

# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

Scientific Journal

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# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

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## EDITORIAL

Welcome to the eight volume of the *Interdisciplinary Description of Complex Systems*, and its first issue consisting of five regular articles.

Two articles include utilisation of agent based modelling. In particular, in the article by S. Gil and A. Serrat-Capdevila an agent based model is formulated in detail and applied to simulating water allocations among interested parties in different environment characteristics.

In their article, M. Ngobye et al. consider in detail scope of applicability of multi-agent models characterised by rather simple governing interactions.

Authors A.M. Tsirlin and S.A. Amelkin set analytically solvable mathematical model for treating the characteristics of equilibrium states in a model situation consisting of several interacting entities.

Author R. Correa in his work properties of the model proposed by Godley and Lavoie, specially regarding the role of the Central Bank in well-defined environment. As author emphasises, it is for the first time in the history of any central bank, that the most important lender of last resort in the world has underwritten long-term real activity. Yet, formal models rationalizing this new stance are not to be found.

In his article, R. Fabac treats organizations, a complex adaptive system in environment, which is another complex system. He elaborated the need to include methods originating in the area of nonlinear processes, to enable the organizations' leaderships to efficiently treat the collected vast and interconnected quantity of data in order to achieve objectives set for the organization.

Zagreb, 30 June 2010

Josip Stepanić

# EMERGENCE OF COOPERATION IN A MODEL FOR AGRICULTURAL PRODUCTION

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*Regular article*

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## ABSTRACT

The emergence of cooperation in a model for an artificial farming society is studied here by the use of an agent-based model. The system is composed of an ensemble of  $N$  agents assumed to have equal access to water, whose availability fluctuates randomly in time. Each agent makes two decisions every sowing season regarding: (1) the type of crop mix to plant and (2) whether s/he joins, or not, a cooperative group that allocates water amongst farmers to maximize the production and share revenues equally. Results show that the degree to which farmers choose to cooperate has a strong dependency on the mean water availability. Cooperation seems to emerge as a way of adaptation to uncertain environments by which individual risk is minimized.

## KEY WORDS

cooperation, water uncertainty, farming, agent-based modelling, resource allocation

## CLASSIFICATION

JEL: C63, O13, Q15, Q25

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## INTRODUCTION

Cooperation is a remarkable form of self-organization according to which two or more individuals or components in a system collaborate to obtain a global benefit rather than an individual one. The emergence of cooperation is a very important kind of complex behaviour [1], which has been studied with many different approaches and in a great variety of research fields, as diverse as biology, sociology or economics [2, 3]. In particular, economics encounters an intriguing dilemma regarding how to account for cooperation under the assumptions of selfishly driven actors that is traditional in economic theories [4]. However, in most cases, cooperation does not imply individual sacrifice for the greater good. Cooperative strategies consisting of sharing resources (information, capital, logistics, etc.) will often report better profits than individual competition.

Production systems of a cooperative nature have existed throughout the history of human kind, and both social and economic implications of such systems present some very interesting aspects. One notable example of cooperative production systems in present day is given by the remarkably successful Mondragón Cooperative Corporations, in Spain, whose growth and diversification are remarkable in comparison with some more conventional corporative structures [5]. In particular, cooperation in real farming communities as production systems has inspired many studies. The use of complex systems tools – such as agent-based modelling – in these fields can be exemplified by Steve Lansing’s research, who explored how the cooperative cropping and irrigation systems in Bali function in the absence of hierarchical control [6, 7]. According to Lansing, self-organization and coordination of the farming districts result in temporal and spatial patterns of cropping and irrigation that are very similar to optimal solutions of computer models replicating the system.

The importance of studying cooperation does not reside solely in the understanding of complex behaviour. To better comprehend how different factors promote or hinder cooperation can also prove helpful for the crafting of the necessary institutions to promote or enforce efficient administration of resources [8 - 11]. In current times, when water is becoming an increasingly scarce resource, new organizational schemes – and the corresponding institutions – must be developed [12], which is particularly true for the allocation of water in many regions. In Mendoza, at the feet of the Andes in Argentina, the problem of water scarcity is an increasingly important one [13]. Cultivation of olives and grapes for wine production are most prominent and of very high quality, and it is the most important industry of the region. Being it an area of low precipitation, farmers in Mendoza obtain water from rivers and aquifers in the region. Water is distributed through an extensive network of irrigation channels which are under government control and regulation. As happens in similar systems across the world, inefficient management and current legislation make this system far from optimal, since water allocation is rigidly organized, and the rights to water access are not transferrable [14]. With this system, vineyards might suffer from lack of water while farmers who grow olives might have it in excess, which often leads to inefficient production, or even to illegal obtention and marketing of water.

The work developed in the present paper aims at evaluating how cooperation can emerge in a farming community in response to climate variability and uncertainty. For this, and intending to refer to a situation similar to that given in Mendoza, we analyze a simple yearly time-step agent-based model of an artificial farming community, in which cooperation between individuals can emerge as an opportunity to better adapt to environmental variability and uncertainty. Farmers are modelled as agents that make choices about the crop mix they sow, and about whether they want to be part of a cooperative that shares water and revenues.

Decisions are modelled probabilistically and are susceptible to change at each time step as probabilities are updated based on previous outcomes. It should be clarified that our intention is not to accurately reproduce any real world situation in a quantitative way. Rather, we intend to explore the possible equilibrium states that are reached through a process that is simple enough to be analysed, in order to test the validity and implications of the assumptions made. Finally, since this model does not include any feedback loop between the behaviour of the agent ensemble and the environment itself, we in turn refrain from considering possible processes of adaptation to a changing environment, taking into account only the fluctuations within stable envelopes of uncertainty. In this way, our model presents a niche where cooperation in a social system allows for a better adaptation to the variability of the environment.

## **MODEL**

In our model we consider water as the only varying resource playing a role. However, since no mechanisms for the obtention of water, or ways in which it is used are taken into account, the availability of other resources can be conceptually encompassed within what is referred to as water, as long as they can be allocated at will. This could include, for example, limited energy supply, fertilizers and pesticides with fluctuating prices, access to machinery, etc. Water will be modelled as a random Poisson variable with a defined mean value  $\Omega$ , which can be interpreted as associated to an average yearly flow in a river. Thus, at every time-step, a random value  $\omega(t)$  will be drawn from a Poisson-distribution with mean  $\Omega$ , meaning that each virtual farmer receives a share  $\omega(t)$  of diverted flows from the river in year  $t$ . Changes in values of  $\omega(t)$  represent yearly variability of river flows and its random treatment is justified by the low interannual correlation of flows in real rivers. Thus, for every realisation, the mean value  $\Omega$  will be kept fixed.

In our approach, we refrain from taking into account any effect of the production output on the eventual market where it is traded. Instead, we omit macroeconomic considerations, and consider the price of the product to be an external factor that we hold fixed. Since we will not refer to any specific units of measure, it follows that within this framework, the concepts of output and revenues are interchangeable. Furthermore, we take the costs to be accounted for in a production function, which then defines the profit as a function of the available water.

We consider an ensemble of  $N$  identical agents representing farmers in a community. A homogeneous community implies not only that farmers' information and predicting capabilities – null in this case – are the same, but also that all farmers' fields are of equal size and characteristics. The community diverts water from the river and divides it by the number of farmers. Thus, all farmers receive the same amount of water every year, which is delivered through irrigation canals and distributed to the agent's fields.

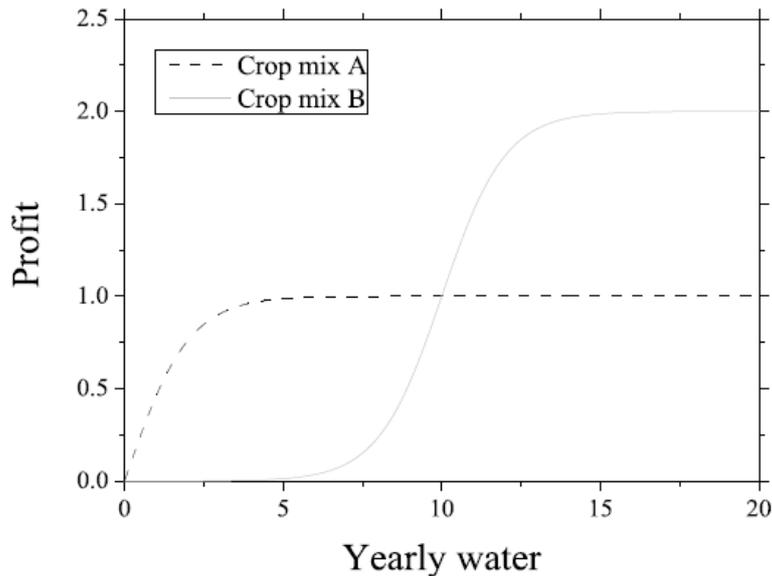
Each farmer faces two decisions every year: (a) plant crop mix A or crop mix B; and (b) join – or not – a cooperative group that allocates water amongst farmers to maximize the production and share revenues equally. Because these choices are modelled probabilistically, each farmer is characterised at each time-step by two probabilities  $P^c(t)$  and  $P^m(t)$ , with which the state of each farmer at time  $t$  will be chosen. At any time step  $t$  representing a year, the state of the  $i$ -th farmer is defined by two binary random variables that can take any of two values  $-1$  and  $1$ . These variables are  $c_i(t)$ , which will represent the crop mix that the farmer will sow and farm that year, and  $m_i(t)$  representing the production and marketing strategy that the farmer will adopt (cooperative or individualist). The particular value of these variables each year is determined randomly with probabilities  $P_i^c(t)$  and  $P_i^m(t)$ , respectively.

For simplicity, only two possible crop mixes are considered, namely crop mix A and crop<sup>1</sup> mix B. Every time step, each farmer decides which of these two crop mixes to farm, which

for farmer  $i$  amounts to taking a state with  $c_i(t) = 1$  with probability  $P_i^c(t)$ , or a state with  $c_i(t) = -1$  with complementary probability. Therefore,  $c_i(t) = 1$  means that farmer  $i$  will plant crop mix A on year  $t$ , and  $c_i(t) = -1$  means that the farmer will plant crop mix B. Each crop mix produces a certain profit  $f$  at the end of the year, which is a function of the water  $\omega$  received that year. These production functions for each crop mix considered are given in Fig.1, where both yearly water  $\Omega$  and profit  $f$  are given in arbitrary units which will be maintained throughout this article<sup>2</sup>. It can be seen that crop mix A represents a safe choice, since it will generate a moderate profit with little water, while crop mix B can produce bigger profits (twice as much for large amounts of water) but requires an important amount of water to produce a significant profit in comparison with crop mix A. In these terms, variables  $P_i^c(t)$  might be interpreted to represent the risk aversion of farmer  $i$ . Both crop mixes return the same revenues for  $\omega_0 = 10$ , which we consider a moderate amount of water.

In addition, farmers select whether they will adopt an individualist production strategy or if they will cooperate with other farmers. An individual production strategy will imply that a farmer will harvest the profits of the selected crop mix corresponding to the available water per farmer  $\omega(t)$  that year, according to the curves in Figure 1. All farmers adopting the cooperative strategy, on the other hand, will group all the water received and redistribute it amongst themselves in a way as to produce maximum total profit in accordance with the crop mixes selected by them. This total profit is later divided in equal shares between all cooperative farmers. Each year  $t$ , each farmer will select one of these production strategies. With probability  $P_i^m(t)$ , farmer  $i$  will be in a state with  $m_i(t) = 1$ , corresponding to a cooperative strategy, and with the complementary probability  $1 - P_i^m(t)$  the farmer will go to a state with  $m_i(t) = -1$ , which corresponds to an individualist strategy. For this reason we refer to  $P_i^m(t)$  as the cooperativity of farmer  $i$  at time  $t$ .

Depending on the success or failure of her/his previous choices, each farmer will change the probabilities with which s/he makes these choices the following year. The probabilities  $P_i^c(t + 1)$  and  $P_i^m(t + 1)$  with which the  $i$ -th farmer will select a crop mix and a production strategy respectively on year  $t + 1$  will depend in her/his performance on year  $t$  in the following way



**Figure 1.** Production functions for both crop mixes considered, as a function of water  $\omega$ . Crop mix A can be thought of as a safe crop mix with moderate returns, while crop mix B presents more risks but can deliver higher revenues. The two curves coincide at  $\omega_0 = 10$ .

$$\begin{aligned} P_i^c(t+1) &= P_i^c(t) + c_i(t) \cdot F(\{f_j\}, f_i), \\ P_i^m(t+1) &= P_i^m(t) + m_i(t) \cdot F(\{f_j\}, f_i), \end{aligned} \quad (1)$$

where  $\alpha$  and  $\beta$  are positive coefficients that regulate the rate of change of probabilities, and  $F(\{f_j\}; f_i)$  is a function that determines how successful production was for farmer  $i$  over year  $t$ . Function  $F(\{f_j\}; f_i)$  should be positive if the profit of farmer  $i$  was satisfactory, and negative if it was unacceptable. In this way, if performance in the previous year was favorable, the selection of this state will be accordingly favored in the future, reinforcing previous behavior that proved successful, whereas if performance was poor, the probabilities of selecting this state will be diminished. Clearly, the fact that equations (1) are linear on the evaluation function  $F(\{f_j\}; f_i)$  means that the probabilities  $P_i^c$  and  $P_i^m$  are unbounded. To maintain normalization, these two probabilities will be restrained to the interval  $[0; 1]$ , meaning that they will be set to 0 or 1 when they grow beyond these limits. In this way, even though these probabilities are modified taking into account only what occurred in the previous year, farmers accumulate experience along the entire simulation.

Function  $F(\{f_j\}; f_i)$  represents the criteria with which farmers evaluate their performance, and can in principle depend on the performance of all farmers simultaneously. A reasonable example for this would be the case in which farmers compare their performance to the average performance of the entire community. Although this approach seems somewhat natural and avoids arbitrariness on our part, we believe it is unrealistic. We think farmers should not evaluate their performances in terms of how poorly others perform, but rather in comparison to how much better they could have performed had they made a different decision. This should be decided in terms of a measure of opportunity costs of each farmer, and not of global variables. At the same time, full potential is not clear a priori, and comparison to excelling players can elucidate this. Thus, we implement an evaluation function of the form

$$F(\{f_j\}; f_i) = \frac{f_i}{\max\{f_j\}} - a, \quad (2)$$

where  $a$  is a parameter between 0 and 1 that can be interpreted to be a *satisficing parameter*, in reference to the concept introduced by Herbert Simon [15]. In this way, when  $a$  is close to 1, only those farmers who have the best performance will be satisfied, but when  $a$  is slightly larger than 0 any performance will be acceptable. In this sense, parameter  $a$  may represent a measure of bounded rationality of agents. In (1) it is assumed that each farmer knows the value of  $\max\{f_j\}$ , a consequence of the fact that maximal profits are in practice renowned, in analogue to other aspects of our society in which people and corporations with maximal profits are well known, sometimes inducing legendary proportions.

Another way of understanding the use of parameter such as  $a$  in the evaluation function is that, in any model that tries to be realistic, changing crops and joining or dissolving cooperative corporations must have associated costs. Thus, even when a farmer knows that there is a better strategy than the one he is adopting, he might still not change his strategy because of these costs. In this sense,  $1 - a$  can be seen as a measure of how tolerant a farmer can be to making sub-optimal profits, taking into account the costs associated with changing his strategy. We chose  $a = 0,5$  for our initial simulations, and then study the dependence on this parameter for a more general understanding of its role. Heterogeneity between farmers could be introduced by setting different values of  $a$  – as well as of  $\alpha$  and  $\beta$  – but in this paper we will maintain the premise of a homogeneous population.

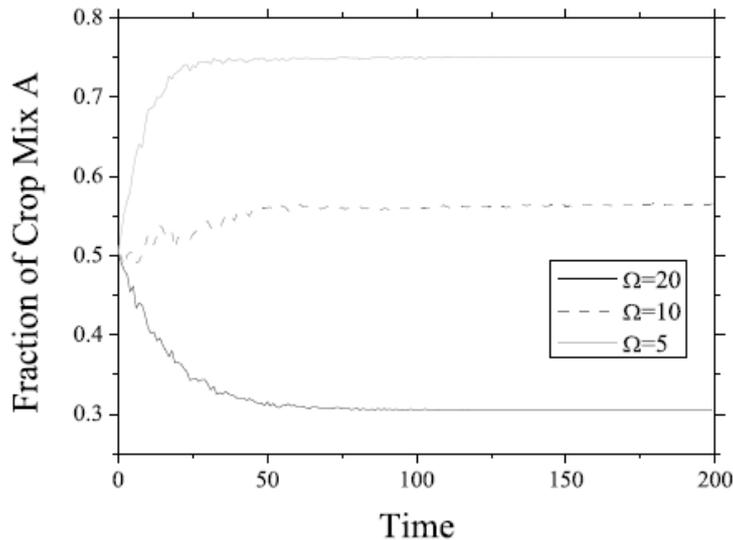
The model is thus defined, and its implementation is as follows: in each yearly time step farmers select a crop mix to sow according to their risk aversion, and a production strategy according to their cooperativity.

Then the year unfolds, yielding a random amount of water from a Poisson distribution with mean  $\Omega$ . All farmers obtain their corresponding profits from their harvest as given by their production strategy. In terms of these profits, each farmer will update her/his risk aversion  $c_i$  and cooperativity  $m_i$  following (1), and a new time step begins. The free parameter  $\Omega$  can determine the conditions of the climate in terms of how it compares to the production functions of both crop mixes. Specifically, if  $\Omega \ll \omega_0 = 10$ , we can understand that we are in a situation of water scarcity, and crop mix B is very inconvenient. On the other hand, when  $\Omega \gg \omega_0 = 10$ , water is abundant, and crop mix B is very likely to produce better results than crop mix A.

We finally note that for individualist farmers, the interaction between farmers is very weak, only introduced through the value of the maximum profit  $\max\{f_j\}$ . On the other hand, when the cooperative strategy is selected, the interaction between farmers becomes strong, and the crop mix selection of every farmer influences the performance of all other cooperative farmers.

