

# FINITE-TIME APPROACH TO MICROECONOMIC AND INFORMATION EXCHANGE PROCESSES

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## **ABSTRACT**

Finite-time approach allows one to optimize regimes of processes in macrosystems when duration of the processes is restricted. Driving force of the processes is difference of intensive variables: temperatures in thermodynamics, values in economics, etc. In microeconomic systems two counterflow fluxes appear due to the only driving force. They are goods and money fluxes. Another possible case is two fluxes with the same direction. The processes of information exchange can be described by this formalism.

## **KEY WORDS**

finite-time processes, information exchange, microeconomics

## **CLASSIFICATION**

JEL: D03, D83

#### INTRODUCTION

Problems on extreme performance of resources exchange processes at finite time have been investigated intensively during last half of a century. Thermodynamic systems have been investigated carefully. That is especially valif for thermomechanic systems, namely heat engines, refrigerators and heat pumps. A lot of papers are published on economic approaches of these problems [1]. Analogies between economic and thermodynamic systems were discussed in papers of K. Martinás [2–4] who formulated a new optimisation approach to description of resources exchange processes. That theory enables us to consider the aforementioned problems from a point of view of a general theory of finite-time processes of resources exchange [5]. In this paper there are no discussions about analogies and differences between economic and thermodynamic systems. Based on the characteristics of finite-time economic systems we introduce another kind of resource exchange systems, namely systems of information exchange.

## RESOURCES EXCHANGE IN ECONOMIC SYSTEMS

Let us consider a process of resources' sale in an economic system. That system consists of at least two economic agents, and at least two type of resources. One resource plays a role of money, thus it is called "money". Other resources are "goods". Fluxes of money and goods during the process of sale are opposite in direction.

Driving force of the process is the difference of the good's value made by the economic agents. Let us show that the only driving force induces both fluxes: the good and money ones.

In thermodynamics there are cases when the same driving force can influence several flows. If the system is near equilibrium state then intensities of flows correspond to Onsager relations  $J_i = \sum_j a_{ij} \cdot x_j$ , where  $J_i$  is the *i*-th flow,  $x_j$  is the *j*-th driving force,  $a_{ij}$  are Onsager coefficients,  $a_{ij} = a_{ji}$ .

So, existence of temperature gradient can induce both the heat and the mass flows. Similarly, difference of chemical potentials  $\Delta\mu$  ( $\Delta\mu/T$  is a driving force for mass exchange process) induces heat exchange between subsystems.

Therefore, difference in values of resources induces two opposite flows, one of the good and the other of the money. Assume that these flows are interconnected by Onsager relations. It means that there exists another driving force of goods and money movement in the economic system.

Because the driving forces are derivatives of wealth functions [4] then the only possible vacation for another driving force is difference of economic temperatures (derivatives of wealth function with respect to money). So, we have two possibilities:

- 1. to postulate that difference of economic temperatures leads to economic exchange (not only money flow).
- 2. to postulate that only one driving force in economic system causes two opposite flows, namely good and money flows.

The second postulate is more natural because if we have n resources then difference of economic temperatures implicates simultaneous flows of all resources, which is not observed. Therefore, during the economic exchange agents tend to a state with equal values of the resource costs. The only exception is the cost of money, as there is no real economic driving force for the elimination of the difference of economic temperatures (liquidity) [4] except some "imaginary states".

### SYSTEMS OF INFORMATION EXCHANGE

The existence of two flows induced by a single driving force allows us to propose further that there exists systems in which a single driving force induces a vector of flows. Intensities of components of such a vector are proportional. In economics that vector consists of two flows. Their directions are mutually opposite. In this section we consider another system in which the dimension of the vector of flows is again two, but the flows have the same direction.

That system is a system of information exchange. Let us consider a system consisting of two agents. One of them gives information flow to another.

There are three kinds of information [7]:

- (i) semantic information (knowledge),
- (ii) syntaxic information,
- (iii) pragmatic information.

Let us explain these using the following example.

## **EXAMPLE I.**

Let me introduce you the agent A. She wants to obtain an answer to a question "Is it worth to read a book R?". She uses a computer recommendation system to get an answer [8]. She needs only one bit of information, either "yes" or "no".

This information is a bit of knowledge which is necessary for A. To obtain that one bit she uses internet and in reality obtains 1 Mbyte of information, including the interface of the recommendation system, elements of computer design and other very convenient tools, which are however not necessary for A. So, 1 Mbyte is total (syntaxic) information obtained by A. For using the recommendation A (possibly) needs to pay, so A is in economic relation and increases her wealth. That increment corresponds to a value of pragmatic information. Information as a capital including human capital is a set of knowledge and, furthermore, the "know-how" has been a subject of investigation elsewhere [9, 10]. Here we consider knowledge not as a capital, but as a good. In this sense we introduce a wealth of the agent A due to her knowledge. Dependence of that wealth on A's knowledge is denoted as S(I). Note that S depends solely on a semantic information I, because syntaxic information is a common good.

This example allows us to introduce various types of information as characteristics of an agent. To formalise description of information exchange systems, let us utilise one more example.

