRODUCTION

If we want to define empires, usually we turn to geography. Empires are characterized by huge territories, often occupied from other countries, but they are also considered as regions with strong political influence, great military power, etc. The country, which places some of its subsystems (economy, army, bureaucracy, religion, culture) outside its border, becomes an empire [1]. This definition makes distinction between three geographical categories. The first, where the centres of the subsystem can be found, are called centres in the model. The second category is where subsystems of the centre can be placed for a longer time; these are called surroundings. The last category refers to, what the centre cannot reach or where its subsystems are not permanently present. It is called periphery. The territory of Europe can also be divided into these categories. Centres and surroundings are coherent geographical units, while the borders of these units are the periphery.

In this article I am going to introduce the above mentioned categories, then integrate Krugman's theory into these categories, and finally examine how the so-formed model can be applied to the classical empire theories.

PARTS OF THE GEOGRAPHICAL UNIT

CENTRES

It is evidence that all the European empires have (see Figure 1.) and used to have centres [1]. These centres evolved on the most advantageous geographical territory of each geographical unit. This is where most of the population can live (mainly plains and basins). Krugman's theory about the connection between demography and economy says that bigger population induces more intensive economic activities, i.e. bigger investments. As a consequence of this process the centre attracts the population from the surroundings [2]. Therefore a self-generating process starts, implying an increasing concentration of the economic activities and the population. If one of these factors changes drastically (for example, the population changes because of plague, war, or the economy because of war, lack of resources), this self-generating process collapses. However, the geographical factors change very slowly, and the human resources is not mobile enough to move, as result, it gives the chance to restart the self-generating process again and again.

SURROUNDINGS

The other part of the geographical unit is the surroundings (see Figure 1) which each centre possesses. These are the places where the subsystems are placed by the centre. The self-generating process of the centre needs more and more resources and the surroundings are able to satisfy the growing demands of the population. Therefore the inhabitants of the surrounding are interested in this conquest. The subordination of the surroundings to the centre changes in time, depending on the power of the centre. When the power collapses, the surroundings can become free, which is mainly realised by the strongest person or the strongest alliance of the surroundings. However, the centre can revitalise fast so the freedom of the surroundings takes only a short time.

PERIPHERIES

Peripheries (see Figure 1) can be found at the border of the geographical units. The main characteristic of this territory is that they are real physical dividing lines between the units, for example, mountains, rivers, and seas [2]. The status of the territories depends on the power of the centre, too. Generally, the periphery is independent of the centres. But while the centre has enough power, the peripheries sometimes lose their independence, which cannot take long, because geography does not give the chance for a long conquest (at given level of technology).

Having a closer look at peripheries, we can differentiate between two types. The first one is when geographical factors are too severe for trade. This territory must be poor, and unable to hold a large population, for example the Balkan. The other type's main characteristic is, when the geography is severe for a longer conquest, but enough suitable for trade (for example, fords, mountain passes). These territories, here called "trading peripheries", are in a much more advantageous position, because they trade between the units' centres; for example, Austria, Switzerland, Netherlands and Denmark. This provides capital for the local people which they invest into infrastructure (for bigger trade) and into industry. Trading resources and other goods towards the periphery leads to the birth of industry. The abundance of goods determines that the only competitive form is the highly-specialised industry, i.e. producing goods which are not easy to be substituted by producers of other areas. According to Krugman's theory the self-generating process starts with the growth of economy and population. Thus these peripheries become some kind of centres.

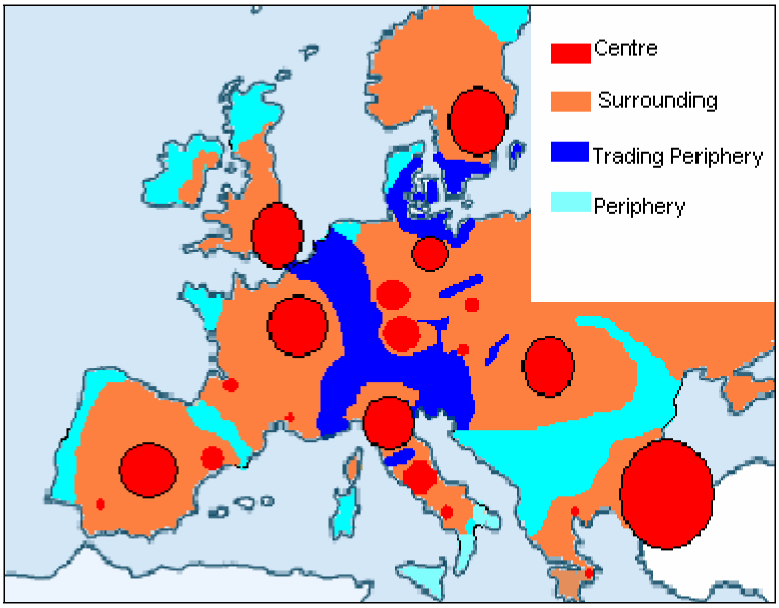

Figure 1. The geographical categories of Europe in the 20th century.