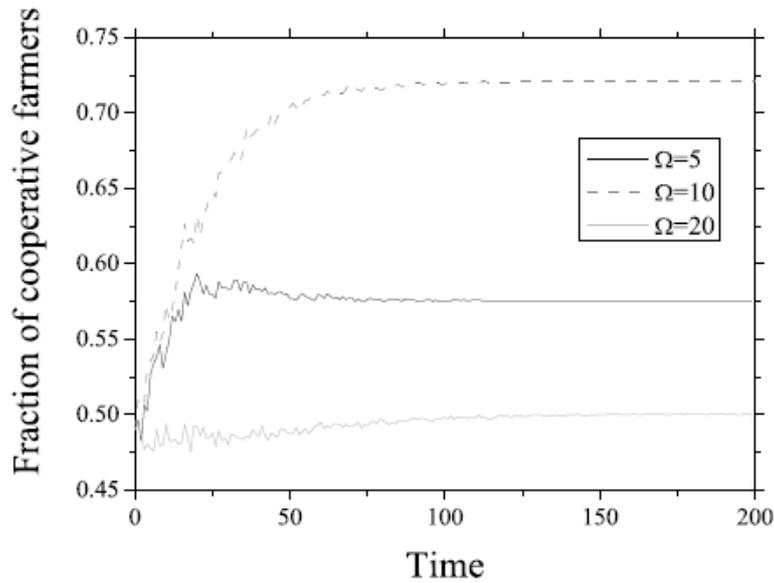
## RESULTS

We analyze the behaviour of a system of 2500 farmers, setting initial conditions for probabilities and states of all farmers randomly with uniform probabilities, and we fix the values of the constants  $\alpha$  and  $\beta$  both as 0,01. Since these parameters modify the rate at which risk aversion and cooperativity change, we can expect that when they are small enough, the choice of parameters will be equivalent to a rescaling of the measurement of time. Thus, the long-term results will become statistically independent of the particular choice of values for parameters  $\alpha$  and  $\beta$ , as long as they are both of the same order of magnitude. As mentioned before, the choice of the  $a$  parameter, characterizing the farmers evaluation of performance, will be set at  $a = 0,5$  in subsequent analysis, except when stated otherwise.

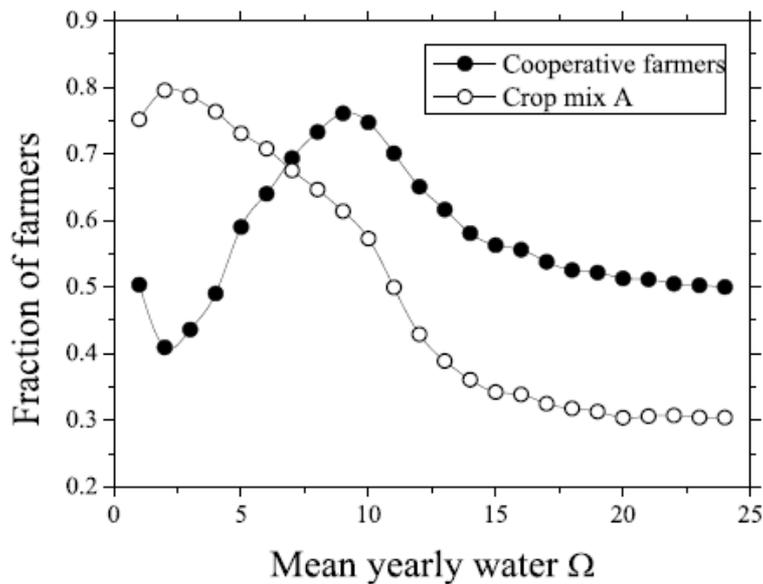


**Figure 2.** Time evolution of the fraction of farmers that farm crop mix A for three different regimes of water availability.

In Fig. 2 we observe the time evolution for the fraction of farmers that select crop mix A as a function of time for three different values of the mean yearly water. We readily see that the system reaches steady values for this fraction in all three cases after 100 steps approximately.



**Figure 3.** Time evolution of the fraction of farmers that adopt a cooperative production strategy for three different regimes of the mean water availability  $\Omega$ .

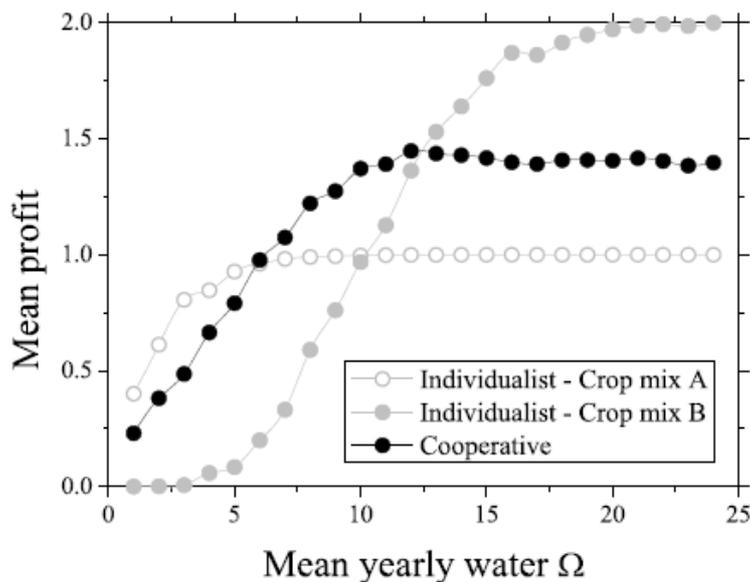


**Figure 4.** Fraction of farmers that select crop mix A and fraction of farmers that adopt a cooperative production strategy, both in the stationary state, as a function of the mean yearly water  $\Omega$ .

This steady value varies strongly with the mean available water. Fig. 3 also shows the time evolution for the same three values of  $\Omega$ , this time in terms of the fraction of farmers who select cooperation as their production strategy. Again, we can see that this fraction reaches a steady value for each value of the parameter  $\Omega$ . Thus we can say that there is a stationary state for this system, which for our choice of parameter values for  $\alpha$  and  $\beta$  is attained in approximately 100 time steps. It is also worth noting, that the dependence of the steady value with  $\Omega$  shows to be non-monotonic. We therefore turn our attention to the study of this stationary state as a function of  $\Omega$ .

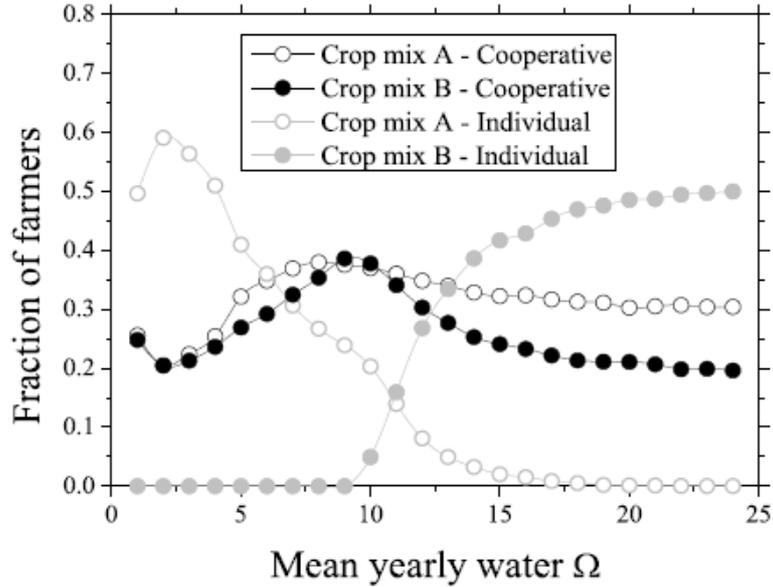
Fig. 4 shows both the fraction of farmers that select crop mix A and the fraction of farmers that adopt a cooperative strategy in the stationary state, as a function of the mean yearly water  $\Omega$ . Each point of the curves has been obtained by averaging over 20 realisations. As we have discussed above, we can see that when the available water increases, the selection of crop mix A becomes less convenient, since more profit can be obtained from crop mix B. Therefore, less farmers select crop mix A when more water is available. However, it is interesting to point out that this fraction does not vanish for large values of  $\Omega$ , as would be expected. This means that, even when crop mix B promises to give much higher revenues, a significant amount of farmers still choose crop mix A, which will most likely return half of the profit that could otherwise be made. We will see later that this is only the case for cooperative farmers.

As noted before, the fraction of cooperative farmers presents a non-monotonous dependence on the parameter  $\Omega$ , having a maximum near the value  $\Omega \approx 10$ , which is the value for which the profit of both crop mixes is the same. For this value, no crop mix presents obvious advantages in terms of average available water. Therefore, uncertainty on which crop mix is more convenient is highest. It is also interesting to note that the fraction of cooperative farmers is typically above 0,5, meaning that, usually, the majority of farmers decide to cooperate. This can, of course, be very sensitive to our choice of parameter  $a$ , and is not to be taken as a general result.

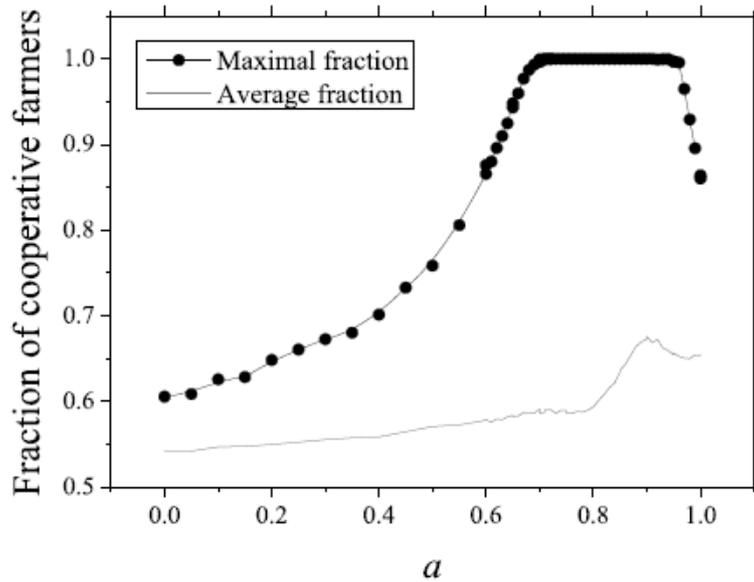


**Figure 5.** Mean profit of different strategies and crops mixes for different regimes of water availability. The points in the curves have been obtained by averaging over farmers, time and different realizations.

In Fig. 5 we can see the different mean profits as a function of the mean yearly water, where the mean profits have been obtained averaging over time in the stationary state, and over 20 different realizations. As suspected, for abundant water regimes, it is most convenient to select crop mix B, while for water scarcity, crop mix A is more suitable. In regimes where the convenience of either crop mix is less evident, adopting a cooperative strategy yields optimal results, which leads to the fraction of cooperative farmers having a maximum. However, it is remarkable that cooperation gives suboptimal revenues in regimes of water abundance and scarcity. To gain better insight on this, we study the fraction of farmers in all possible states when the system has reached stationarity as a function of parameter  $\Omega$ . These results are



**Figure 6.** Fraction of farmers selecting different crop mixes and strategies in the long-run as a function of the mean yearly water.



**Figure 7.** Maximal and mean values of the fraction of cooperative farmers as a function of satiscing parameter  $a$  for the range  $1 \leq \Omega \leq 25$ . For each value of  $a$  in the evaluation function  $F(\{f_j\}; f_i)$ , the fraction of cooperative agents in the stationary state is calculated for all values of mean water  $\Omega$ , and the maximum and the mean are extracted.

shown in Fig. 6. Here we can see that, while individualist farmers choose the more convenient crop mix when the water is either scarce or abundant, a significant fraction of cooperative farmers do not. Some cooperators may plant crop A when there is enough water for crop B, and vice-versa, some farmers in the cooperative choose crop mix B in regimes of water scarcity, being a consistently bad choice. Since revenues are shared equally between all cooperative farmers, the performance of a single farmer does not affect that farmer's profit significantly. In this situation, the persistence of less productive farmers makes cooperation inefficient.

Finally, in an attempt to reduce arbitrariness in our model, we analyse how valid some of these conclusions are for different values of the satiscing parameter  $\alpha$  in the evaluation

function  $F(\{f_j\}; f_i)$ . In Fig. 7 we can see both the maximum and the average fraction of cooperative farmers for different values of  $a$ . For each value of  $a$ , the stationary state of the system is analysed for a range of values of mean yearly water  $\Omega$  between 1 and 25. For this range, the maximum and the average fraction of cooperative farmers are extracted and plotted as functions of  $a$ . We can see that for large values of  $a$ , that is, when only the highest performances are acceptable, cooperativity can reach very high levels, and in several cases it can be the strategy adopted by the entire system. However, the mean value of the fraction of cooperative farmers increases only slightly, indicating that this high level of cooperativity occurs only for a narrow range of climate conditions, namely, in the vicinity of  $\Omega \approx \omega_0$ . This supports the idea that cooperativity seems to be the most convenient strategy in regimes of uncertainty, regardless of how permissive one is with other farmers' inefficiency.

## CONCLUSIONS

We have developed and studied a simple agent-based model for production in an artificial farming community, in which farmers make two decisions regarding their production every year, namely, the kind of crop they will farm, and whether they will produce and market their harvest by themselves, or join a cooperative corporation sharing both resources and profits with other farmers in the cooperative. Their decisions are made probabilistically in terms of the variables that characterize each farmer. These change according to past experience, reinforcing the probabilities of selecting strategies that were successful in the past.

Analysing the behaviour of this system under different regimes of the parameter that characterises the climate (water availability), we have observed that the number of farmers that adopt cooperative strategies maximizes when climate conditions make it least obvious which crop mix to select. In these regimes, it has also been seen that the mean profit of farmers who decide to cooperate is accordingly higher than the profit of those who decide to produce individually. In this sense, we can understand that cooperation is the optimal strategy in situations of uncertainty. In other words, cooperation in our system serves as an operative way to minimize risks, allowing for a more efficient way of allocating resources to minimize losses.

However, when one of the crop mixes is clearly more convenient, the presence of farmers in the cooperative that select the wrong crop mix makes cooperation inefficient<sup>3</sup>. Since profits are shared equally amongst all cooperative farmers, some might still have an acceptable performance despite selecting a clearly inconvenient crop mix. Unproductive farmers might then be satisfied with sharing profits produced by other farmers, and they become a burden for the cooperative. In turn, farmers in the cooperative that do make the right choices will have returns not too much lower than the ones they could be having farming individually, and will not have enough incentive to quit the cooperative. Therefore, the profit of cooperative farmers is sub-optimal in regimes of abundance and scarcity of resources. In other words, bad crop choices can be residually consistent inside a cooperative, and while the sharing of water and revenues can act as a shield towards risk, it can blind farmers with respect to what is convenient and therefore reduce competitiveness.

It is important to note that this is not a case that would fit the paradigms of the Free-Rider problem (Tragedy of the Commons) [16, 17], since farmers choosing suboptimal crops are not benefited from it. Nevertheless, it is clearly a situation of Pareto inefficiency, since choosing a more appropriate crop mix would yield benefits for every cooperative farmer, without reducing the profits of any other [18].

Farmers adopting individualist strategies seem to adapt much better to water availability extremes, in the sense that when water is either abundant or scarce, individualist farmers always select the crop mix which is clearly more convenient.

## **FUTURE WORK**

Many generalisations and extensions of this system can readily be made. One of them is the inclusion of a water market in which farmers adopting individualist production strategies could trade excess water for profit. In this way, farmers who selected crop mix B and get caught in a draught could still make a profit by irrigating with water that farmers who selected crop mix A might have in excess. Water marketing systems, where transfers are made between users, are developing fast in some parts of the United States and Europe, attempting to allocate water more efficiently in times of scarcity and change.

Also, the inclusion of a market for harvests could make the model more realistic. In this way, even when water is excessive, crop mix B might not be the most convenient if every farmer is using it, since the market would be saturated and the price of A would increase. The variability of prices according to the production would introduce another risk factor that could lead to different results.

Finally, an interesting addition to the system could be that of considering a spatial distribution for the farms. In this system, cooperation could be allowed between neighbouring farmers, permitting the possible emergence of several cooperative corporations which would compete amongst each other.

## **REMARKS**

<sup>1</sup>Although this is a drastic limitation of the model, we would like to point out it approximates the situation in Mendoza, where a clear distinction can be made between high-quality wine grapes, which demand very specific amounts of water, and other crops whose irrigation requirements are far less strict.

<sup>2</sup>The equations for the production functions of crop A and B are  $f^A = \text{th}(\omega/2)$  and  $f^B = 2 \cdot (1 + e^{10-\omega})^{-1}$ , respectively. These functions were chosen arbitrarily because of their shape.

<sup>3</sup>It should be noted that by efficiency, we refer to the capability of making optimal strategic choices and not to the operation of the cooperative. In our model, cooperatives are operationally efficient since they do indeed distribute water in a way as to produce maximum profit.

## **ACKNOWLEDGMENTS**

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## Izviranje kooperacije u modelu poljoprivredne proizvodnje

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### SAŽETAK

Izviranje kooperacije u pojednostavljenom društvu poljoprivrednika proučavano je modeliranjem pomoću agenata. Model se sastoji od N agenata jednakog pristupa vodi, čija dostupnost nasumično fluktuiru u vremenu. Svaki agent donosi dvije odluke svake sezone sijanja obzirom na: (1) vrste koje će zasijati te (2) hoće li se pridružiti kooperativnoj grupi koja raspodjeljuje vodu među poljoprivrednicima tako da maksimiziraju proizvodnju i jednako raspodjele prihode. Rezultati pokazuju kako je stupanj u kojemu se poljoprivrednici odlučuju za kooperaciju znatno ovisan o dostupnosti vode. Rezultati upućuju na izviranje kooperacije kao način adaptacije na nepredvidljivu okolinu pri kojem se minimizira individualni rizik.

### KLJUČNE RIJEČI

kooperacija, nesigurnost vode, modeliranje pomoću agenata, raspodjela resursa

# MATHEMATICAL MODELS AND EQUILIBRIUM IN IRREVERSIBLE MICROECONOMICS

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## ABSTRACT

A set of equilibrium states in a system consisting of economic agents, economic reservoirs, and firms is considered. Methods of irreversible microeconomics are used. We show that direct sale/purchase leads to an equilibrium state which depends upon the coefficients of supply/demand functions. To reach the unique equilibrium state it is necessary to add either monetary exchange or an intermediate firm.

## KEY WORDS

irreversible microeconomics, mathematical models, thermodynamics

## CLASSIFICATION

JEL: C63, D01, D83

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## MODELS OF IRREVERSIBLE MICROECONOMICS

Microeconomics includes the subject of the interaction between economic agents. Each agent is a set of individuals. Number of individuals is huge in real economic systems and it is not possible to control each of them. Averaged parameters can be used as controls in these systems. Prices are parameters of that kind. The same situation is in thermodynamic systems where parameters averaged on the set of particles can be controls. These parameters in thermodynamics are temperatures, concentrations etc. That is why microeconomic systems as thermodynamic ones can be considered as macrosystems [1]. In this sense they are analogous to thermodynamic systems. A state of the economic agent is described by stocks of resources. One of the resources should act as money: this resource is called a *basic resource*. Each economic agent needs money.

The system is *isolated* if there is no resource to exchange between the system and its surroundings. The system is closed if the system and its surroundings can exchange only one resource and this resource is basic one. An economic system is open if there is more than one resource to exchange with the system's surroundings.

We mention some facts which should be accounted for in a mathematical model:

- there are several processes in economic systems. They are the exchange of resources, consumption, and production. We assume that behaviour of economic agents is rational. It means that states of the agents are comparable. Thus, we can introduce a *wealth function* depending on the economic agent state [2] and postulate that the wealth function is an objective function for an economic agent.
- the lower the price the lower the intensity of sales of economic agents. There exists a price when the intensity is equal to zero. This price is called the agent's value of the resource. It means that if the price is greater than the value then the agent sells the resource, and if the price is greater than the value then the agent buys the resource. If the price is equal to the value then there is no exchange process; the agent is in equilibrium state with its environment.
- resource flows between economic agents in an isolated system without consumption and production processes run down in time and tend to zero. In the extreme case all economic agents appraise resources equally.
- it is impossible to increase the wealth of an economic agent due to resources exchange with another agent so that the stocks of the latter at the beginning and the end of the process are the same.
- extraction of money is possible in a closed system with the help of an intermediate firm only. In this process the firm should buy and sell the resources at different prices.

