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MICROMACHINING –REVIEW OF LITERATURE FROM 1980 TO 2010

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Review article

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ABSTRACT

Trend of miniaturization of products and consequently its components nowadays can be evident in almost every production field. To accomplish requirements imposed by miniaturization micromachining proved to be a satisfied manufacturing technique. Herein the term micromachining refers to mechanical micro cutting techniques where material is removed by geometrically determined cutting edges. The aim of this review article is to summarize existing knowledge and highlight current challenges, restrictions and advantages in the field of micromachining.

KEY WORDS

miniaturization, non-MEMS, micromachining, size effects

CLASSIFICATION

JEL: L63, O31
PACS: 87.85.Va

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INTRODUCTION

The trend of micro-miniaturization of the products and its parts has already become forceful in industry, especially in field of micro electromechanical system (MEMS) or micro system technology (MST). In MEMS manufacturing techniques such as *photolithograpy, chemical-etching, plating* and *LIGA* are used, as shown in Figure. They are very well known in semiconductors or microelectric manufacturing and used for large volume production, mainly sensors and actuators made of silicon or limited range of metals.

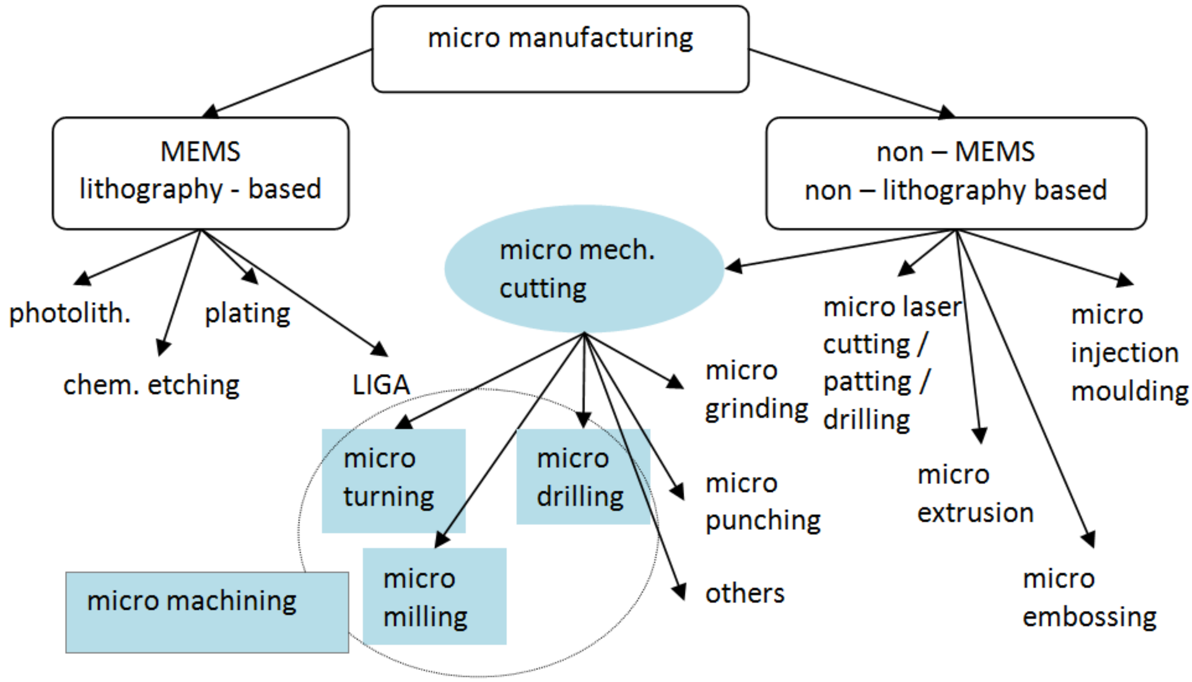

Figure 1. Classification of micro manufacturing techniques.

However, in the last two decade new category of micro manufacturing techniques have been developed, known as non-MEMS or non-lithography-based micro manufacturing. Non-lithography-based micro manufacturing include techniques such as *micro EDM, micro mechanical cutting, micro laser cutting/patting/drilling, micro extrusion, micro embossing, micro stamping* and *micro injection moulding* (Figure). These manufacturing techniques are fundamentally different from MEMS micro manufacturing in many aspects [1]. Non-lithography-based micro manufacturing can produce high-precision three dimensional products using a variety of materials and possessing features with size ranging from tens of micrometres to a few millimetres. Table 1 shows the fundamental differences between MEMS micro manufacturing and micromachining.