This model implies that there must be a connection between population and economy. On the basis of Figure 2 and 3, the concentration of the population and the economy seems to be connected to centres and trading peripheries. The surroundings and mainly the periphery are poorer and have smaller population.

Since the geographical territory changes very slowly and the population of the regions is highly immobile, the structure seems to be constant in time [3]. It provides the answer to the question why the centres of the empires are the same for hundreds of years.

THE LIFE OF EMPIRES

The territory of Europe can be separated into the categories mentioned above, i.e. into centres, surroundings and peripheries. All of these parts have their own "life cycle" similar to those described in biology [4]. In the long run only the centres can expand their natural borders, because only these territories have the basic of demography, economy, culture, and army to turn into an empire [1]. However, not only do the centres have this basis but also the

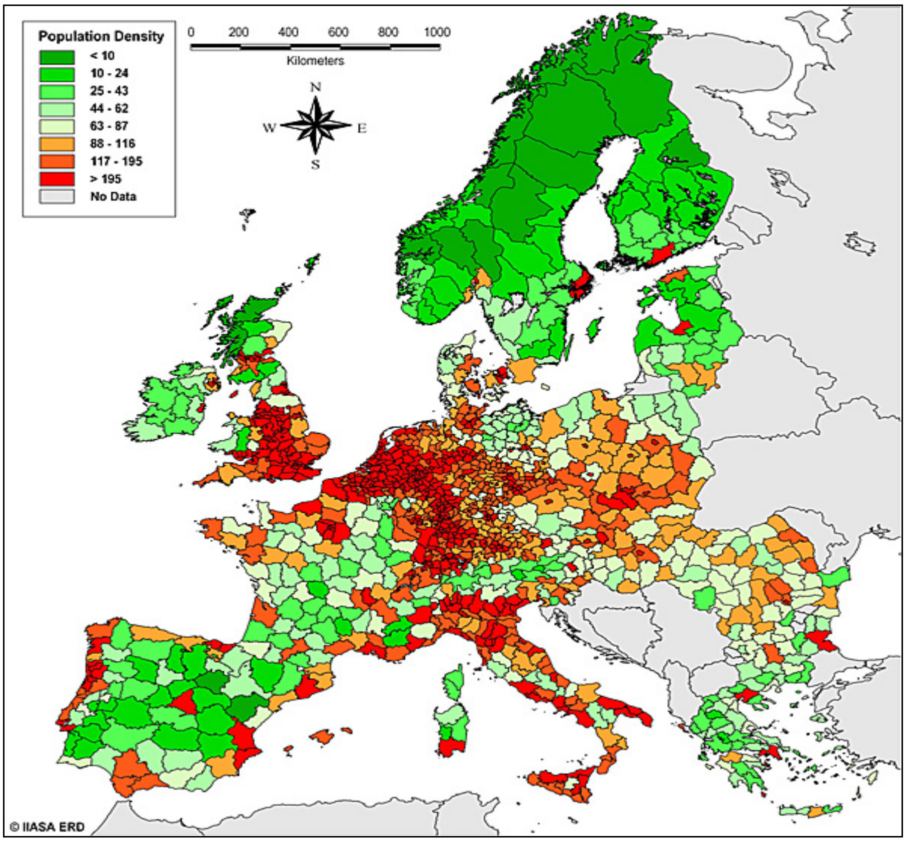

Figure 2. The population of Europe in 2005., (from http://www.iiasa.ac.at/Research/ERD/DB/mapdb/map_9.htm).