These facts need an adequate mathematical description. Some features are familiar to thermodynamic systems, others are different. There are many published works devoted to thermodynamic approaches to economic processes [3 - 8], but irreversible microeconomics is at a nascent stage. Many problems are yet unsolved, e.g., the determination of the optimal regime of the intermediate firm under different conditions.

Two features of economic systems are important. One of them is called the voluntary principle. It means that it is not possible to donate any amount of a resource. Each transfer of a resource should be indemnified by another resource.

The other feature is the following. Spontaneous processes of interaction exist in any economic macrosystem. These processes do not demand any changes in the neighbourhood. However, some external influence is necessary to return the system to its initial state. These processes

are irreversible. We need a value characterizing the degree of irreversibility of such systems. The value reaches its maximum at the equilibrium state like entropy in thermodynamics.

Let us denote by  $N = (N_1; \dots, N_k)$  the vector of resources stocks and by  $M$  the amount of the basic resource. For all economic agents  $M$  has a common dimension. In other words, all agents use the same currency as money. Values  $N$  and  $M$  are extensive values: if we divide a homogenous economic agent the values divide in the same proportion. Intensive values exist as well. They are the value of resources  $p = (p_1, \dots, p_k)$ . The values depend on resources stocks. A common assumption is that if several economic agents are in equilibrium state then the values do not change. So, values are assumed to be homogeneous functions of zero degree.

Value of the resource  $p_i$  is the minimal (maximal) price in units of the basic resource when the economic agent is ready to sell (buy) the resource with the infinitesimal small intensity.

In exchange processes economic agents can play the role of other seller or buyer. They are characterized by demand functions. The demand function for resource  $i$  shows the intensity of purchasing the resource with respect to its value  $p_i$  and price  $c_i$ . Demand function  $n_i(p_i, c_i)$ :

$$\text{sign}[n_i(p_i, c_i)] = \text{sign}(p_i - c_i), \quad (1a)$$

$$n_i(p_i, c_i) = 0 \quad \text{if } c_i = p_i, \quad (1b)$$

$$\frac{\partial n_i}{\partial c_i} < 0 \quad \& \quad \frac{\partial n_i}{\partial p_i} > 0, \quad i = 1, \dots, k, \quad (1c)$$

determines the kinetics of resource exchange processes. The value of  $n_i$  can either be positive when the agent buys the resource or negative when the agent sells the resource. Recall that the dimension of both price and value is the ratio of the basic resource to the resource units.

There are three kinds of economic agents:

- the values of some economic agents depend on the stocks of resources. Let us call such agents *passive economic agents* or *economic agents with finite capacity*. The larger the stock of a resource the less its value. The larger the stock of the base resource the larger the value of all other resources.
- then there are agents whose values do not depend upon the stocks of resources. Let us call them *economic agents with infinite capacity*, *economic reservoirs* or *markets*. Note that the introduced terms do not purport absence of free will of individuals. It was postulated that the agent can consist of a large number of individuals. Due to the law of large numbers averaged behavior of the individuals allows one to introduce regular laws like supply/demand functions. These laws can either depend on amount of accumulated stocks of the resources or not. It is occasioned by scale of the agent. In the case of enormous scale of the agent resource fluxes cannot change the stocks significantly, that is why values of such an agent are either constant or changed due to exogenic factors.
- Intermediate firms are economic agents who can change their values voluntarily. These agents either set prices or intensities of resources flows so as to maximize the intensity of extracted basic resources. This firm works like the working fluid of a heat engine. It can establish contact with other economic agents and its relationships are characterised by controls.

## **WEALTH FUNCTION AND ITS CHARACTERISTICS**

Macrosystem approach to economic agents allows us to introduce a function  $S$  depending on  $N$  and  $M$ . Each agent wants to maximize this function depending on its economic agent type. Following [2] we shall call it a *wealth function*. In early works [3, 5] this function was called a structure function. The wealth function and the values of resources are interrelated by the following ratios:

$$p_0 = \frac{\partial S}{\partial M}, \quad p_i = \frac{1}{p_0} \frac{\partial S}{\partial N_i}, \quad i = 1, \dots, k, \quad (2)$$

The wealth function is homogenous of degree one. It means that if we multiply all arguments by a constant then the result will be multiplied by the same constant. Therefore, we can rewrite it using the Euler theorem:

$$S(N, M) = p_0 \left( M + \sum_i p_i M_i \right) = p_0 U(N, M), \quad p_0(N, M) > 0. \quad (3)$$

The proof of the existence of the wealth function was provided by L.I. Rozonoer and is published in the Appendix of review [7]. In [9] the proof was extended to the general case of  $k$  resources. The term *capitalization* (the  $U$  function) was introduced as an internal estimation of the cost of all resources collected by an economic agent. The principle of voluntary exchange does not permit any exchange process if the wealth of at least one economic agent decreases during the process.

In order to prove the existence of entropy in thermodynamic systems we need to introduce an integrating multiplier which has the same dimension  $K^{-1}$  for any subsystem. The dimension of the integrating multiplier  $p_0$  introduced for the economic system depends on the dimension of the wealth function. The dimension of the wealth function can be different for different agents because during the exchange process the value of money will not be equal. We will show that this feature leads to the dependency of the equilibrium state on the kinetics of exchange (the *demand function*).

One interesting case of economic system occurs when there are several currencies there. Equilibrium state in an isolated system with foreign exchange market corresponds to equal values of currencies. If the market is an economic reservoir then  $p_0$  for each economic agent determines by equilibrium price of corresponding currency at the market. It means that in this case value  $p_0$  does not depend on state of the economic agent.

In the following sections we will consider equilibrium states in isolated and closed systems and in systems including intermediate firms.

## DIRECT EXCHANGE OF RESOURCES

In an isolated system the total amount of resources does not change as there is neither consumption nor production. Let us assume that

- the amount of money as a proportion of the amount of other resources is constant. In other words, the sum of intensities of money fluxes equals zero,
- during the voluntary exchange of resources economic agents with relatively high values  $p_{ij}$  ( $i$  denoting a resource and  $j$  an economic agent) buy the resource and those with relatively low values sell the resource.

In that case, we can introduce value of cost of resources  $F = \sum_i p_i N_i$ . During exchange process wealth function does not decrease. Rate of  $F$  changing does not decrease, too. So, we have the following expression:

$$\sigma \equiv \frac{dF}{dt} = \sum_j \sum_i p_{ij}(N_j, M_j) \frac{dN_{ij}}{dt} \geq 0, \quad (4)$$

where  $\sigma$  can be interpreted as the sum of the rates of capitalizations of economic agents when the values of money  $p_{0j}$  are constants.

Equilibrium state is stable. In this state value of resources cost is maximal and resources fluxes  $n_{ij} = dN_{ij}/dt_i$  are equal to zero.

Let us consider a system where the exchange of resources takes place. If resources fluxes are not infinitely large the equilibrium state is reached asymptotically. However, the equilibrium is not unique. It is a function of the parameters of the exchange kinetics.

In the equilibrium state the following balance equations are true:

$$\sum_{j=1}^n \overline{N_{ij}} = N_{i0}, \quad i = 1, \dots, k, \quad (5)$$

$$\sum_{j=1}^n \overline{M_j} = M_0. \quad (6)$$

Here  $\overline{M_{ij}}$  and  $\overline{N_{ij}}$  are equilibrium values of money and resources stocks. Adding equilibrium conditions to equations (5) and (6):

$$p_{ij}(\overline{N_j}, \overline{M_j}) = \overline{p_i}, \quad i = 1, \dots, k; \quad j = 1, \dots, n. \quad (7)$$

There are  $(k + 1)n + k$  unknowns in the system of equations whereas the number of equations (5-7) is  $k(n + 1) + 1$ . Taking into account that  $n > 2$  we have more unknowns than equations. That is why the equilibrium state is determined not uniquely. The equilibrium state depends on the kinetics of exchange.

A consequence of the characteristics of functions (1) is that the capitalization of each economic agent cannot decrease. So, the following inequalities are true.

$$\sum_{i=1}^k (\overline{p_{ij}} \overline{N_{ij}} - p_{ij0} N_{ij0}) + \overline{M_j} - M_{j0} \geq 0, \quad j = 1, \dots, n. \quad (8)$$

We can solve for the equilibrium state if we add equations for the kinetics of resources exchange between the  $j$ -th and  $v$ -th agent:

$$n_{ijv}(p_{ij}, c_{ijv}) = -n_{ijv}(c_{ijv}, p_{ij}), \quad (9)$$

$$m_{jv} = \sum_{i=1}^k c_{ijv} n_{ijv}(p_{ij}, c_{ijv}) = -m_{vj} \quad j, v = 1, \dots, n. \quad (10)$$

These equations allow us to derive the dependence of price on time. The dependency of the system state on time can be found from the differential equations,

$$\begin{aligned} \frac{dM}{dt} &= -\sum_{i=1}^k \sum_{v=1}^n c_{ijv} n_{ijv}(p_{ij}, c_{ijv}), \\ \frac{dN_{ij}}{dt} &= \sum_{v=1}^n n_{ijv}(p_{ij}, c_{ijv}), \quad j = 1, \dots, n; \quad i = 1, \dots, k, \end{aligned} \quad (11)$$

with given initial states. The solution of these equations tends to the equilibrium state as  $t \rightarrow \infty$ .

As an example, let us consider the case of a scalar resource exchange between two agents. At each moment of time, the price of the resource is determined by the equation

$$n_1(p_1, c) = n_2(p_2, c), \quad (12)$$

This price should fulfill the inequality

$$p_1 \geq c \geq p_2, \quad (13)$$

to meet the principle of voluntary exchange. So, if the equations of resource exchange kinetics are linear

$$n_1(p_1, c) = \alpha(p_1 - c), \quad (14)$$

$$n_2(p_2, c) = \alpha(p_2 - c), \quad (15)$$

then the price can be found from the equation

$$c(p_1, p_2, \alpha_1, \alpha_2) = \frac{\alpha_1 p_1 + \alpha_2 p_2}{\alpha_1 + \alpha_2}. \quad (16)$$

Furthermore

$$\frac{dM_1}{dN_1} = \frac{dM_2}{dN_2} = -c, \quad dN_2 = -dN_1, \quad (17)$$

$$\begin{aligned} N_1(0) &= N_1^0, & M_1(0) &= M_1^0, \\ N_2(0) &= N_2^0 = N_0 - N_1^0, & M_2(0) &= M_2^0 = M_0 - M_1^0. \end{aligned}$$

These conditions allow us to express  $c$ ,  $M_1$ ,  $M_2$ ,  $N_2$  as functions of  $N_1$ . The change in capitalization can be calculated as

$$\Delta U_1 = \int_0^{\bar{N}_1} [p_1(N) - c(N)] dN, \quad (18)$$

$$\Delta U_2 = \int_0^{\bar{N}_1} [c(N) - p_2(N)] dN, \quad (19)$$

for each agent and

$$\Delta U = \Delta U_1 + \Delta U_2 = \Delta F = \int_0^{\bar{N}_1} [p_1(N) - p_2(N)] dN, \quad (20)$$

for the system as a whole. In equations (18 - 20) values  $p_1$ ,  $p_2$  depend on  $N_1$  only, because other variables are functions of  $N_1$ . In an isolated system, the total amount of money does not change, that is,  $\Delta U = \Delta F$ .

Let us use the Edgeworth diagram to illustrate the model. This diagram consists of two sets of equipotential lines of the wealth functions for both agents. One set of the lines is rotated through  $180^\circ$ , initial point of the system of axes is at point  $(M_1^0, N_1^0)$ .

Note that differential

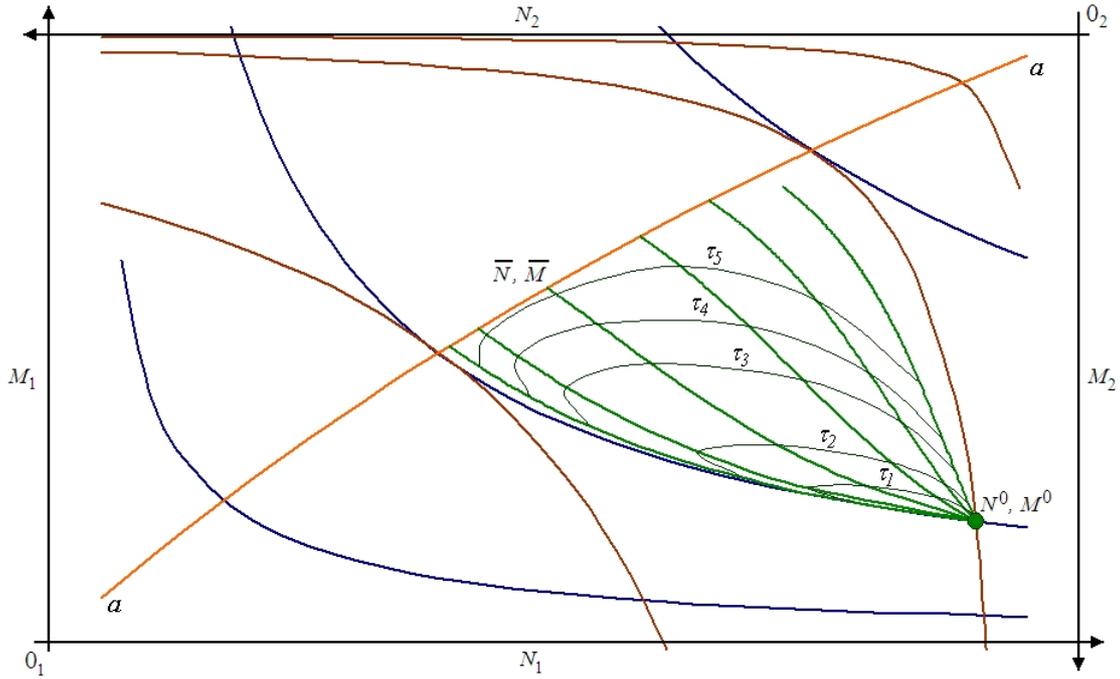
$$dS = \frac{\partial S}{\partial N} dN + \frac{\partial S}{\partial M} dM = 0,$$

holds along an equipotential line.

According to (3) a slope of a tangent to an equipotential line equals to  $\partial M = \partial N = -p$ . It means that all points where two equipotential lines touch each other meet equilibrium conditions (7). A set of these points is the equilibrium set (contract line). However, it is not impossible to reach any point of the contract line from the given initial state during the finite period of time.

*A point on the contract line is accessible if conditions (13) are satisfied. These conditions guarantee that the wealth function and capitalization cannot decrease for both agents. It is not possible to move along the contract curve without decreasing the wealth of one agent. So the set of equilibrium states is a Pareto set.*

The equilibrium state depends on the equations of resource exchange kinetics. The state cannot be reached in finite time. Sets of attainability for linear laws of kinetics and trajectories of exchange in the Edgeworth diagram are shown in Fig. 1.



**Figure 1.** Edgeworth diagram of sets of attainability for linear laws of kinetics and trajectories of exchange.

## PROFITLESS AUCTION SALE

Let price  $c$  be fixed at the value  $c = p$ . Then the trajectory of the system state is a straight line. The slope of the line is  $c$ . It should be chosen to equal the values of the resources at the equilibrium state. In this case the capitalization of both agents will be constant. The corresponding economic process is a process of profitless auction sale.

The equilibrium conditions consist of the balance equations (5) and equations (7). Prices  $c$  should be fixed on the right-hand side. The following equations are true for money stocks:

$$\overline{M}_j = M_j(0) - \sum_{i=1}^k \overline{c} [\overline{N}_{ij} - N_{ij}(0)], \quad j = 1, \dots, n. \quad (21)$$

From these equations we obtain (6) after taking (5) into account.

In sum, we have  $k(1 + n) + n$  equations in as many unknowns. That is why the equilibrium state is determined. An illustration of profitless auction sale is the market for electrical energy where power stations and consumers propose their supply and demand dependencies respectively and a dispatcher assigns the equilibrium price to equalize supply and demand. The equilibrium state in the resource exchange at a profitless exchange corresponds to the maximum of the sum of wealth functions. The problem is to maximize the sum of two wealth functions subject to the resources constraint conservation laws. The Lagrangian is

$$L = S_1(\overline{N}_1, \overline{M}_1) + S_2(\overline{N}_2, \overline{M}_2) + \lambda_1(\overline{N}_1 + \overline{N}_2) + \lambda_2(\overline{M}_1 + \overline{M}_2).$$

The stationary conditions are

$$\frac{\partial S_j}{\partial N_j} = \lambda_1, \quad \frac{\partial S_j}{\partial M_j} = \lambda_2, \quad j = 1, 2. \quad (22)$$

Because

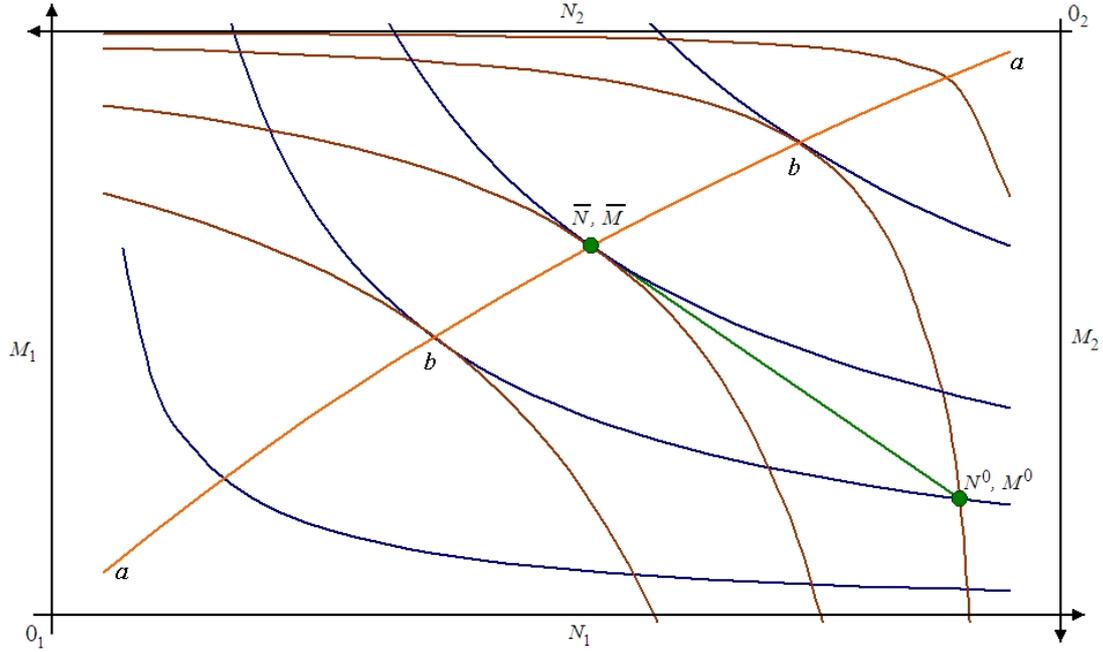
$$\frac{\partial S_j}{\partial M_j} = p_{j0}(\bar{N}_j, \bar{M}_j), \quad \frac{\partial S_j}{\partial N_j} = p_{j0}(\bar{N}_j, \bar{M}_j) p_j(\bar{N}_j, \bar{M}_j)$$

both the values of the resources and the values of money should be equal for all agents. The result holds if both wealth functions have the same dimension.