Micromachining refers to mechanical micro cutting using geometrically determined cutting edge(s) (micro turning, micro milling and micro drilling, etc.) performed on conventional precision machines or micromachines. Although lithography-based manufacturing can achieve smaller feature size, micromachining has many advantages in terms of material choices, relative accuracy and complexity of produced geometry. Moreover, it is a promising technology for bridging the gap between macro and nano/micro domain [1, 2], as can be seen in Figure.

Although micromachining techniques are similar to conventional (macro) machining manufacturing techniques, simple scaling of parameters or process model cannot be applied due to size effects. There are two research approaches taken to deal with size effects. These two approaches overlap in some areas and attempt to address similar issues, such as cutting

Table 1. Comparisons between MEMS-based process and micro machining (adapted from [1]).

	MEMS – based process	**Micro mechanical machining**
Workpiece materials	Silicon, some metals	Metals, alloys, polymers, composite, technical ceramics
Component geometry	Planer or 2.5D	Complex 3D
Assembly methods	None or bonding	Fastening, welding, bonding
Relative accuracy	$10^{-1} – 10^{-3}$	$10^{-3} – 10^{-5}$
Process control	Feedforward	Feedback
Machine size	Macro	Macro or micro
Production volume	High	High or low
Production rate	High	Low
Total investment	High	Intermediate or low
Applications	MEMS, microelectronics, some planner micro parts	Various applications requiring 3D micro components

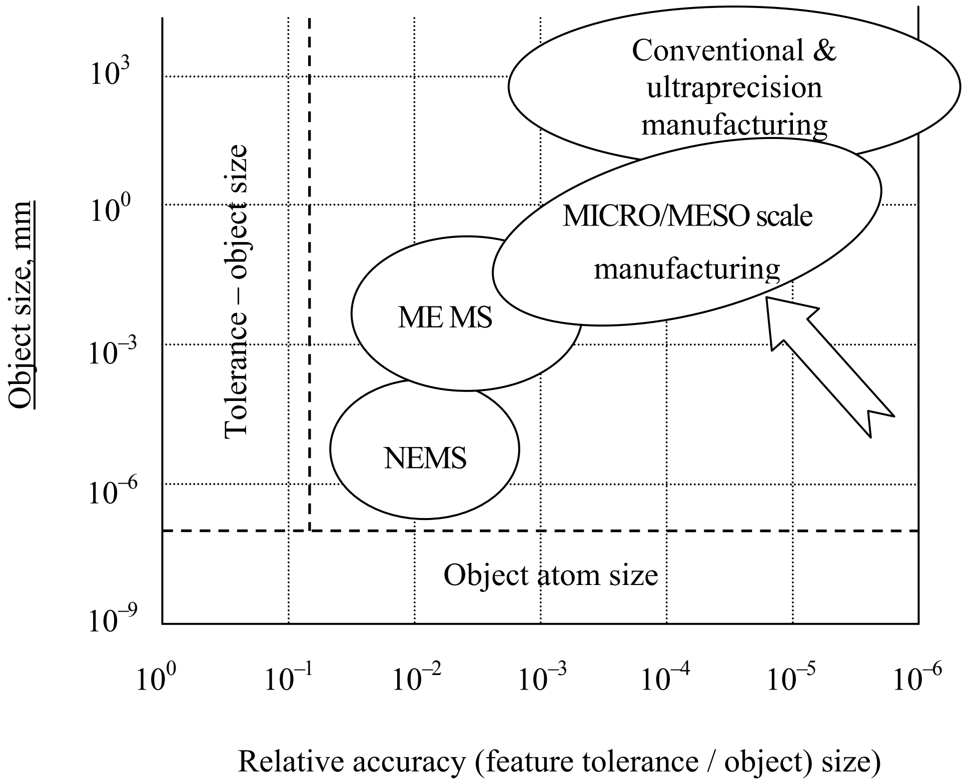

Figure 2. Micro manufacturing size/precision domains in relation to other existing technologies (adapted from [1]).

tool edge size effect, minimum chip thickness, etc. [1]. One approach is based on minimization of the conventional machining process, tooling and equipment with an emphasis on their scaling down effects. Macro models are adapted to micro cutting with consideration of the size effects. The other approach, covered in this paper, find its origin in ultra-precision machining, with the emphasis on cutting mechanics. This approach is similar to diamond cutting research, but studies micro cutting, with more emphasis on tool geometries, material crystalline orientation and micro structures. Key aspects that have influence on micromachining process are shown in Figure 1.