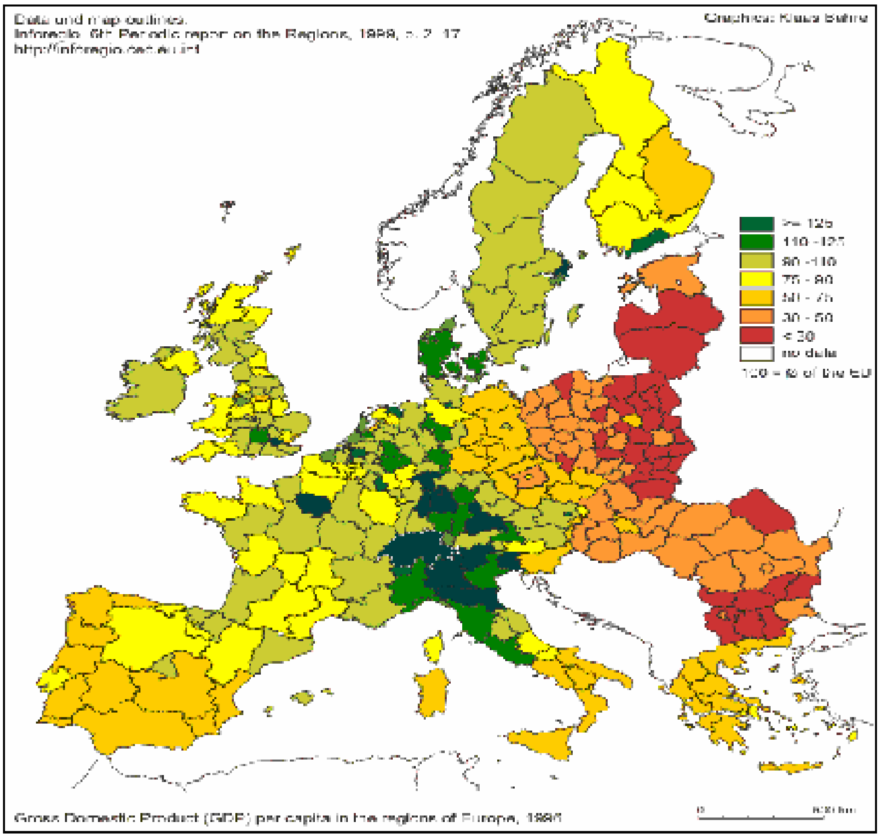

Figure 3. The state of development in Europe in 1999 (from http://janklaas.de/images.php).

trading peripheries can become empires. That is the reason why the history of empires seems to be connected to centres and to trading peripheries.

BUILDING THE BASIS: THE DEFENSIVE STAGE (STAGE 1)

The starting point of the process is chaos. The economic and religious elite is willing to do everything to preserve their positions, thus they need political stability. They make use of a political group to serve their own needs. With their supporting this political group gets into power. The reason for this alliance is that the political elite needs social and pecuniary assistance. After affirming their stability, the political elite can provide financial support to the religious and economic elite. With the alliance the political elite has the possibility to build a bureaucracy and an army whose leaders will later be part of the elite too. The elite also needs a culture to legalise their power (see Figure 4) [5].

As a result of the state's stability investments launch. It seems to be demographically proved that after the critical periods the growth of the population starts. On the basis of Krugman's theory the self-generating process comes into existence. The high incomes and the large number of people make the army, the bureaucracy and the culture stronger [6].

THE OFFENSIVE STAGE (STAGE 2)

When this background is strong enough, the centre or the trading periphery starts to conquer the surroundings. This is necessary because this self-generating process needs more and more resources. The military conquest is not the only form of expansion: every subsystem can be placed outside the border [1]. At this stage the whole elite is interested in conquest, thus there is no conflict between the participants; the expansion is unbroken (see Figure 4).

As far as the surroundings are concerned, at this stage their leading elite has two options: resistance or integration. They have no good chance to resist for a long time, because the centre has more power. But integrating the surroundings' elite to the empire's elite is more successful, because being the member of the empire's leading elite gives larger possibilities.

THE COLLAPSE (STAGE 3)

Geography restricts the conquest because geographical circumstances constrain the speed of information flow and the transport of recourses. When the centre reaches the periphery, the political elite cannot organise the defence and the bureaucracy here and further away, assuming a given level of technology. After some defeats the political elite recognises that any further expansion is an illusion, which breaks the unity within the elite and generates conflicts. The economic and social basis of the empire is shaken. The result of these crisis's that the empire collapses leading to chaos and civil war. The centre losts the periphery and sometimes also parts of the surroundings (see Figure 4) [7].