Price  $c$  is equal to the value  $p$  in the equilibrium state. This value is used in equation (7). Conditions (5 - 7) should be supplemented with the following equation:

$$\bar{M}_2 = (N_0^2 - \bar{N}_2) \bar{p} + M_2^0. \quad (23)$$

It allows us to determine the final state (Fig. 2).



**Figure 2.** Edgeworth diagram of final state in the example of two-state profitless auction sale.

## EXCHANGE BY BARTER

We have a system of agents. Each agent has stocks of several resources. The initial amounts and their values are known. During the exchange process the quantity of money of each economic agent is fixed. The kinetics of resources exchange is given by

$$\frac{dN_j}{dt} = \sum_{v=1}^n A_{jv} \Delta p_{jv}, \quad v \neq j, \quad j = 1, \dots, n, \quad (24)$$

where  $A_{jv}$  are the kinetic coefficients,  $\Delta p_{jv}$  are differences in the values of the resources. In equilibrium  $\Delta p_{jv} = 0$  according to (7). These equations and balances (5) determine equilibrium distribution of resources.

The following proposition is true for isolated systems.

**Statement 1:** *At the fixed distribution of money, the distribution of other resources in the equilibrium state corresponds to the maximum of total capitalization of economic agents subject to*

$$F(M_0, \bar{N}) = \sum_i \sum_j p_{ij}(\bar{N}_{ij}, M_j^0) \bar{N}_{ij} \rightarrow \max_{N_{ij}}, \quad (25)$$

$$\sum_i \overline{N_{ij}} = N_i^0, \quad \overline{N_{ij}} \geq 0, \quad i = 1, \dots, k; \quad j = 1, \dots, n. \quad (26)$$

If the equilibrium stocks of all resources are positive then this maximum is equal to

$$F^*(M_0) = \sum_i \overline{p}_i N_i^0, \quad (27)$$

where  $\overline{p}_i$  is the equilibrium value of the  $i$ -th resource.

## MONETARY EXCHANGE MARKET

Let us consider a system containing a monetary exchange market. Each agent can change his currency for a basic resource. The exchange rate is considered to be constant and independent of the intensity of currency fluctuations. It means that the monetary exchange market is an economic reservoir. So, the value  $p_{0j}$  of the basic resource is constant to.

The existence of a monetary exchange market influences both the equilibrium state as well as the set of possible states of agents significantly.

Condition

$$p_{0j}(N, M) = k_j, \quad j = 1, \dots, n, \quad (28)$$

connects stocks of basic and other resources and, therefore, restricts the states of each agent. The differential of capitalizations

$$dU_j = dM_j + \sum_i p_i(N_j, M_i) dN_{ij},$$

is a total differential on this set. If we express  $M_j(N_j)$  from equation (28) and substitute this expression to  $p_{ij}$  then the following  $n(n-1)/2$  relations hold for values  $\tilde{p}_j = p_j(N, M(N))$ :

$$\frac{\partial \tilde{p}_{ij}}{\partial N_{ij}} = \frac{\partial \tilde{p}_{ij}}{\partial N_{ij}}, \quad i, j = 1, \dots, n. \quad (29)$$

Equilibrium in the closed system with a monetary exchange market is unique. It is determined by conditions (5), (7) with (28) additionally. The sum of reduced wealth functions

$$S = \sum_j S_j(\overline{N}_j, \overline{M}_j) \cdot k_j^{-1}, \quad (30)$$

with conditions (5) reaches the maximum in equilibrium state.

## SYSTEMS WITH AN INTERMEDIATE FIRM

Consider a system consisting of  $n$  economic agents. Exchange between agents is possible only through an intermediate firm. The initial state and wealth function of each agent is known. The problems are:

- to determine optimal time dependence of prices to maximize profit of the intermediate firm,
- to determine the maximal amount of basic resource which can be accumulated by the intermediate firm (profitability of the system).

First, we assume that the duration of the process is not restricted. For an isolated system the extracted resource is

$$E_\infty = \sum_{j=1}^m [M_j(0) - \overline{M}_j] \rightarrow \max, \quad (31)$$

where  $\overline{M}_j$  is the amount of the basic resource of the  $j$ -th agent in the equilibrium state.

The profitability of the system will emerge from the equilibrium state. The value of the basic resource  $p_{0j}$  of each agent is positive. Hence the maximal amount of the basic resource extracted from the system in conditions of voluntariness corresponds to equations

$$S_j(\overline{N}_j, \overline{M}_j) = S_j[N_j(0), M_j(0)], \quad j = 1, \dots, n. \quad (32)$$

These equations can be called conditions of reversibility of the process.

The values of each resource for each agent are equal in equilibrium:

$$p_{ij}(\overline{N}_j, \overline{M}_j) = \frac{\partial S_j}{\partial N_{ij}} \left( \frac{\partial S_j}{\partial M_j} \right)^{-1} = \overline{p}_i, \quad i = 1, \dots, k; \quad j = 1, \dots, n. \quad (33)$$

Conditions (32), (33) together with the balance relations

$$\sum_{j=1}^n \overline{N}_{ij} = \sum_{j=1}^n N_{ij}(0), \quad i = 1, \dots, k. \quad (34)$$

determine both the equilibrium state of the system and its profitability  $E_\infty$ . So we can state that *equilibrium in the system with the intermediate firm is determined by the initial state of the system and does not depend on the kinetics of resource exchange*. There is, thus, a difference between economic systems with and without the intermediate firm.

In an economy with two agents and a scalar resource the states of the agents move along the corresponding level curves of the wealth functions. The distance between the equilibrium states is equal to the profitability of the system.

If the duration of the process is restricted then the value of the resource extracted by the intermediate firm is less than the profitability.

## SYSTEMS WITH AN INTERMEDIATE FIRM CONTAINING AN ECONOMIC RESERVOIR

In a system with an economic reservoir equilibrium values are fixed for all agents and equal to the value of the reservoir. The number of equations here is  $m \cdot n$

$$\overline{p}_{ij}(\overline{N}_j, \overline{M}_j) = p_{ri}, \quad i = 1, \dots, k; \quad j = 1, \dots, n. \quad (35)$$

Here  $p_{ri}$  is the value of the  $i$ -th resource of the economic reservoir.

If there is no intermediate firm the remaining  $n$  equations can be written using the kinetic equations of exchange between agents and between agents and the reservoir. If an intermediate firm exists conditions of exchange reversibility should be added.

$$S_j(N_{j0}, M_{j0}) = S_j(\overline{N}_j, \overline{M}_j) \quad j = 1, \dots, n. \quad (36)$$

Let us use the Edgeworth box to depict the exchange of the scalar resource between two agents. The equilibrium state corresponds to the two points in the diagram. The values of the wealth functions of agents do not change for a system without an intermediate firm. But the distance between the points is composed of extracted profit and the amount of the resource obtained from the reservoir.

It can be shown that the maximal profit of the intermediate firm corresponds to the value  $p_r$  such that the second derivatives of the basic resource with respect to  $N_j$ , calculated along equipotential curves are equal in absolute value and opposite in sign.

## CONCLUSION

Equilibrium conditions are obtained for different structures of a system. In some cases, the equilibrium state depends on the kinetics of resource exchange, in others on the initial state alone.

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## MATEMATIČKI MODELI I STANJE RAVNOTEŽE U IREVERZIBILNOJ MIKROEKONOMIJI

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### SAŽETAK

Razmatran je skup ravnotežnih stanja sustava koji se sastoji od ekonomskih agenata, ekonomskih rezervoara i tvrtki. Korištene su metode ireverzibilne mikroekonomije. Pokazano je da izravna kupoprodaja vodi na ravnotežno stanje koje ovisi o koeficijentima funkcija ponude i potražnje. Za postizanje jedinstvenog ravnotežnog stanja potrebna je ili kupnja novcem, ili tvrtka za međutransakcije.

### KLJUČNE RIJEČI

ireverzibilna mikroekonomija, matematički modeli, termodinamika

# REGIME-CHANGES IN A STOCK-FLOW-CONSISTENT MODEL

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## ABSTRACT

We explore the dynamical properties of the Godley-Lavoie model with a focus on Central Bank horizons. The stability properties of modes of regulation are traced from a regime of private bank money to the current crisis with the Central Bank levers of short-term bonds issue to the emerging policy regime of long-term bonds as built-in stabilizers.

## KEY WORDS

financial crises, long-term bonds

## CLASSIFICATION

JEL: E12, E42

## **INTRODUCTION**

Capitalism is a mode of regulation. Its stability properties are not given and there is no stationary state to which it converges [1]. In response, institutions, especially codes of conduct pertaining to the common good, evolve and are embodied in empirical rules. Collective beliefs are represented in formal apparatus like the monetary and fiscal powers of the state. But generally, social norms are informal codes that evolve especially when individual actions cause negative side effects on others [2]. They serve the function of restraining egoistic impulses in favour of superior collective outcomes. Norms prescribe and proscribe behaviour. Below, we will be concerned with the mechanisms of accumulation that ensure coordination between the real and the financial sphere. Cooperation between the relevant stakeholders must be tacit in a social equilibrium and we contend that individual-choice theories are a less than satisfactory level of explanation. Thus, we demur from regarding institutions as solutions of evolutionary games. Intertemporal equilibrium and stability are implied. Conventions, in this sense, may do no more than deliver a temporary equilibrium. They do not eliminate the ineluctable uncertainty under which agents must operate. Only, their representations of their dilemmas are transformed [3]. In evolutionary game theory, agents are backward-looking and naive. Their strategies, time after time, depend on past states of the world. On the other hand, cooperation can be supported over time by agents like the state that provide what the last authors referred to call “form-giving operations”. By delivering objects around which social relations can coalesce, only governments, say, can invest in form-giving policies across space and time. Indeed, the agenda of generalized Darwinism is being scrutinized critically [4, 5]. In political economy, variation is not blind nor selection passively endured, but actively pursued. In social evolution, the relative success of a “phenotype” does not depend on its capacity to reproduce but rather on the ability to assess and learn. The new principles of evolutionary biology of relevance for evolutionary economics are the systemic character of the genome which embraces cooperation, self-organization, and consciousness and the persuasive evidence that evolution is punctuated. For instance, the “new institutional” theories of finance are founded on the assumption that the macro implications of finance have to be derived from the study of the interaction between banks, financial markets, firms, and households [6]. One distinction that operates is that between liquidity and credit. The former pertains to the total volume of loanable funds available in the system, the latter to the total volume of loanable funds the banks are willing to provide. During a wave of pessimism, liquidity is rife but disbursement is low. In euphoric times, even if liquidity dries up, financial institutions will break the barriers of credit to borrowers.

Our thesis, then, is that the interdisciplinary algebra of complex systems must be forward-looking. The strategy must embrace novelty and surprise. Thus, while initial conditions matter in the particular solution of differential equations, they do not contain the path that societies may follow. In the case of the recent financial-real crisis, while many argue, after Hyman Minsky, that “it” happened in the past and will continue to happen again, the particular aspect of the present meltdown in the form of the construction of complex contracts requires special attention. Our interest lies in the revolutionary features of Federal Reserve policy to mitigate the recession. For the first time in the history of any central bank, the most important lender of last resort in the world has underwritten long-term real activity. Yet, formal models rationalizing this new stance are not to be found. We emphasize that our formulation below is a contribution to the heterodox literature on the subject. On the other side, the fungibility of the neoclassical model is being exploited and models are being written to include central bank balance sheets. “Forward-looking” must not be confused with

expectation formation by rational agents. Our model is structural, free of microeconomic optimization exercises. Variety will be generated by the state equations.

An early and unique treatment of the importance of stock-flow norms in the macroeconomic accounting process was the W. Godley and F. Cripps (1983) classic [7]. For instance, a variable like the fiscal stance played an important role in the steady state properties of the model. Recently, a completely self-contained and coherent stock-flow-consistent follow up is to be found in the W. Godley and M. Lavoie (2007) work of scholarship [8]. The role of norms is relatively muted here and with good reason. The notion of practices like the normal rate of capacity utilization in non neoclassical economics is slippery [9]. It is not generally the case that feedback mechanisms that pull the economy back to a normal state when it is out of kilter exist. Norms are only given at a particular point of time. Consequently, long-run analysis in the sense of fully-adjusted positions at normal rates is not likely to be meaningful. Attention should rest on short- or medium-run equilibria or provisional equilibria. They arise from the equality of savings and investment or between aggregate demand and supply. Furthermore, existing norms are different from targets. In truth, we can say no more than the rate of capacity utilization remains within an acceptable range in a given time frame. With radical uncertainty and the irreversibility of decisions about the capital stock, concepts like Harroddian instability might not be tractable. On the other hand, in the language of Régulation Theory drawing on the work of Pierre Bourdieu, habitus (our norms), while absorbed in extant practices, are also generative [10]. Change and even crises occur when habitus and the components of the economic fabric get desynchronized. The role of the state is immense here. The financial arms of the government, for example, can initiate a change in the rates of exchange of different forms of capital. The reactions of the old habitus no longer apply. New norms must apply to the newly constituted field. The mode of regulation coagulating since the early nineties has been christened “enlightened neoliberalism” [11]. The context is one in which the economic slowdown has been aggravated by the general hike in commodity prices. Industrial capital has dissolved its pact with labour and aligned itself with finance capital. Under the Fordist regime monetary policy was the handmaiden of fiscal policy [12]. Under financialization, the inflation rate is the sole variable of interest to the monetary authorities. The role of the state in the new dispensation is yet to be defined. In the sections that follow, the stock-flow norms that are both common across societies and that distinguish them are the degree of monopoly, unit costs, government expenditures and the direct tax rate.

All regimes are common in being successively obsessed with and then downplaying inflation in analytical discussion. There are universal constraints on the depletion of natural resources and food but Keynesian scholars focus on costs which vary from one country and region to another. At the moment, deflation is the common enemy and pulling economies out of the trap of inactivity the universal agenda. Quantities are back on the drawing board, prices have been relegated to the sidelines. Thinking about contemporary recessions in the mainstream has moved beyond sticky-wage treatments of the labour market to generating outcomes without any opportunities for Pareto-improving worker-employer bargains [13]. The focus is on job creation rather than job loss. The incentive for job creation is the margin between the productivity of a new worker and the wage to be paid, both as discounted present values. As regards termination, workers do not lose their jobs because their wages are too high. They are fired or exit because the opportunity cost in the market at large exceeds their productivity in their current job.

The Godley-Cripps and Godley-Lavoie architectures consist of sets of identities. We tease movement out of these relationships to provide a primary difference equation in the next section. The equation combines production and finance, micro and macro. The equation is stretched over the next two sections to correspond to changed monetary conditions. The first

two configurations are unstable. The introduction of the mechanism of the long-term bond provides a mild stability.

## PRIVATE BANK MONEY

According to the circuitistes, firms draw on their credit lines with banks when they have to make payments. Financial intermediaries grant loans,  $L$ , and simultaneously create deposits,  $M$ . The deposits are transferred to the accounts of the employees. Output that has been produced but not yet sold constitutes an increase in inventories,  $\Delta I$ . The increase in inventories is investment in working capital and the finance at this stage is initial finance. In sum,

$$\Delta L = \Delta M = \Delta I.$$

In an economy with only private banks, total gross production  $Y$  can be defined as the sum of payments to workers  $WB$ , the interest payments made by firms on their debts  $r_{t-1}^L L_{t-1}$ , and amortization funds [8]. The attraction of the framework emerges in the first few pages of the books in the inclusion of this fund in the accounts. Most texts ignore it because they are less than exacting in their accounting principles. Indeed, a complete model with amortization funds could generate inventory cycles with the importance of banks in the damping of those oscillations. Since our interest lies elsewhere, we exclude this component to define

$$Y = WB + r_{t-1}^L L_{t-1}$$

More familiarly, total income is the sum of wages and profits  $\Pi$ . That is,

$$Y \equiv WB + \Pi.$$

Now, the historic costs of current-period sales,  $HC$ , is defined as [7]

$$HC = WB - \Delta I.$$

The profit mark-up, therefore, is

$$\lambda \equiv \Pi / (HC + r_{t-1}^L L_{t-1}).$$

Finally, we introduce an identity in real terms [8]. Assume that firms produce ‘widgets’ which are identical physical objects. Then, in real terms,  $Y$  translates to production,  $y$ . In that case, unit costs  $UC = WB/y$ .

Putting all the equations and definitions together, we derive a markup-on-loans difference equation with the non homogenous part being the wage bill.

$$L_t = (1 + r_{t-1}^L - r_{t-1}^L / \lambda) L_{t-1} + UC \cdot y.$$

As indicated earlier, our fundamental equation combines the microeconomics of price and wage setting with the institution of relationship banking. Only the pure French circuit delivers a stationary state of the above system. Indeed, the initial disbursement of loans is the stationary state when there is no degree of monopoly and unit costs considerations are absent. Interest rates on loans charged by banks are a pathological draught at the moment of the closure of the circuit and should be equal to zero. With positive values of the above variables there is no meaningful solution.

Post Keynesians have been unappreciative of the possible historically contingency of banks in the sense we understand them [14]. In recent times, the demand for loans has risen to finance non-GDP growth. Bank finance has not gone hand-in-glove with real production. At the same time, the prospects for classical banking in the USA, in one prognosis, are rosy [15]. Since the nineties, the banking system has been steadily consolidating, capitalising on the removal of barriers to interstate banking. Canada and most European countries underwent their consolidation and merger phase in the late nineteenth and early twentieth centuries. The

outcome is few large banks along with a spread of community banks and small banks catering to local needs. The demise of Bear Stearns and Lehman has pushed other investment banks to merge with commercial banks. Thereby, they come under the rubric of federal regulation. The emergence of these universal banks, the claim goes, will return the system to the pre-Glass-Steagall institutional milieu and closer to the European model.

## GOVERNMENT MONEY AND PORTFOLIO CHOICE

To the extent that holders of government debt are not the same individuals as taxpayers whose payments will service it, or, to the extent that current holders do not internalise the cost even if the ultimate debt-service liability falls on them, government debt is an asset to the public [16]. Having more or less of it, consequently, affects the optimum holding of portfolios of other assets and liabilities not excluding the desired holding of physical capital stock. Indeed, the case for Central Bank (CB) liabilities is fundamental in the context of the current debate about the future of central banks. CB liabilities are the unit of account of an economy. Even in the instance when the commercial banks' demand for reserves is nil, CBs can influence interest rates if they pay a charge on reserves with them [17].