#### **EXAMPLE II.**

Let us consider a closed system consisting of two agents A and B. A is a professor who reads a lecture to a student B. During the lecture, A transfers to B flows of both the syntaxic and the semantic information with intensities g and q, respectively. A stock of information of A can be divided into the following parts [11]:

- (i) Both agents know it  $(I_0)$ ;
- (ii) A wants to teach B  $(I_1)$ ;
- (iii) A does not want to tell it to anybody because it is know-how  $(I_2)$ ;
- (iv) A wants to know it and tries to find that information (*J*).

It is evident that

$$\frac{\partial S}{\partial I}\Big|_{I \in I_0} = 0, \quad \frac{\partial S}{\partial I}\Big|_{I \in I_1} < 0, \quad \frac{\partial S}{\partial I}\Big|_{I \in I_2} > 0, \quad \frac{\partial S}{\partial I}\Big|_{I \in J} < 0.$$
(1)

Note that if A gives B one bit of knowledge, then that bit is transferred from set  $I_1$  to set  $I_0$ . Let us introduce value of knowledge  $v = \partial S/\partial I$ . Intensity of syntaxic information g then depends on values of both agents. In the simplest case this intensity can be written in a linear form

$$g = -\alpha(v_B + v_A). (2)$$

In our example the flow of syntaxic information depends on interest of the student (value of knowledge in set J of agent B) and on qualifications of the professor (negative value of knowledge in set  $I_1$  of agent A).

To introduce the intensity of syntaxic information flow we need to find intensity of the corresponding flow of semantic information. However, prior to that we need balance equations. There is no conservation law in processes of information exchange, neither for semantic nor for syntaxic information. Really, if A gives B a bit of information then the stock of A's information does not change. In any process of information exchange the total amount of information in the system will increase. But during such a process the intensity of semantic information flow decreases: B can obtain a quantity of knowledge during A's lecture, and this quantity is not larger (unfortunately, sometimes it is much smaller, even if B obtains all syntaxic information from the lecture) than a quantity of information given by A.

Let us introduce redundancy of information as ratio of syntaxic and semantic flows' intensities. Then, according to [12, 13], intensity of obtaining the knowledge depends on the existing level of knowledge  $I_0$  and on the redundancy of information  $C_A$ ,

$$C_R = C_R(I_0, C_A).$$
 (3)

If a time period for information exchange  $\tau$  is not finite then for  $C_A \to \infty$  agent B obtains all knowledge given by A, without losses. But the intensity of knowledge flow in that case is infinitesimal, thus we can call it a reversible process of knowledge exchange. At finite time these losses,

$$\Delta = g(v_A, v_B) \left[ \frac{1}{C_A} - \frac{1}{C_B(I_0, C_A)} \right], \tag{4}$$

can be significant.

Value of  $\Delta$  characterises irreversibility during information exchange. So, this value is analogy to energy dissipation in thermodynamics and to capital dissipation in economics [1]. Due to these analogies it can be called a dissipation of knowledge.

Now we have prerequisites to formalise problems of extreme performance of information exchange process. These problems are:

#### PROBLEM I.

The problem of maximum amount of transferred knowledge subject to a given quantity of syntaxic information

$$\int_{0}^{\tau} \frac{g(v_{A}, v_{B})}{C_{B}(I_{0}, C_{A})} dt \to \max_{v_{A}, C_{A}}, \tag{5}$$

subject to

$$\int_{0}^{\tau} g(v_A, v_B) dt = G_0, \quad \dot{I}_0 = \frac{g(v_A, v_B)}{C_B(I_0, C_A)}, \quad I_0^0 = I_0^b.$$
 (6)

#### PROBLEM II.

The problem on minimum of average knowledge dissipation subject to given average intensity of semantic information flow

$$\int_{0}^{\tau} g(v_{A}, v_{B}) \left[ \frac{1}{C_{A}} - \frac{1}{C_{B}(I_{0}, C_{A})} \right] dt \to \min_{v_{A}, C_{A}}, \tag{7}$$

subject to

$$\int_{0}^{\tau} g(v_A, v_B) dt = G_0, \quad \dot{I}_0 = \frac{g(v_A, v_B)}{C_B(I_0, C_A)}, \quad I_0(0) = I_0^b, \quad I_0(\tau) = I_0^e.$$
 (8)

Efficiency of information exchange in this formalisation will be

$$\frac{q_B}{q_A} = 1 - \frac{\Delta}{q_A} \,,$$

i.e. its value is determined by the value of knowledge dissipation.

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## MIKROEKONOMSKI PROCESI I PROCESI IZMJENE INFORMACIJA U PRISTUPU KONAČNIH VREMENA

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## **SAŽETAK**

Pristup konačnih vremena omogućava optimiranje režima procesa ograničenog trajanja u makrosustavima. Pokretačka sila tih procesa je razlika intenzivnih varijabli: temperature u termodinamici, vrijednosti u ekonomiji itd. U mikroekonomiji se javljaju dva suprotna toka uzrokovana samo jednom pokretačkom silom. To su tokovi robe i novca. Druga mogućnost je dva toka istog smjera. Procesi izmjene informacija mogu biti opisani tim formalizmom.

## KLJUČNE RIJEČI

procesi u konačnom vremenu, izmjena informacija, mikroekonomija