REVITALISATION (STAGE 1 OR 4)

Naturally, the end of a process is the beginning as well. As it has already been mentioned, the geographical circumstances change very slowly and the centre's and the trading periphery's population is not mobile enough to move, thus the life cycle process restarts at the same place [1].

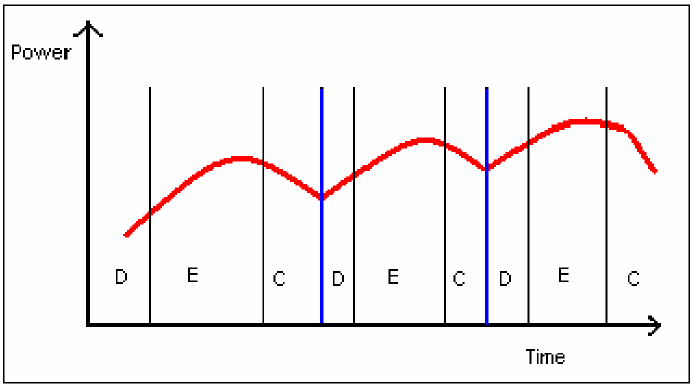

Figure 4. Cyclical development of empires.

THE IMPORTANCE OF TECHNOLOGICAL CHANGE

So far a given level of technology has been assumed, but technological development has a vital role [3]. New technology gives the chance for a dynamic improvement of the centres. In a fortunate situation a new technology makes them able to conquer and to keep further territories and peripheries as well. With a new technology the information flow and the transport of resources can be assured. From the view of the centre, the enlargement of the geographical field is the result of technological evolution (Figure 5)[1].

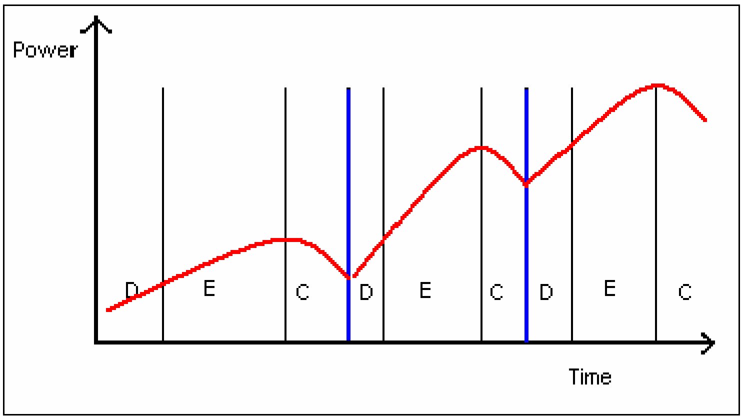

Figure 5. The result of the technological changing.

APPLYING THE MODEL IN HISTORY

Nowadays Europe's geographical units are independent and mainly national countries. The time needed for the development of these countries was different in each case. In England the integration finished in the 11th century, in France and Spain in the in the 15th century, but in Italy and Germany only in the 19th century, which depends on the homogeneity of the geographical territory [8]. In countries with homogeneous geographical units the centre was able to conquer the surroundings fast just like in England, or in Sweden [9]. In these countries the geographical circumstances were also advantageous to unify different dialects, which led to the early integration of the language.

However, in those countries where the geographical territory consist of smaller units [9], the annexation was slower, just like in France (Garrone, Provance) or Spain (Catalonia, Andalusia). These small units all have their own centres and peripheries with different dialects. From these centres the one with the most power had the chance to conquer the others with the help of new technology, for example Ile-de-France with the mass-army [10].

In Germany and in Italy the small geographical units were nearly equal concerning territory and population so that is the reason why the integration took longer time. In these cases only a drastic technological change (the industrial revolution) gave the chance for integration [10]. The centres could expand only step by step because of the systematical change of power. However, the expansion was not untroubled. Europe (like Italy and Germany) consists of nearly equal units so the border between the units gives the maximal line of the conquest. Still, each centre has its own progress line, which results in the fact that the centres are not at the same level of progress. While one centre is at an offensive stage, the other can be at a defensive one, which modifies the development of the life cycles (Figure 6).