We introduce government bills,  $B$ , which pay an interest rate  $r$ . We ignore profits from the central bank in the government budget constraint and, consequently, an additional equation in high-powered money. With  $G$  denoting government expenditure and  $T$  direct taxes [8],

$$B_t = (1 + r_{t-1})B_{t-1} + (G - T),$$

that is the government must issue fresh bonds to finance its deficit [8] and

$$T = \theta(Y + r_{t-1}B_{t-1}),$$

where  $\theta$  is the direct tax rate. Both income from work and returns from assets are taxed.

With taxes endogenous the equations above combine to

$$B_t = (1 + r_{t-1} - \theta r_{t-1})B_{t-1} + \theta[\lambda L_t - \lambda(1 + r_{t-1}^L)L_{t-1}] - \theta(1 + \lambda)UC \cdot y + G,$$

(see the appendix for details). Our fundamental equation is now extended to include the items of the government budget constraint. The system is unstable. Note, for later reference, that the law of one interest rate does not hold and the policy rate is different from the market rate. The solution given by the null issue of government bonds is unsatisfactory. It is important for the CB to fix the rate of interest on its liability as a policy variable. There is no equilibrium value for a fiat unit of account [17]. However, the counterpart condition is institutionally interesting. According to the arithmetic, the level of government expenditure must be of an order in excess of the markup and unit costs in combination to ensure increasing loan disbursement. Directly, the equation describes the prospects of the recession, USA, 2007- [18]. There has been a fall in net lending to the private sector. Receipts in the form of new loans have been volatile and falling through 2008. The expectation is that gross lending will continue to fall below repayments for some time.

## LONG-TERM BONDS AND CAPITAL GAINS

More government debt to service presumably implies the responsibility to raise more revenue. At the same time, a large and liquid market for securities without default risk improves the efficiency of the market for private placements [16]. The market for corporate bonds in the USA operates in a manner different from thirty years ago when, due to legislative constraints, the Treasury was effectively unable to issue long-term bonds. Indeed, the time might be ripe for central banks to embrace a neo-Schumpeterian perspective and, instead on fixing short-term targets, address long-term issues in the accumulation of assets [19].

In a related development, during the last decade, governments have begun to partially fund pension fund obligations, recognizing that implicit pension debt represents a possible inter temporal fiscal constraint [20]. Furthermore, governments are alive to the threat that financial markets may penalize them in sovereign debt markets if this pension debt is not well managed. Consequently, new government pension funds have adopted portfolio investment strategies not different from private sector asset management. The success of these arrangements in providing long-term welfare rests on financial maturity as well as the capacity to adapt and innovate. Ultimately, they are social institutions and must represent the interests of the stakeholders. It is possible to design well-governed pension and retirement income institutions, public, private, and multinational, capable of hitting the golden mean between functional efficiency and stakeholder representation. The package might be implemented with different institutional amendments.

We introduce a third component of wealth, long-term government bonds, which provides an opportunity to introduce capital gains into the model. Now the accumulation of household wealth  $V$  can be defined as

$$V_t - V_{t-1} = (YD_t - C_t) + CG,$$

where  $YD$  is disposable income,  $C$  is consumption and  $CG$  is capital gains defined as follows where  $BL$  distinguish long-term bonds from the earlier.

$$CG = \Delta p_t^{BL} \cdot BL_{t-1}.$$

The Haig-Simons definition of income as consumption plus the change in wealth has been introduced [8]. In a world with fewer options, the excess of income over consumption would increase bank deposits. That is,

$$\Delta M = YD_t - C_t$$

Putting it all together we make a detour from our main equation to examine the following difference equation in asset prices

$$p_t^{BL} \cdot BL_{t-1} = p_{t-1}^{BL} \cdot BL_{t-1} + \Delta V - \Delta M,$$

(the systems treatment is available in the appendix). The dynamics of an economy driven by the search for capital gains is plagued by the problem of unit roots. The conditions for stability are delicate. Returning to the main course, we have a government budget constraint with the fiscal levers exogenous and the addition of an additional asset [8]. That is to say,

$$p_t^{BL} \cdot BL_t = p_t^{BL} \cdot BL_{t-1} - [B_t - (1 + r_{t-1})B_{t-1}] + (G - T).$$

In a manner identical to the earlier, with a loop describing taxes, we have [8]

$$T = \theta(Y + r_{t-1}B_{t-1} + BL_{t-1}).$$

Similarly, the equations combine to deliver

$$p_t^{BL} \cdot BL_t = (p_t^{BL} - \theta)BL_{t-1} - [B_t - (1 + r_{t-1} - \theta r_{t-1})B_{t-1}] + \theta[\lambda L_t - \lambda(1 + r_{t-1}^L)L_{t-1}] - \theta[(1 + \lambda)UC \cdot y] + G,$$

(for the last time, the appendix has the formal details). Our master equation is stretched to absorb the contemporary innovation of long-term bonds. Along with the weapon of a tax rate, an additional direct weapon is the price of long-term bonds. Recall, the reciprocal of the price is the interest rate. The system decomposes with the stationary solution for the level of long-term bonds determining the quantum of short-term bonds and loans outstanding. The contrast with the earlier regime is that the quantity of short-term bonds now is positive. Only those instruments should be chosen that augment systemic stability and dampen economy-wide negative spillovers. An illustration is the use of the discount window recommended by Minsky. He supported discount window operations in institutions like life insurance

companies as long as they were occupied in ‘to the asset’ financing [21]. We have already noted that the Fed, in a first, has been providing credit directly to financial firms during the current crisis rather than disbursing liquidity rapidly to the market through open-market purchases of Treasury securities [15]. As theorists in the Social Structure of Accumulation (SSA) tradition have reminded us, the context for investment in durable assets is designed by state action [22]. Businessmen could as well turn to financial assets instead of plant and machinery in the absence of appropriate policy. Perversely, the suggestion is that the CB lender of last resort operations should be as unpredictable as possible. So-called constructive ambiguity is the defining privilege of the sovereign [23]. The first challenge faced by Central Banks during the current financial meltdown was the changing maturity composition of banks’ net demand for funding liquidity. There was an increase in the net demand for term funding relative to overnight funding. To a large extent this portfolio switch was due to the massive reintermediation of conduits. Such behaviour reflects the counterparty risk inherent in a deleveraging of the conduits. We end by recalling the evolutionary tone struck at the beginning. So-called evolutionary policy-making is the attempt, through concerted action, to anticipate and manipulate selection outcomes. The growth of the wealth of nations might also create “spontaneous disorder”. There is need for procedural welfare interventions. Policy problems are never given but continuously created in a process that involves deliberate and selective communication with specified agendas.

## CONCLUSION

Commercial banking is institutionally fragile. Consequently, financial intermediation has been supported through history by CBs. The latter, in turn, have used bank rate policy to great effect. However, the current financial crisis has highlighted the tenuousness of short-term measures. The outcome might be one in which central bankers will grasp the future with all the weapons at their command. The nascent regime of accumulation is one in which the bridges between a perilous present and an uncertain future will be constructed by the two macroeconomic organs of the state working in tandem. We have offered some macro analytics of the instrumentality of the long-term bond with price-setting CBs. However, the activity results in a non-null level of short-term debt. It has also transpired that the tax rate continues to be a sharp stabilizing scythe. The policies will have to be fine-tuned to deal with the particularities of regimes. The institutional specifics include the degree of monopoly and unit costs.

## ACKNOWLEDGMENTS

Wynne Godley passed away on May 13 this year. This paper is dedicated to his memory. The inputs of two anonymous referees to an earlier draft were most helpful. The usual disclaimers apply.

## APPENDIX

### GOVERNMENT MONEY AND PORTFOLIO CHOICE

In dynamical system terms, the master equation of the section is

$$\begin{bmatrix} B_t \\ L_t \end{bmatrix} = \begin{bmatrix} (1 + r_{t-1} - \theta r_{t-1}) & 0 \\ -\frac{(1 + r_{t-1} - \theta r_{t-1})}{2\theta\lambda} & (1 + r_{t-1}^L) \end{bmatrix} \begin{bmatrix} B_{t-1} \\ L_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{(1 + \lambda)UC.y}{\lambda} - \frac{G}{\theta\lambda} \end{bmatrix}$$

Since the trace of the coefficient matrix is positive and the determinant is positive, the equilibrium is an unstable node.

The stationary solution is given by  $B = 0$  and

$$-r_{t-1}^L = \frac{(1 + \lambda)UC.y}{\lambda} - \frac{G}{\theta\lambda}$$

## LONG-TERM BONDS AND CAPITAL GAINS

The story of capital gains is

$$\begin{bmatrix} p_t^{BL} \\ V_t \\ M_t \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{BL_{t-1}}{3} & 1 & 0 \\ \frac{BL_{t-1}}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1}^{BL} \\ V_{t-1} \\ M_{t-1} \end{bmatrix}.$$

The eigenvalues are lined along the main diagonal. The spectral radius is unity. The origin is attractor only if that number is strictly less than one.

The complete model is given by

$$\begin{bmatrix} BL_t \\ B_t \\ L_t \end{bmatrix} = \begin{bmatrix} \frac{(p_t^{BL} - \theta)}{3p_t^{BL}} & 0 & 0 \\ \frac{(p_t^{BL} - \theta)}{3} & \frac{(1 + r_{t-1} - \theta r_{t-1})}{2} & 0 \\ -\frac{(p_t^{BL} - \theta)}{3\theta\lambda} & -\frac{(1 + r_{t-1} - \theta r_{t-1})}{2\theta\lambda} & (1 + r_{t-1}^L) \end{bmatrix} \begin{bmatrix} BL_{t-1} \\ B_{t-1} \\ L_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{(1 + \lambda)UC.y}{\lambda} - \frac{G}{\theta\lambda} \end{bmatrix}.$$

The equilibrium is a saddle point. Only the eigenspace associated with long-term bonds might be stable. Indeed, let us take the situation  $p_t^{BL} - \theta > 0$  to begin with. As the price of long-term bonds begins to fall (the rate of interest on long-term bonds increases) and direct tax rate begins to rise, a qualitative change occurs at  $p_t^{BL} - \theta = 0$  called a saddle-node or fold bifurcation.

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## **PROMJENA REŽIMA U KONZISTENTNOM MODELU ZALIHA I TOKOVA**

R. Correa

Odsjek za ekonomiju, Sveučilište u Mumbaiju

Mumbai, Indija

### **SAŽETAK**

Istražena su dinamička svojstva modela Godley-Lavoie s težištem na središnjoj banci. Svojstva stabilnosti načina regulacije su praćena od režima privatnog novca banke do aktualne krize s stabilizatorima središnje banke u vidovima od kratkoročnih obveznica do izvirućeg režima dugoročnih obveznica.

### **KLJUČNE RIJEČI**

financijska kriza, dugoročne obveznice

# COMPLEXITY IN ORGANIZATIONS AND ENVIRONMENT - ADAPTIVE CHANGES AND ADAPTIVE DECISION-MAKING

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## ABSTRACT

The features of complexity are ever more present in modern organizations and in environments in which they operate, trying to survive and be as competitive as possible.) In the processes of, the so-called emergence, the formal organizational structure, designed purposefully and with a plan, is going through a change due to complexity and the need for adaptation. As a result, there is a variety of new informal groups. At the same time, the intended structural changes and business process changes occur because of the perception that the leadership and senior organizational management have of the strategic situation. Managers in modern organizations often use business intelligence (BI) systems when making important business decisions. These systems offer support to the decision-making by gathering and processing relevant data and information about the company performance, but also about the data on conditions in close and remote environment. A modern company is characterized by the complex adaptive system, but the environment in which it operates together with other business subjects (agents) is also complex. Consequently, the requirements for appropriate or optimal decisions and successfully completed activities are hard to meet. Given that expected future events and circumstances often occur in nonlinear mechanisms, the decisions made by following the models of traditional predicting and planning are not satisfactory. This calls for new approaches to decision making and acting.

## KEY WORDS

complexity, complex adaptive systems, business intelligence, decision-making

## CLASSIFICATION

JEL: D21, D73, L22

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## **INTRODUCTION**

Organizations are social entities oriented to certain goals. They are characterized by a designed structure and coordinated activities, and are open in terms of closeness with their environment. Organizations contain collection of resources, categorized as human and material resources that can be coordinated and managed to perform certain tasks. Processes and activities are determined in order to execute tasks, which enable organizations to continuously realize their goals.

One of the most prominent metaphors in the treatment of organization stems from the so-called system approach. It has been well known for many years and has origins in cybernetics and application to complex technical systems. People make efforts to apply certain rules of behaviour of technical systems to organizations. First of all, there is the concept of the systems management, at which efforts were made to apply certain characteristic concepts of control, feedback, measurement of system's performance, etc., to organizations as primary social systems.

## **ORGANIZATIONS AS COMPLEX SYSTEMS**

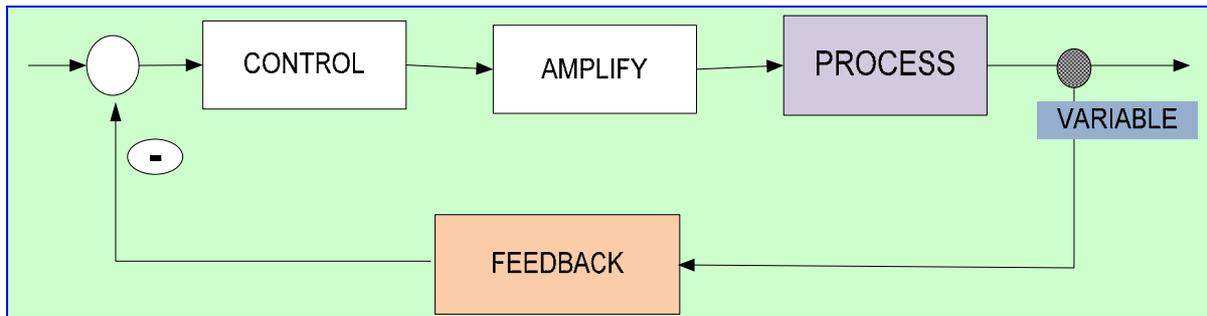
In view of organization, the system approach extends to the theory of complexity, which, in terms of social systems, primarily affirms the important concept of the so-called complex adaptive systems (CAS). The idea of the complex system denotes presence of many independent entities, agents that behave in accordance with their objectives, and perform mutual interactions. At that it is important to observe that the complex system does not allow simple reduction, as is the case with a multitude of unconnected elements. That is why it is sometimes difficult to understand the behaviour rules of the complex system since it is not possible to set up a simple and yet satisfactory model. This issue is an important task for the managers as they are faced with concrete challenges in their organizations on a daily basis. CAS are characterized by several key attributes that can be concisely described by terms reflecting the behaviour of these systems: complexity, agents, emergence self-organizing, adaptability, nonlinearity.

Complexity as a characteristic feature occurs and grows when interdependence of the elements within the system becomes relevant. In such systems each part or agent has significance of its own, and removal of certain element from the system leads toward destruction of the existing system's behaviour [1; p.9]. CAS are open systems whose components are firmly interrelated and have the ability of self-organizing and dynamics. There are also certain local rules that apply to these components or agents. The dynamics is present because of interrelations, interactions and influences of numerous agents. As a result, CAS are subject to constant and discontinuous changes [2].

The aforementioned interactions among system's elements may result in occurrence of certain higher levels of organization, cores of new structures, and this phenomenon is called emergence. Elements or agents in organizations are individuals, organizational units, groups and so on. The occurrence of well-known informal organizational groups that significantly distort the structure defined by the purposeful design of organization, can be explained by the complexity conditions. Agents connect in accordance with their specific goals and interests. However, in real organizations they often connect at the expense of real, declared organizational goals. One desired scenario is the situation when self-organizing is motivated by learning within the organizations with a purpose of adapting the structure to external challenges and thus improving performances of the system itself.

Organizational adaptation to environment with the option of changing its structure is an important phenomenon in both theory and practice of the organizational design and organizational changes.

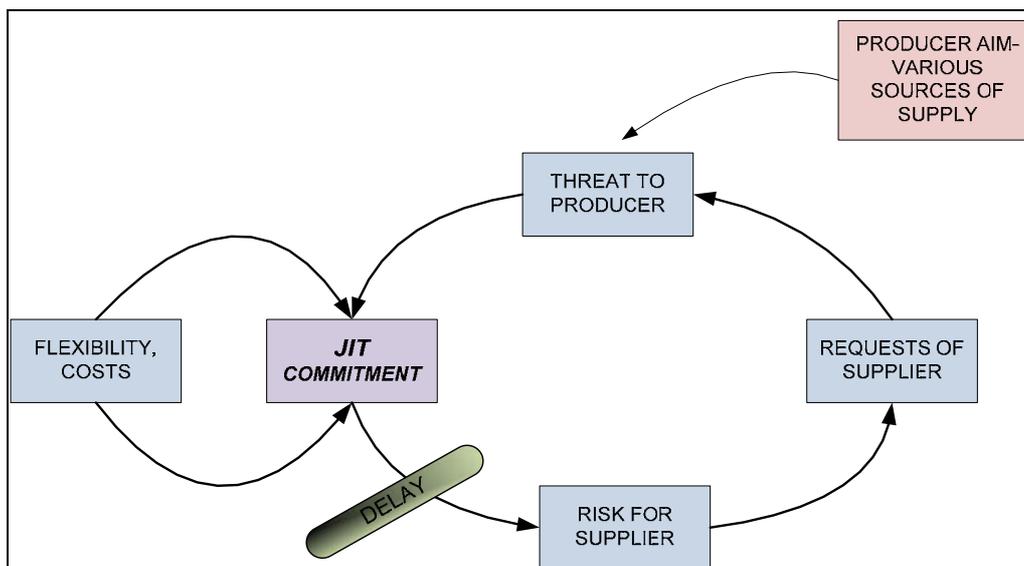
According to the system theory, the effects of the process balancing in the traditional control paradigm are achieved by means of negative feedback (Fig. 1). The behaviour of the system can be controlled by sending the output results relative to certain desired values back to the input segment of the process development. Such mechanism may serve to control the behaviour of social systems and it represents a contribution to the organizational theory studies.



**Figure 1.** Negative feedback (an example).

While the negative feedback acts as a stabilizer of the system, the positive feedback activates the process of amplifying that may lead toward instability after a certain time. Nonlinearity is a phenomenon that can be explained in different ways. Due to numerous connections and interactions in CAS, the outcomes of processes and events are nonlinear with regard to the values of input variables. In the environment of organizational activities nonlinear processes are mainly unwanted because they decrease the possibility of control and adequate responses to impacts and events in the environment. Nonlinear occurrences imply circumstances of disproportional relative changes in the input-output states of the processes, for example, if some company is successfully increased their production but this phenomena does not have consequences in proportional growth of their profit, due to the saturation of markets.

However, when applied to the creation of responses to the challenges of environment, nonlinearity may be useful and desirable. Organizations represent adaptive and intelligent



**Figure 2.** Commitment to „Just in time“ system – influences [5, p.105].

entities since they can take actions that were not pre-planned, and the final outcome is not just a simple sum of isolated individual efforts. The actual performance is also a result of the included nonlinear processes [3, 4]. The systems that possess distinctive CAS attributes demonstrate emergent rather than deterministic behaviors. The type of control in these systems is self-organizing and, to a lesser extent, centralized and hierarchical control.

The amplifying mechanism in the case of CAS is often joined with the stabilizing process that includes conditions of limitation and, consequently, keeps the growth inside certain regular boundaries. Such pattern for CAS was illustrated by Senge in the example of introduction of Just in Time System in business (Fig. 2).

Improvements achieved by implementing the JIT system, such as lower costs and other benefits, stimulate the manufacturers to commit themselves to this approach. However, the demands for a prompt reaction to the needs for supplies urge suppliers to fight for their exclusive position. This scenario exposes the manufacturer to risks because he would prefer the option of having multiple sources of supply [5]. As a result of the second loop, commitment toward the JIT is undermined. In the end, the commitment is expected to remain on certain reasonable, but no too high levels.

The complexity within organizations and in their environments, with the described phenomena included, leads to reluctance to organizational changes. In order to maintain competitiveness and survive in a potentially worst-case scenario, organization must change. There is a variety of theories dealing with organizational changes, and analyses are being made of the factors that influence changes, of the type and comprehensiveness of changes, of the effects and tasks of the strategic leadership, impacts of the changes on employees, etc.

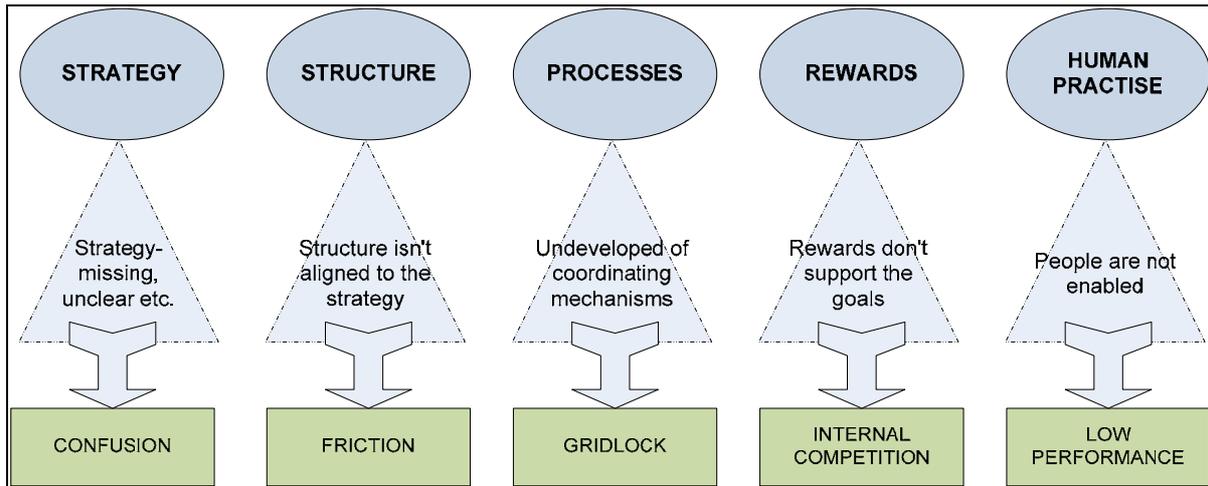
## **ORGANIZATIONAL CHANGES**

*Organizational design* is the activity through which managers and other responsible authorities select and manage structure, processes and culture as main organizational aspects. An organization should select a design that will enable it to successfully control activities that are essential for achievement of its goals.

Organizational changes are relatively frequent, which is not the case with strategic changes, because the latter are about organizational design, and they encroach upon the domains of structure, strategy, key processes and culture. Organizational change is a process used by organizations to redesign their structure, processes and culture with an aim to move from the current state toward a future desired state in order to increase their effectiveness and efficiency [6; pp.10-11]. Organizations effect changes because they want to see their mission accomplished, focusing on objectives that range from survival to dominance. Aware of the process of changes initiated by the management, through formal channels, they are encouraged by certain factors from within the company and the business environment. Common internal factors of changes for the companies fall into the categories of: strategy changes, organizational growth process, life-cycle of the organization. The main external drives of organizational changes are within the domains of the market dynamics, new technologies, and socio-economic trends.

Organizational changes can be observed with respect to five key elements of organizational design. Mismatches between and within these key elements of the organizational architecture result in occurrence of distorted performance (Fig. 3). These circumstances in turn require the intervention and changes in the domain of key elements and their relationships.

Organizational changes in the domain of structure are particularly important. According to one definition, organizational structure represents a system of mutual relationships and



**Figure 3.** Effects of mismatch of organizational design elements [7; pp. 4-5].

connections in the company (organization). Structuring of an organization involves distribution of power (authority) as well as responsibility to existing organizational units and individuals. Suitable organizational structure provides support to the successful implementation of strategy and strategic plans, whereas unsuitable structure obstructs it [8; p.107].

There is a variety of types of structure, and most common versions are: simple structure, functional structure, divisional, project structure, matrix, and so on. When selecting and analyzing a suitable structure it is important to take into account the so-called dimensions of organizational structure, such as: complexity, centralization, formalization. When selecting a suitable design and organizational structure one can choose among a number of possible combinations of types and characteristics of the structure dimensions. In reality, inside the organizational structure, apart from the one formally declared and implemented, there also exists the informal structure. This formation usually emerges over the time, as a result of interests and interactions between agents, and it promotes their specific goals and aspirations.

Self-organization is a process typical of the complex adaptive systems in which components of the system communicate with each other in a way that can be described as spontaneous. These phenomena occur in practice in real organizations. Parts of the system are adapted and coordinated to produce certain common behavior. Creative organizations are developed through the crisis and no-stability phases and they create a new, more complex form of inside order in an unexpected way. In this context, new strategic directions to a greater extent emerge and to a lesser extent are planned [9; pp.240-241]. So in addition to changes in the organizational structure the process of self-organization is also responsible for changes in the field of strategy.

The changes that are due to the complexity phenomena do not always produce positive effects. Organizational adaptation i.e. structural learning result in the changes of the organizational structure and these changes are generally for the better. However, due to the new distribution of agents with empirical and other knowledge, certain negative patterns occur so that the agents in new environments do not have satisfactory interactions and responses in relation to their knowledge [2, 3]. So the process of knowledge management is definitely degraded to a certain extent, due to organizational adaptation.

With an aim to maintain successful performance CAS are able to change their structure, strategy and processes. That is why different types of organizational flexibility are recognized and required [10, 11]:

1. flexibility of processing (related to business processes improvement),

2. flexibility of planning (enables fast reaction to unexpected events, addressed to the adaptation of organizational strategy),
3. flexibility of resource allocation,
4. hierarchical flexibility (regarding the relations of power and decision-making, along with allocating resources is addressed in the adaptation of the structure of the organization).

From the listed organizational flexibility types it is obvious that flexibility covers main elements of the organizational design (Fig. 3). The changes of organizational strategy, the purpose of which is adaptation to the requirements imposed by environment, occur by analogy with the changes in the structure, thanks to mutual interactions of interconnected agents. Casual ingredients of strategy then emerge, something that is not proclaimed or defined as a method, but in reality they emerge, crystallize and act. In modern paradigms of strategic management and organizational theory increasing attention has been focused on business processes, so its changes are particularly important for the organizations. Range of changes goes from those incremental, often related to automatization and implementation of information technology, toward the reengineering of business processes (BPR). Radical operation using the full range of techniques and tools as well as with cross-functional characteristics makes BPR demanding performance that however often gives dramatically improvements. BPR is the creation of entirely new and more effective business processes, without regard for what has gone before [12; pp.4].

In addition to affecting the structure within the limits of organizational borders, contemporary environment and business conditions influence creation of new possibilities for expanding boundaries. Business processes can go across the number of organizations and in this they support connections of diverse systems. In the view of market as a broader environment with complexity attributes, agents present here are in fact different organizations with their specific goals and interrelations. Self-organizing and new structures are created in the form of strategic alliances, clusters, networks, virtual organizations.

Different organizational changes of the strategic levels mentioned above represent to a large extent adaptation to new and different circumstances. We consider adaptation as a process or state of changing, aimed at fitting in and adjusting to a new environment, or to different conditions and the resulting changes. Adaptation of a system sometimes requires modifications.

## **ADAPTATION AND LEARNING**

As for adaptation of an organization to its environment, and the changes that occur as a consequence of that adaptation, it needs to be noted that there are several approaches covering this area. They have roots in the fields ranging from psychology to cybernetics. In that respect Senge mentions learning organizations, Stacey elaborates dynamism and non-linearity of behavior, Forrester et al. successfully applied the theory of systems several decades ago. Apart from them, there is a whole range of authors who were investigating the near domain of knowledge management.

Argyris considers types of learning and organizational learning. In his view, adaptation is connected with learning in the way that it is close to the so-called 'single-loop learning' [13; pp. 115-124]. According to Zack [14], the ability of an organization to learn, accumulate knowledge from its experiences, and reapply that knowledge is in itself a skill or competence that may provide strategic advantage.

The *organizational leadership* in the learning organizations must institutionalize and improve the process of *knowledge collection*. Knowledge collection involves the process of observing the external environment and the internal process of performance measurement. It also

involves various initiatives, such as launching of programs in the domain of development of technology, science and so forth. The cumulative *modification of the process*, which results in the growth of organizational competences ensures promising responses to future crises caused by environmental phenomena. This is the second key ability of successful learning organization. In the context of strategic decision-making organizations can also be considered as *interpretation systems*. Organizational interpretation is defined as a process of understanding events and creating mutual understanding and conceptual schemes among members of senior management.

Changes within an organization call for learning of something new, adjusting to a new way of carrying out operative activities. Changes demand application of the newly learned knowledge and performance in a new way. In that sense learning is not just a process of acquiring knowledge based on experience. It also implies a component of action taking. Kolb provides a well-known model of experimental learning, which includes four stages closed in the learning cycle [15; pp.9-14, 16]:

- a) concrete experience,
- b) reflective observation,
- c) theoretical concepts,
- d) practical experimentation.

Adaptive organizational changes manifest themselves over time in reduced magnitude of effect of destabilizing events that occur in the environment, and also in the accelerated and successful restoration of the system to the good state [17]. The requirements for successful adaptive organizational changes can be summarized within the following five categories [18; p.536]:

- 1) distinctive features of organization in the process of system changes must be in line with the company's strategy,
- 2) in most of the cases the process of changes must be iterative and dynamic because adaptive changes occur under the circumstances of uncertainty and external conditions that are also subject to changes,
- 3) adaptive changes call for learning about the ways of achieving the required structure, processes and organizational behaviour,
- 4) the support to organizational changes must be provided by as many stakeholders as possible (owners, managers, employees, clients),
- 5) adaptive changes must be effected on all organizational levels, but most of the responsibility lies on the management.

The leadership and top managers have a specific role in organizational changes. When observing the organizational metaphor of the so-called 'flux and transformation' as opposed to, for example, the metaphor of 'political system', one can notice a significant difference in the expected roles of leaders and required traits of leaders. Traditional metaphors saw leader as the main designer, the one that implement changes, a skilled orator. It used to be a person with a vision, someone who is familiar with project management, who supervises and controls. In recent metaphors, those close to the theory of complexity, the leader is "facilitator of emergent change", he makes connecting possible, amplifies issues [15; pp.122-123]. So, leaders appear as particularly relevant agents within organization that are complex adaptive systems in which various interactions take place.

When we talk about successful adaptable changes we need to mention the concept of "adaptive cycle" that promotes three properties important for the development of the system in the future [19]. They include wealth, controllability and adaptation potentials. Wealth

implies the system potentials that determine the range of possible options in the future. Thus are determined the limits of the possible. The inner controllability of the system relates to the degree of interconnection between the process and control variables, and indicates the degree of flexibility (or rigidity) of the control. At the same time, controllability indicates the maximum level at which the system is capable of controlling its own fate. As opposed to the vulnerability of the system, the adaptive capacity is determined as a measure of elasticity in response to unexpected disorders or shocks. Such unexpected external impacts can also change the level of internal control [20].

The concept of complexity imposes and identifies new principles in the organizational design and in the behavior of the system, both its parts and its wholeness. It also incites the building of preconditions for successful responses of the organization to potential atypical events and impacts. As seen by a number of prominent theoreticians, complexity and its dynamics represent a barrier to the learning as we perceive it within a traditional organizational theory. Numerous entities with their interactions, positive and negative feedbacks, create non-linearities and unexpected phenomena, when it is hard to make conclusions following certain rules in force. Complexity slows down agents' learning (individuals and organization) because the feedback on the effects of decisions and actions taken also includes delays, errors, limited perception. Some processes that occurred are irreversible; new rules apply to new circumstances. Variables simultaneously change, and it is hard to decide what their mutual relationship is.

Given that the learning of decision-makers in the conditions of complexity is slowed down and made difficult, the decision-making is growingly demanding, and the mistakes made in that process are likely to be more frequent and more serious. As a result, more attention is being paid to the structuring and implementation of a system that provides support to decision-making.

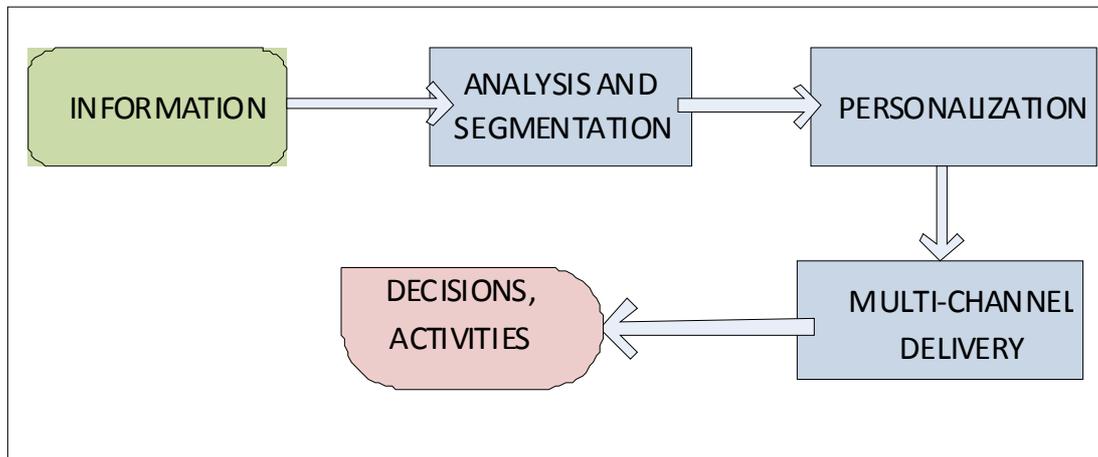
## **DECISION-MAKING IN ORGANIZATIONS – BUSINESS INTELLIGENCE SYSTEM**

Modern business environment implies complexity, which cannot be reduced in a simple way; therefore there are a number of factors to be considered in order to make appropriate decisions. To make sure that decision-makers have the required and correct information when deciding, it is necessary to build support systems that are called business intelligence systems (BI). They are tasked to collect and structure information and ensure targeted service for decision-makers in terms of providing them with information, processing of relevant parameters, variables and factors in the domain of business activities and environment of an organization.

Among numerous definitions we have singled out the one by Moss and Atre [21] who claim that business intelligence is primarily architecture and collection of integrated operative applications and applications to support decision-making, as well as the database that provides businesspeople with an easier access to relevant data. From the technical point of view, business intelligence is a system that automatizes collection of data from different sources, processes them, transforms them and delivers them to end users. Business intelligence is a response to the growing need for information and analytical tools indispensable for [22]:

- transformation of data into information,
- better management of daily operations by using relevant and updated business data,
- faster decision-making, based on relevant and updated information.

Collecting of information is a demanding job that requires time and engagement of resources. On occasions it is necessary to make decisions very fast, because a timely decision is worth more than a quality decision made too late. According to some authors, modern organizations spend as much as 80 % of time in collecting information and 20 % in analyses and decision-making [23]. Special value of a good business intelligence system is in that it may reduce both the time for decision-making and the time for data collection.



**Figure 4.** Components of the business intelligence model (modification from [24; p.26]).

Business intelligence helps transforming collected data into quality information needed in decision-making. While traditional systems that provide support to decision-making tended to ignore personalization of information, business intelligence takes it into account (Fig. 4). Through analysis and segmentation these systems direct data at individual employees by using, in general, a larger number of channels. Once he received information, the manager, as the entity that is in interaction with other components of the system, takes certain actions or makes appropriate decisions.

People who come from military circles, intelligence community, diplomacy, but also a large part of non-professionals equate business intelligence with espionage-related activities. For this particular part of the entire field of BI it can be said that it is close to the sub-category called competitive intelligence. Prescott and Miller [25] define the concept of business intelligence that comprises five types of intelligence: *Competitive intelligence*, *Customer intelligence*, *Market intelligence*, *Technical intelligence* and *Partners intelligence*.

Nowadays the requirements of complexity are more and more prevalent, and they impose the need for modernization of the concept of BI. Obstacles to learning and decision-making are also largely present in the classical BI systems. That is why more suitable solutions are being sought to support decision-making. One of the promising concepts that provides support to adaptation and modified decision-making is the so-called *adaptive system of business intelligence* [26]. This system is designed to meet the needs of complex conditions found in the environment.

## ADAPTIVE BUSINESS INTELLIGENCE

Strategic decision-making and consideration must always take into account the future, and work out possible scenarios that will dominate the world of tomorrow. Consequently, it is necessary to anticipate expected changes and states of certain factors, variables and parameters by means of different methods. Particularly valuable scenario includes analysis because it opens the way to the estimation of discontinuity and the results of nonlinearity, which encompasses the effects of the presence of complex circumstances. Scenarios are

actually prudent, but also speculative stories intended to incorporate the concrete world of today into the envisaged set of future circumstances [27; p.45].

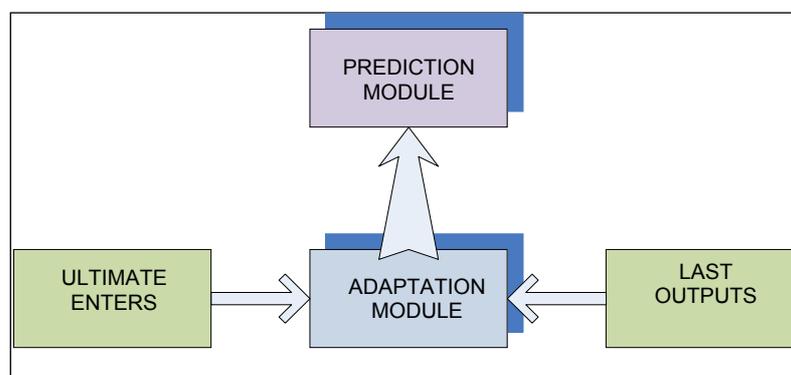
Organizational decisions that need to be implemented through operative actions are predominantly made on the basis of the current data or, at best, on the basis of linear predictions for situation in a short-term period. Due to the complexity of the environment and nonlinearity, those decisions are likely to be far from optimal. One of the ways of providing support to decision-making that meets the needs of new demands is the so-called system of adaptive business intelligence. The whole concept is based on the system of classical business intelligence. It employs the same infrastructure and techniques, but it is upgraded with specific new modules. These are system components that make possible *prediction*, *optimization* and *adaptation*.