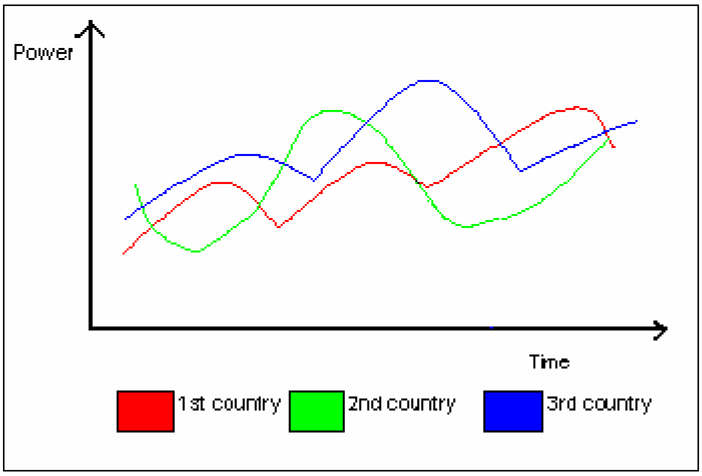

Figure 6. The development course of empires in reality.

CIVILISATIONS

If we accept the theory that a continent can be separated into units and peripheries, it is worth examining the theory at a higher level as well. We assume that the units are civilisations [11], while the borders, such as oceans, mountain ranges, deserts are stricter dividing lines. These units must have centres such as the Blue-Banana in Europe, the Chinese-plain in China, the Ganges-Basin in India. If the centre exists, the civilisations must have rhythmical changes in their life cycles. When the centre is at an offensive stage, the leading elite tries to place the subsystems outside the border of the civilisation. It can be exemplified by colonisation and crusades. It also means that those territories that are in the advantageous situation to trade between the civilisations become trading peripheries. This explanation can provide the solution of the problem why some countries such as England, Japan or nowadays Ireland have been able to achieve a magnificent economic growth.

CONCLUSION

On the basis of geographical aspects, a region can be divided into a centre, surroundings a periphery and a trading periphery. These categories determine various kinds of development. I have attempted to integrate into one model the categories of a region, Krugman's theory and the classical empire theories. The so-formed model justifies 1. the capital concentration and modernization ability of trade 2. the economic efficiency of certain regions, which can help to decide which regions should be preferred in case of subsidy allocation.

This model also draws the attention to the fact that historical processes are much more determined by geography than it is usually assumed.

In further research, it would be beneficial to apply this model on a global level, which may help to examine the process of globalisation.

REMARK

[1]It seems that the only fix point in the model is the economic elite. However, the technological change destroys its stable position. A new invention always results in a new elite, that recognises the importance of the new technique. The old economic elite must revitalise, otherwise they lose their power. The new elite of the economy is often interested in changing of the political system, because a new group can serve their needs more efficiently.

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CIKLIČKI RAZVOJ CARSTAVA – PRIMJENA KRUGMANOVE TEORIJE

V. Eszterhai

Sveučilište Eötvös Loránd
Budimpešta, Madžarska

SAŽETAK

Proučavanje razvoja carstava duge je tradicije u povijesnim znanostima. Međutim, razumijevanje smjerova razvoja nailazi na poteškoće ako se zanemari zemljopisno okruženje. Smatram kako se do kompleksnijeg razumijevanja povijesti carstava može doći proučavanjem zemljopisnog okruženja i Krugmanove ekonomsko-zemljopisne teorije.

KLJUČNE RIJEČI

carstvo, središte, periferija, trgovinska periferija, ciklički razvoj

MANUSCRIPT PREPARATION GUIDELINES

Manuscript sent should contain these elements in the following order: title, name(s) and surname(s) of author(s), affiliation(s), summary, key words, classification, manuscript text, references. Sections acknowledgments and remarks are optional. If present, position them right before the references.

SUMMARY Concisely and clearly written, approx. 250 words.

KEY WORDS Not more than 5 key words, as accurate and precise as possible.

CLASSIFICATION Suggest at least one classification using documented schemes, e.g., ACM, APA, JEL, PACS.

TEXT Write using UK spelling of English. Preferred file format is Microsoft Word. Provide manuscripts in grey tone. For online and CD-ROM versions, manuscripts with coloured textual and graphic material are admissible. Consult editors for details.

Use Arial font for titles: 14pt bold capital letters for titles of sections, 12pt bold capitals for titles of subsections and 12pt bold letters for those of sub-subsections.

Include figures and tables in the preferred position in text. Alternatively, put them in different locations, but state where a particular figure or table should be included. Enumerate them separately using Arabic numerals, strictly following the order they are introduced in the text. Reference figures and tables completely, e.g., "as is shown on Figure 1, *y* depends on x …", or in shortened form using parentheses, e.g., "the *y* dependence on *x* shows (Fig. 1) that…".

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