Adaptive BI system (ABIS) includes the adaptation mechanism in the form of a sub-module, and it should be structured so that it can [26]:

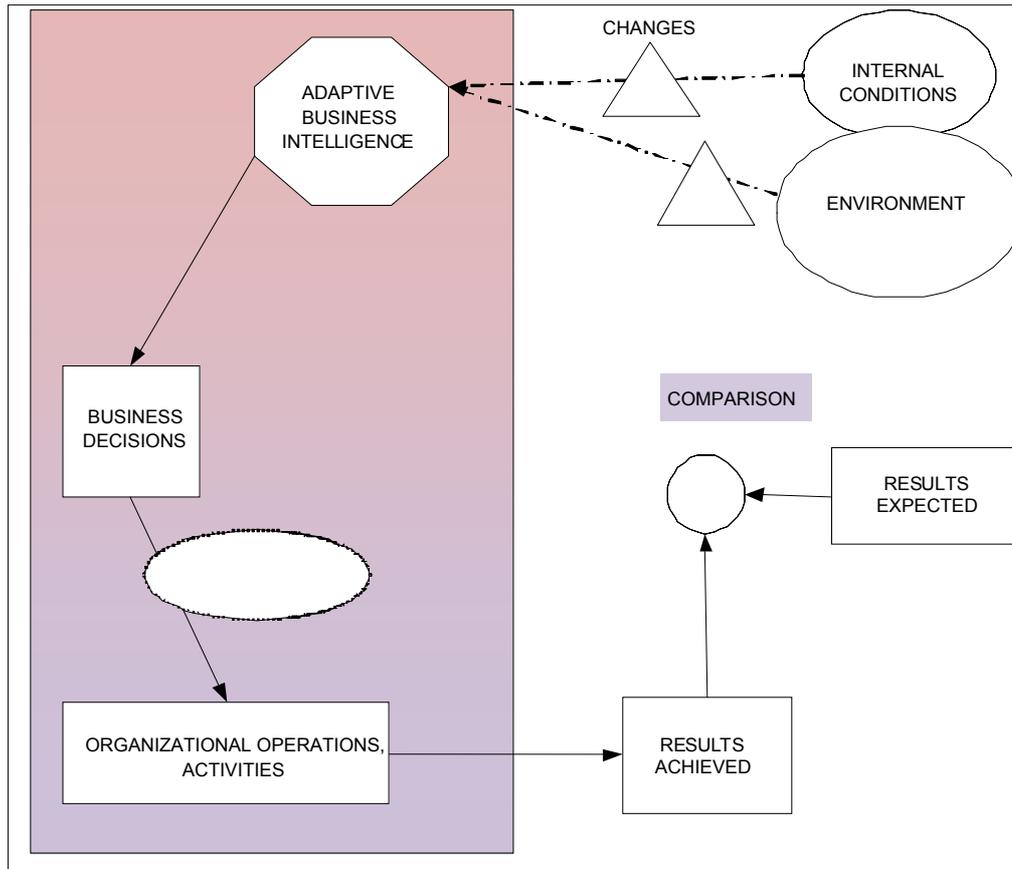
- perform data search (available data are prepared and analyzed in detail),
- use prediction models, at which prediction module is built on the basis of results of data search,
- contain optimization module based on prediction modules,
- upgrade prediction module and thus contribute to more precise prediction of changes in environment, etc.

The structure of the aggregate model is such that it contains certain modules that are intended for specific purposes. The prediction module within the system may contain several sub-models and its basic function, depending on input data, is to generate the output, i.e. provide prediction with certain accuracy. The prediction model needs to be „trained“ first, and this is done by employing historical data [26]. For this activity of prediction various approaches, methods and tools can be used (forecasting, regressive analysis, neuron networks, decision-making trees, etc.). The purpose of optimization, which is performed in a separate module of adaptive BI, is to detect the best solution of all potential and accessible solutions. Sometimes we restrict ourselves to the search for a set of satisfactory solutions. Of course, we can test sensitivity of the solutions. Optimization methods that are suitable for use can be found in the category of “classical” methods (linear programming, dynamic programming, etc.) and/or the category of “contemporary” methods (genetic algorithms, neural networks, etc.). Optimization module must be capable of recommending the best solution that is based on the outputs of prediction modules.

Adaptive business intelligence system has to be adjustable in the way that it is capable of learning and adapting to the changes that occur in the environment. At that the adaptation sub-model has the leading role (Fig. 5). The higher level of adaptation implies the learning from data, but also from own predictions and errors. To detect discrepancies between the



**Figure 5.** Adaptive part of the ABIS [26; p.44].



**Figure 6.** Framework of functioning of adaptive organization with adaptive BI system.

predicted results and the real results adaption module compares certain predicted and real values. If there is an error, adaptation module must be modified (adapted) so that it can be reduced. Adaptation module can be „adjusted“ to the changes in the environment by changing certain rules in the process of concluding [26]. It is possible to build it so that it is constantly adapted through measuring and monitoring of own errors in predicting.

In reality adaptive organization that implements adaptive business intelligence system functions in effect as described in Fig. 6. The cyclic development of adaptations within an organization in a range of time iterations results in occurrence of interpretations, decisions, implementations of decisions, and evaluations of achievements. Decisions, their quality and the subsequent achievements come back in the form of feedback to the adaptability module. The BI system delivers the requested information accordingly, and creates knowledge. Data on environmental factors, business results, situation in the organization, and data on expected changes (prediction), initiate the decision-making system and activities in the way that is corresponds more with the condition of complexity. That is why the responses and behavior of this system are sometimes non-standard, because it operates in the process of adaptation.

As regards the functioning cycle, what follows after managers' decision are actions. A trend in the contemporary organization theory that has become growingly relevant is known as “action perspective”, and this stream represents the analytical option that needs to be taken into consideration in addition to the already recognized realistic decision-making perspective [28; p.258]. At that one can start from the viewpoint that organizations are best understood as action generators [29].

It is particularly important to establish if such learning and adaptive framework function properly. Here we will focus on less formal considerations. In fact, such system of activities inevitably implies the generating of errors at more than one place, and then they propagate in iterative points of time in the process. Errors occur due to:

- inaccuracy of perception and interpretation of internal data, external environment, and data on changes,
- (im)precisions of prediction (forecasting) of the model in the BI system,
- limited rationales of decision-makers,
- interpretations of decisions on what actions should involve,
- (im)precisions of actions relative to implementation of decisions, etc.

**Table 1.** Errors in the organization.

<b>Time</b>	$t_1$	$t_2$	$t_3$
<b>Event</b>	data, information	decisions	Actions
<b>Output</b>		$f_1(data)$	$f_2(decisions)$
<b>Errors</b>	Er(data)	Er (decisions)	Er (actions)

Errors that occur as a result of imperfection of decision-makers, i.e. their limited rationality, are well known to all those who act in practice, or study issues of decision-making. It can be said that decision-makers are bounded rational in that they are only partially aware of the information available and are not able to fully analyze it [30; p.26]. What is common to the majority of errors is that they propagate through the system and if they occur in the initial parts and phases, their size is likely to assume other, usually larger dimensions.

$$er(data) \rightarrow er(data) + er(BI) + er(interpretation(data)) + er(decision making) \rightarrow (1) \\ er(data) + er(BI) + er(decision making) + er(interpretation(decisions)) + er(actions)$$

Errors (Table 1.) also occur in the correction-related activities (optimization, adaptation) since the functioning of the adaptive BI system itself is not quite precise. Of course, the intention is to reduce all errors, especially those that are likely to be serious and which occur at the onset of the functioning of the cycle. The approximate equation (1) illustrates how initial errors assume new dimensions and summarize with them.

Throughout this cycle (*perception – decision-making – action*) errors occur in different phases, and through propagation they generally initiate their own growth. The system of the so-called adaptive business intelligence should provide support and enhanced recommendations for decision-making in general. This will contribute to the better and more adaptive decision-making. In order to enhance its properties in general, the organization such as CAS must build up its flexibility, especially in the domain of processing, hierarchy, and planning. The adaptive BI system only partly supports the overall adaptability. Errors that we analyze eventually result in non-optimal *organizational behaviour*, less perfect *actions* and business operations. Therefore, to enhance organizational adaptations even more, improvements must encompass the whole “learning and doing” cycle rather than just the decision-making phase. Mental maps changes are essential to occur through the whole organization [5], in all relevant processes.

There is another circumstance present here that contributes to enhanced adaptability. The interaction of a large number of agents involved in the making of modified decisions gives an impulse to the enhancement of adaptability. That will stimulate additional processes of self-organizing of the higher levels that are characteristic for CAS.

## CONCLUSION

If we look at an organization in terms of the metaphor of the complexity theory, we perceive it as a complex adaptive system (CAS). It contains a great number of independent entities, agents that behave in accordance with its goals, and their relationship is that of mutual interaction. Due to connections and interactions there occur higher levels of organizing, cores of new structures, self-organization, and emergence.

The complexity in organizations and their environments call for organizational adaptation in the form of well designed and yet spontaneous changes of structure, process, and strategy. The conditions of complexity require a different design, new leadership, and more advanced decision-making. Adaptation of an organization to its environment is linked with the practice of organizational changes. Generally, *organizational change* is motivated the move the ongoing situation towards certain desired situation in the future that is aimed at increased efficiency and competitiveness. Organizational changes call for learning of new knowledge. However, in operative terms, they call for adaptation to new ways of performing operative activities.

Learning is a prerequisite for organizational changes, whereas complexity is a barrier to learning as perceived by the traditional organization theory. Numerous entities with their interactions, and processes of positive and negative feedbacks incite emergence of nonlinearity and unexpected outcomes. At that adoption of regularity and legality is slowed down and rendered more difficult. Leaders have a significant role in the launching and implementation of complex organizational changes. They must be successful in scanning and interpreting of environment, and they must motivate people to accept adaptive changes. For the better adaptation organization must develop flexibility, especially in the domain of structure, execution of processes, planning and allocation of resources.

Decision-making, as one of the crucial phases in the cycle of adaptive learning is raised to a higher level by means of support system that reduces the burden of complexity. The widely adopted support is provided by means of the business intelligence (BI) systems, which collect information, structure them and provide decision-makers in organizations with relevant information. To ensure quality decisions and higher adaptability more advanced concepts are structured, such as the so-called adaptive business intelligence. This system is based on the classical business intelligence that is upgraded with specific new modules intended for *prediction, optimization and adaptation*. Although this approach ensures better conditions for decision-making and better chances of success, it is not easy to minimize the problem of propagation of errors through the decision-making system and acting upon those decisions.

To enhance the ability of adapt in general, organization such as CAS must work on its flexibility in terms of design solution. The adaptive BI system partly steps up the level of overall adaptability. Errors occurring here in the cycle from perception to action eventually result in non-optimal *organizational behaviour*. Therefore, adaptation elements present only in the stage of decision-making, by means of the BI system, are not enough. For a more successful organizational adaptation improvements must encompass the entire organizational *learning and doing* cycle and all segments and levels – all the way to the action itself – operative, tactical and strategic.

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## KOMPLEKSNOŠĆ U ORGANIZACIJAMA I OKOLINI – ADAPTIVNE PROMJENE I ADAPTIVNO ODLUČIVANJE

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### SAŽETAK

Karakteristike kompleksnosti sve su izraženija u modernim organizacijama i okolini u kojoj one djeluju, nastojeći opstati i biti kompetitivne koliko je to moguće. U procesima izviranja, osmišljena organizacijska struktura se mijenja zbog kompleksnosti i potrebe za adaptacijom. Rezultat toga je raznovrsnost novih neformalnih grupa. U isto vrijeme, predviđene strukturalne promjene i promjene poslovnih procesa odvijaju se zbog procjena strateške situacije od strane vodstva i višeg menadžmenta u organizacijama. Menadžment u modernim organizacijama često rabi sustave poslovne inteligencije pri donošenju važnih poslovnih odluka. Ti sustavi pružaju potporu odlučivanju putem prikupljanja i procesiranja značajnih podataka i informacija o svojstvima kompanija, ali također i podataka o uvjetima u bližoj i daljoj okolini. Moderna kompanija karakterizirana je kao kompleksni adaptivni sustav. Također je kompleksna i okolina u kojoj ona djeluje s drugim poslovnim subjektima. Slijedom toga, zahtjeve za pravilno ili optimalno odlučivanje i uspješno zaključivanje aktivnosti teško je ispuniti. Budući da su očekivani budući događaji i okolnosti često dio nelinearnih struktura, odlučivanja donesena na temelju modela tradicionalnog predviđanja i planiranja nisu zadovoljavajuća. To traži nove pristupe odlučivanju i djelovanju.

### KLJUČNE RIJEČI

kompleksnost, kompleksni adaptivni sustavi, poslovna inteligencija, odlučivanje

# TYPES AND PRIORITIES OF MULTI-AGENT SYSTEM INTERACTIONS

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## ABSTRACT

Multi-Agent Systems may be classified as containing No Direct Interactions, Simple Interactions or Complex, Conditional Interactions between agents. This paper argues and illustrates that models with simple interactions, even though possibly less fascinating for the Multi-agent system theorists than complex interaction models are, deserve more attention in the Multi-agent system community. Simple interaction models may contain social learning and reciprocal relationships. Maybe most importantly, Simple interaction models enable cross-scale connections by linking local to global actors in their local and global 'life worlds'.

## KEY WORDS

multi-agent systems, social learning, reciprocal relationships

## CLASSIFICATION

ACM Categories and subject descriptors: I.2 [ARTIFICIAL INTELLIGENCE]; Agent Interaction

JEL: C63, C69

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## INTRODUCTION

Multi-agent technology is presently moving from supply and theory-driven explorations to applications in a wide array of scientific and societal fields. In the present contribution, we take our examples from applications of multi-agent systems (MAS) in land use science. We hope and expect, however, that our message may be fruitful in many other areas, too.

The background of our paper is formed by the difference between theoretical and empirical fascinations. The ideal-type theorist is fascinated by MAS because of the capacity of a multitude of simple agents to generate system-level evolution, learning, stability and so on. The ideal-type empiricist is fascinated by MAS because of the capacity of simple agents to potentially generate a fit with reality that supports understanding and prediction of that reality. One example is from collective action (e.g., [1]). The theoretical fascination with collective action stems from that within rational choice theory, collective action involves almost insurmountable dilemmas. What models, in game theory or MAS, may resolve this problem? The empirical scientist, on the other hand, finds that in reality, collective action is everywhere, only with strongly varying degrees of success. Could MAS models explain this variety?

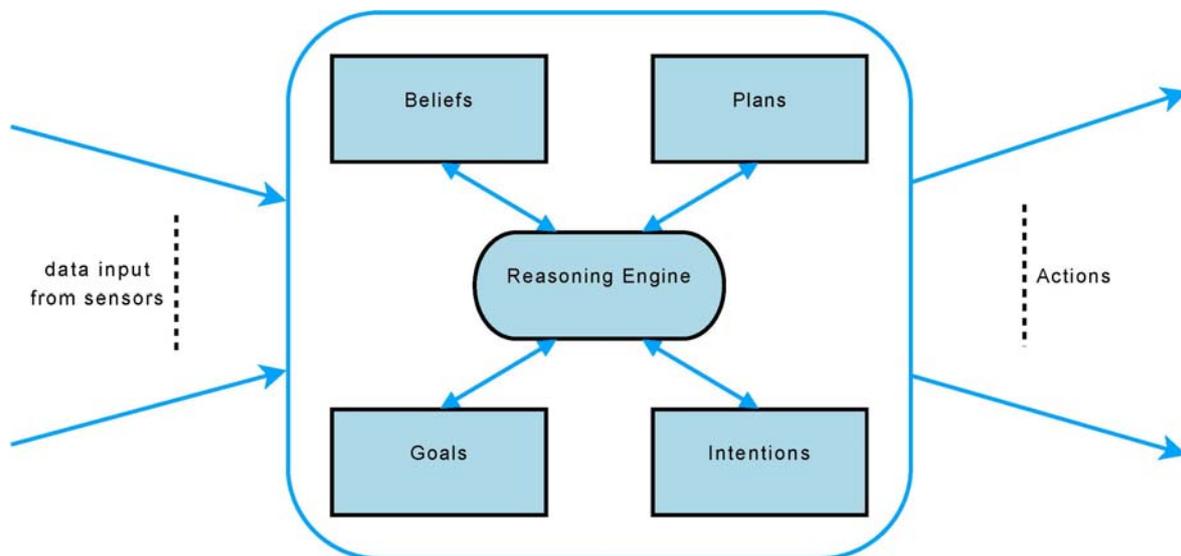
In this paper, we propose a three-tiered ladder of MAS model complexity that captures the tension between theoretical and empirical relevance. The rest of the paper is organized as follows. In section 2 we give an overview on the theory behind the Belief-Desire-Intentions (BDI) model and the Action-in-Context (AiC) framework. The classification of the types of MAS models based on their interactions is described in section 3 and we conclude in section 4 with priorities for MAS models with a primarily empirical ‘versus’ those with a primarily theoretical aim.

## AGENTS AND AGENT THEORY

There is no universally accepted definition of the term agent, however some definitions have been suggested depending on the domain used in the sequel:

- “Most often, when people use the term ‘agent they refer to an entity that functions continuously and autonomously in an environment in which other processes take place and other agents exist [2].”
- “An agent is an entity that senses its environment and acts upon it [3]”. “The term agent is used to represent two orthogonal entities. The first is the agents’ ability for autonomous execution. The second is the agents’ ability to perform domain oriented reasoning.”
- “Intelligent agents are software entities that carry out some set of operations on behalf of a user or another program, with some degree of independence or autonomy, and in so doing, employ some knowledge or representation of the user’s goals or desires.”
- “An autonomous agent is a system situated within and a part of an environment that senses that environment and acts on it, in pursuit of its own agenda and so as to effect what it senses in the future [4].”
- “An agent enjoys the following properties: (i) autonomy – agents operate without the direct intervention of humans or others, and have some kind of control over their actions and internal state, (ii) social ability – agents interact with other agents (and possibly humans) via some kind of agent-communication language, (iii) reactivity – agents perceive their environment and respond in a timely fashion to changes that occur in it, (iv) pro-activeness – agents do not simply act in response to their environment, they are able to exhibit goal-directed behaviour by taking initiative [5]”.

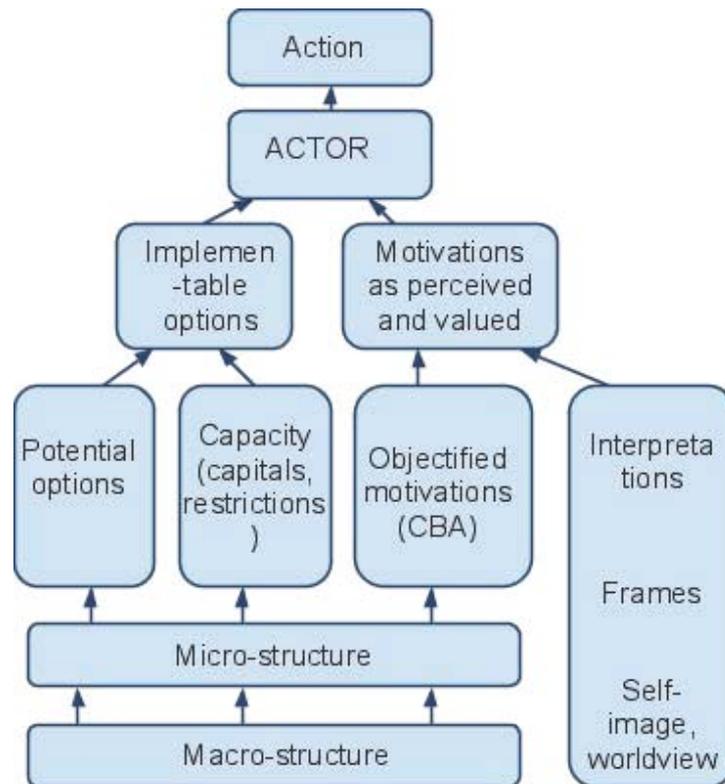
In MAS models, agents are given a structure for decision-making. A well-known agent theory is the Belief-Desire-Intentions (BDI) framework that was first proposed by Bratman [6]. BDI describes ‘beliefs’ as the representation of the agent’s knowledge about the current world/environment and messages from other agents as well as the internal information. ‘Desires’ represent a state that the agent is trying to achieve and ‘intentions’ are the chosen means to achieve the agent’s desires, generally implemented as plans. Thus, an agent is characterised by its beliefs, goals (desires), and intentions – it will intend to do what it believes will achieve its goals given its beliefs about the world. Additional to these three components, a BDI agent is usually assumed to have a plan library – a set of “plans as recipes” that it can use to achieve particular goals given particular preconditions. An intention is formed when the agent commits to a particular plan – a particular sequence of steps to perform – from this set in order to achieve a goal. The steps themselves may be atomic actions, or they may be sub-goals, which can be satisfied by other plans [7]. Because the agent does not need to commit to a particular plan for these sub-goals until the last possible moment, this allows a balance between reactive and deliberative planning. The model is shown in Figure 1.



**Figure 1.** The BDI Model.

Putting together the beliefs, goals, plan library and intentions is the ‘reasoning engine’. This reasoning engine is what drives the agent, updating beliefs, monitoring and updating goals and intentions, selecting plans to achieve goals, and based on the current intentions, selecting the actions to perform. A key feature of the plan library is that although the plans are fixed “recipes” for action, they do not have to be fully specified. For any particular goal, there may be multiple plans to achieve that goal, and while any plan may be fully specified as a sequence of actions, a plan may instead consist of a sequence of sub goals, or a combination of actions and sub goals. In the case that the plan contains sub goals, the agent can delay the choice of how to achieve a particular sub goal until the time that it reaches that stage of the plan. While this does not achieve the full range of adaptability that people display, it does allow considerable flexibility in the agent’s planning, and its resulting behaviour.

Aside from the BDI model is the Action-in-Context (AiC) framework in which De Groot [8] developed an agent structure that expresses a broad rational choice theory just like BDI but contains some differences too, e.g. that the “plans as recipes” are taken up in the core structure (as ‘options’) but that the ‘intentions’ element is skipped in favour of going straight to ‘actions’. Moreover, the structure is multi-layered in order to express the linkages to the



**Figure 2.** The Agent structure of the Action-in-Context framework (AiC).

structure and culture that the agent is part of. ‘Actor’ in AiC equals the ‘Reasoning Engine’ in BDI. The AiC agent model is depicted in Figure 2, describing the agent structure of the Action-in-Context framework (AiC) [8].

Implementable options are all actions the actor has available to reach the action’s objective. Motivations as perceived and valued are the criteria (e.g. economic, social and cultural merits) by which the actor evaluates the action. Potential options are all options known by the actor. Capacity is the difference between potential and implementable options and is composed of positive capitals (economic, social, cultural, physical) and negative restrictions (prohibitions, taboos). Objectified motivations are those easily expressible in simple terms such as money, time or calories, often summarized as the actor’s cost-benefit analysis (CBA). Interpretations are the cultural and psychological colours (quantitatively: multipliers) that attach to these objectified factors. Interpretations are embedded in more general frames of reasoning (which may be contextual, depending on the micro-structure), which in turn are embedded in self-image and world view (e.g. for a farmer, the image of what is a good farmer). Micro-structures are all structures (environments – physical, social and on the web) where the actor makes a difference. Macro-structures are all structures where the actor does not make a difference (e.g. for most actors, the oil market).

Although we will frame most of our examples in AiC terms here, we will not discuss the comparative merits of the agent definitions or two agent models here. The major point to note, however, is that all agent definitions as well as the two agent models allow no direct interaction of agents in multi-agent systems. In a MAS, variable types of agents may all respond to a variable, physical or economic environment without interacting with any other agent, and the system would still be a MAS.

## **TYPES OF MULTI-AGENT MODELS**

In this section, we will distinguish and illustrate three types of MAS models. This is of course not the only way to make MAS classifications, but ours serves the specific purpose of elucidating the differences between theoretical and empirical fascination. The classification is:

1. NI – No direct Interaction between agents,
2. SI – Simple Interactions between agents,
3. CI – Complex, or Conditional, or Collective Interactions between agents.

What we will call ‘simple’ and ‘complex’ here also relates to our purpose. For multi-agent models that focus on land use decisions, it will for instance be relevant to make the MAS spatially explicit, i.e. taking data from and writing data in a GIS (e.g. the MameLuke framework, [9]). This obviously makes the MAS more complex but this is not the type of complexity we want to capture here. What we will call ‘simple’ is the type of interactions between agents that are basically one-way (although they may become reciprocal if one agent responds to the request of the first agent). What we will call ‘complex’ interaction could also be called ‘conditional’ interaction or ‘collective interaction’. Thus, an interaction of “I move to where I see you sitting” is a simple interaction, while “I move if you move” is a complex interaction.

A MAS that contains no direct-interacting agents is an NI MAS, a MAS that contains at least one simple interaction is an SI MAS, and a MAS containing at least one complex interaction is a CI MAS.

### **NO DIRECT-INTERACTION (NI) MAS MODELS**

Overmars et al. [10] elaborate on the question if empirical land use science should remain caught in its current inductive, econometric paradigm, or try and become more deductive (‘theory-led’), e.g. by assuming an actor theory and testing the degree to which we may explain land use with it. In order to illustrate their case, they apply an inductive inference and a MAS model to land use in a part of the Philippines. Expressed through a simplified AiC structure, the MAS model contained different ethnic groups with varying land use options and different preferences (‘interpretations’) for different crops. The crops have different outcomes depending on markets, slopes and other environmental features. The agents are spread out over the GIS map according to their real demography. The agents do not interact, however. They just do their activities with their land without being aware of, let alone interacting with, any neighbour.

Both the inductive inference and the MAS model (without any calibration, i.e. purely deductive) generated a 70 percent fit between inferred/predicted and real land use in the GIS grid cells. This is empirically fascinating, inter alia because induction versus deduction is a basic question in empirical science and because deduction delivers a superior, namely causal rather than statistical, type of knowledge. This also enhances policy relevance. The response to a new crop, for instance, is easy to predict with the MAS, but out of reach of the inductive model (simply because the new crop, by definition, cannot be found in the dataset to do induction with). From the theoretical point of view of developing MAS models, at the same time, the model is utterly dull. Nothing happens in the model, so to speak. One run does it all.

‘Emergence’ is the hunting ground of theoretical MAS modellers. It may be interesting to note, therefore, that even NI MAS models may generate emergence. Take the well-known case of the Paramecium in an aquarium. Starting from a random distribution, these tiny creatures will be found in one corner of the aquarium after a while. Why? Are they attracted to each other (an SI model)? Or do they see food or smell in one corner and move, somewhat

analogously to the NI model of the farmers above? Biological reality is an even simpler NI. The only rule for Paramecium is: “move randomly until you find food”. The pattern emerges out of this simplest of NI rules. For virtual examples, see the Net Logo site, e.g. the termites running about in the Net Logo Termites model [11].

Even some forms of social learning can be modelled in NI MAS. One spatially explicit example is a model written in the AiC-based MameLuke Framework [12]. Farmers here learn from evaluating their own performance but also from those of their neighbours, e.g. looking at how their neighbour’s various crops are doing on slopes comparable to their own, depending on their neighbor’s choices such as the planting date. Thus, farmers learn from each other without direct interaction with each other.

### **SIMPLE-INTERACTIVE (SI) MAS MODELS**

Farmers in the Philippines and anywhere obviously do interact with neighbours and other actors. They discuss crop choices, they are influenced by government and NGO agents; they get news from their children that life in the city is much better than muddling through in the village. Capturing a wider understanding and prediction of land use therefore requires MAS models that contain Simple Interactions such as these.

The interaction that we will give special attention here is how land users are influenced by actions from ‘higher’ actors in a causal chain. These causal chains may be simply called power. We focus on this type of SI because it is strangely neglected in the social sciences and MAS. The usual answer of the social scientist to questions referring to interaction between agents is “social networks!”. Social networks capture a lot, but not power in a straightforward manner, even when networks are constructed of actors that do not necessarily know each other. The president of your university, the government that raises and spends your taxes, the bus company that decides if you have an option to commute by public transport, are they in your social network? Network theory does allow to identify central actors in your network, e.g. through Bonachich’s eigenvector centrality metric, but it does not seem likely that the director of the bus company will be identified as central in your network. Farmers, too, they are influenced by subsidies, by prohibitions, by markets, by extension policies and so on, decided by actors far removed from them. Yet, these actors may often be much more relevant for the explanation of land use than the farmer’s social network that basically contains of other farmers that are in the same boat rather than determining where the boat is going.

Let us have a closer look here, put in AiC terms. We may find farmers all growing maize, while looking at their motivations, growing coffee would in fact be more profitable. It may be the case that the farmers are tenants and that the landlord has prohibited coffee. In AiC terms, the landlord has taken away coffee as an option. The landlord could be called a ‘secondary’ actor behind the farmers as primary ones. Why would the landlord have issued the prohibition? It may be because he fears that allowing any perennial crop implies that legally, farmers have invested in the land, through which they may build up land tenure rights. Who may have influenced this motivation of the landlord? The Ministry of Land may well be a tertiary actor here, refusing as it does to settle the land rights issue either in favour of smallholders or big holders. And in the background of that one may be the World Bank as an actor, arguing that land reform has failed so often and advising to wait for a “willing seller, willing buyer” situation.

Figure 3 depicts the generalized version of this basic idea, called the ‘Actors field’ in the AiC framework. The essence is that actors are connected through actions that influence the options and/or motivations of other actors. Actors expand the range of implementable options of other actors e.g. through Research and Development (R&D) communication such as

agricultural extension. They enhance capacities ('empowerment') through supporting organizations or rural credit schemes. They reduce capacities through prohibitions and permit conditions. They influence objectified motivation e.g. through levies, subsidies, bonds, improving circumstances in public traffic, and so on. They work on actors' cultural interpretations through making things 'hot' or declaring them fit only for the losers (like smoking). In all this, actors are not connected directly (in face to-face contact, seeing each other, forming a social network or community). For the sake of simplicity, we have connected the actors through a reduced version of AiC actors' model with simply 'options' and 'motivations'. Taking the full model works the same way – or BDI for that matter, with actors influencing beliefs, optional plans etc.

The action of the primary ('proximate') actor (e.g. a category of farmers) is influenced by one secondary actor through influence on the primary actors' options (e.g. through the action of knowledge dissemination, a credit scheme or a prohibition), by another secondary actor through influence on the primary actor's motivations (e.g. the government through the action of establishing a fertilizer subsidy), which in turn is influenced by a tertiary actor (e.g. the fertilizer industry lobbying and bribing the government for the subsidy).

The AiC agent structure has been simplified into the Action-Actor-Options-Motivations triangles that may be repeated and expanded in any direction as far as empirical significance or theoretical interests go. (See [13] for an example on tropical deforestation).

Actors fields may be forever expanded, following a general rule that "behind every factor, there is an actor". With students, it is always fun to try find the International Monetary Fund (IMF) behind anything. Actual modelling of actors fields depends on choices of empirical, policy or theoretical relevance. Note that the causal routes reach all the way from the local to the global levels. Yet the whole picture remains 'actor-based'. The World Bank is an agent that lives in a 'world system' of global trade flows, national actors and so on from which it takes its options and motivations. Yet, the actor, not the world system, acts. Note that we model cross-scale interaction here, with each actor taking his options and motivations from his own life world scale (local, regional, global).

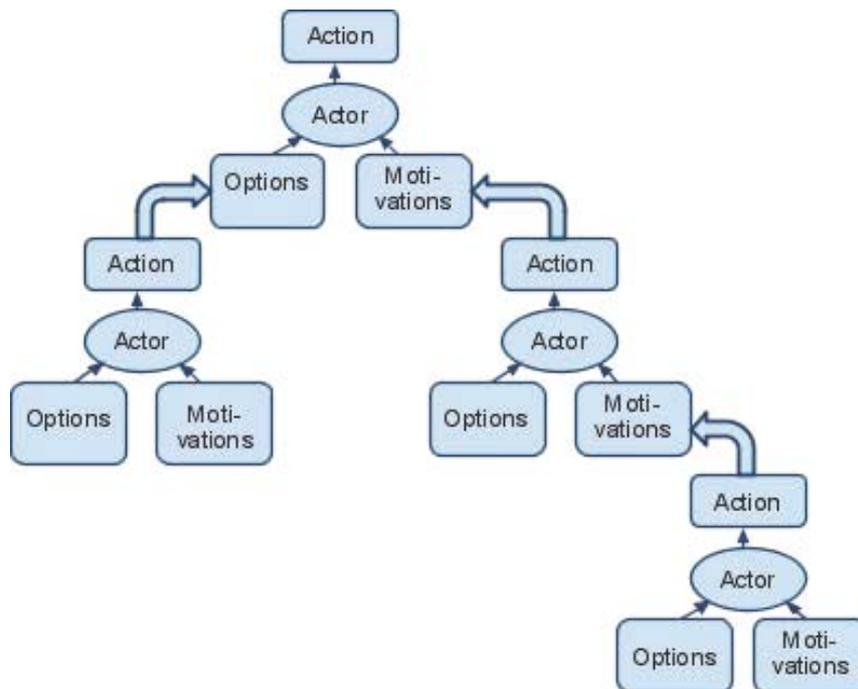


Figure 3. An Actors Field structure in the Action-in-Context framework.

Social learning (from other actors) and the almost endless variety of inter-agent influences captured by the actors' fields are not the only possible types of Simple Interactions. SI MAS models may, for instance, also involve spatial interaction, e.g. a rule of "I settle where I see kinsmen settled" in migration models (e.g., [14]). To a certain degree, also collective action may be taken up in SI models. Not fully, because that will be typically the CI domain, but through simple empirical rules that may be grounded in field work. One such a rule concerning the emergence of farmer cooperative may be, for instance, "if there are fewer than 20 agents, and if these agents have a basic level of trust in each other e.g. through shared ethnic identity, and if their profits through the cooperative are at least twice the total profits of working separately, a cooperative will be formed". Note that this rule contains conditions ('fewer than 20' etc.) but these are not the interactive conditions of the CI MAS models, which in this case might read as 'I will join the cooperative if you do'.

### **COMPLEX-INTERACTIVE (CI) MAS MODELS**

As said, the Complex, Conditional Interactions that characterize the CI MAS models are of the type of "I move if you move" – see for instance the seminal tit-for-tat rule and prisoners dilemma. CI MAS models are typically made to study the emergence of stable societies out of multiple interactions of agents with such rules, preferably without invoking 'non-rational' elements in the actor model such as social norms [15]. The examples and projects here, revolving around concepts such as complexity, trust, self-regulation, agent societies and so forth are too well-known and numerous to necessitate elaboration here.

### **CONCLUSION AND FURTHER WORK**

Complex, conditional agent interactions are the great attractor for MAS and game theorists. And indeed, no-one can be blamed for being fascinated by the intricacies of tit-for-tat, the prisoner's dilemma and the emergence of stable societies out of simple actor rules. Or: what would be a greater achievement than being able to predict the outcomes of multi-country climate negotiations with a MAS? Giddens' [16] theory of structuration, standing in the perennial 'actor versus structure' debate in the social sciences, may be summarized as that actors create and change institutions (i.e. structure – rules, organizations, societies), and that institutions shape and influence actors. This principle positions the CI MAS and SI MAS models with respect to each other. CI models typically focus on collective action, i.e. how agents build institutions, e.g. through trust, collective social capital, stepwise actions, negotiations, reciprocity in multiple encounters, and so on. SI models typically focus on the reverse causality, i.e. how institutions shape and influence agents.

This pattern is informative in the way the CI and SI models inform and link to each other. CI models can approach how institutions are made, while SI models can approach how institutions, once made, impact on actions in causal chains of agents.

For any empirical problem however, the fact remains that the question must be raised, what is needed most for the given challenge? What direction of causality dominates in the given problem? Hence, with Ockam's razor in mind, the question is: can SI MAS models do the job here? As we saw, cross-scale interactions, with interlinked actors all living in their own life worlds up to the global level, are no problem for SI MAS models, as is social learning with actors interpreting what other actors do. As we saw too, even if some questions of collective action are relevant to the empirical problem to some degree, it might be simulated in an SI model in a reduced form, e.g. a field-based rule.

Our main message at this point is that SI models, if fully explored inter alia through the actors' field concept, hold an enormous empirical potential. Our bet would be that 90 per cent

of the land use change in any nation can be approached through SI MAS models that contain the interactions between local, national and global agents. The relatively low level of attention that these models receive, though understandable because of the 'social network' routine in the social sciences and the supply-driven history of MAS, is therefore not an entirely positive thing. If MAS modelling is to live up to its great promise for the empirical social sciences, exploring its potentials to work with simple, SI actor interactions would appear to be of great value.

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## **VRSTE I PRIORITETI MEĐUDJELOVANJA U SUSTAVIMA MNOŠTVA AGENATA**

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### **SAŽETAK**

Sustavi mnoštva agenata mogu biti klasificirani ovisno o tome sadrže li neizravna međudjelovanja, jednostavna međudjelovanja ili kompleksna, uvjetovana međudjelovanja između agenata. Ovaj rad razmatra i ilustrira kako modeli s jednostavnim međudjelovanjima, iako su za teoretičare sustava mnoštva agenata manje fascinantni nego modeli sa složenim međudjelovanjima, zaslužuju više pozornosti u zajednici koja se bavi sustavima mnoštva agenata. Modeli jednostavnih međudjelovanja mogu sadržavati društveno učenje i recipročne relacije. Kao možda i najvažnije, modeli jednostavnih međudjelovanja omogućuju povezivanje više skala putem vezanja lokalnih i globalnih aktera u njihovim lokalnim odnosno globalnim svjetovima.

### **KLJUČNE RIJEČI**

sustavi mnoštva agenata, socijalno učenje, recipročne relacije

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