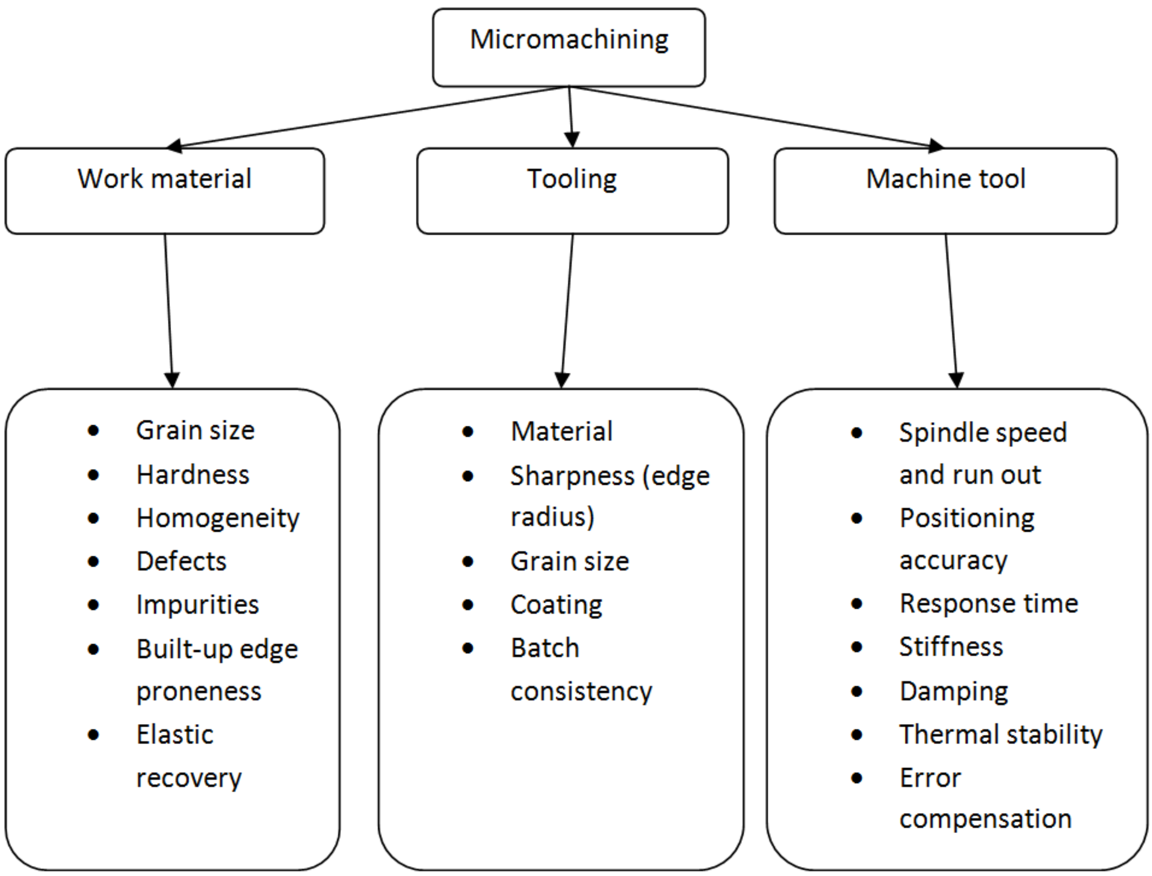

Figure 1. Key aspects in micromachining (adapted from [3])

Although research in micro cutting has been reported since late sixties [3, 4] strong interest in micromachining can be evident from the middle of the last decade, as it can be noticed from Figure 2. However, there is little research papers dealing with materials that cannot be machined easily [3]. Micromachining of materials such as hardened steels, stainless steels, silicones, glasses and ceramics introduces additional problems related to excess tool wear, unpredictable tool failure, low stiffness of the micro tools, surface and subsurface cracks, etc.

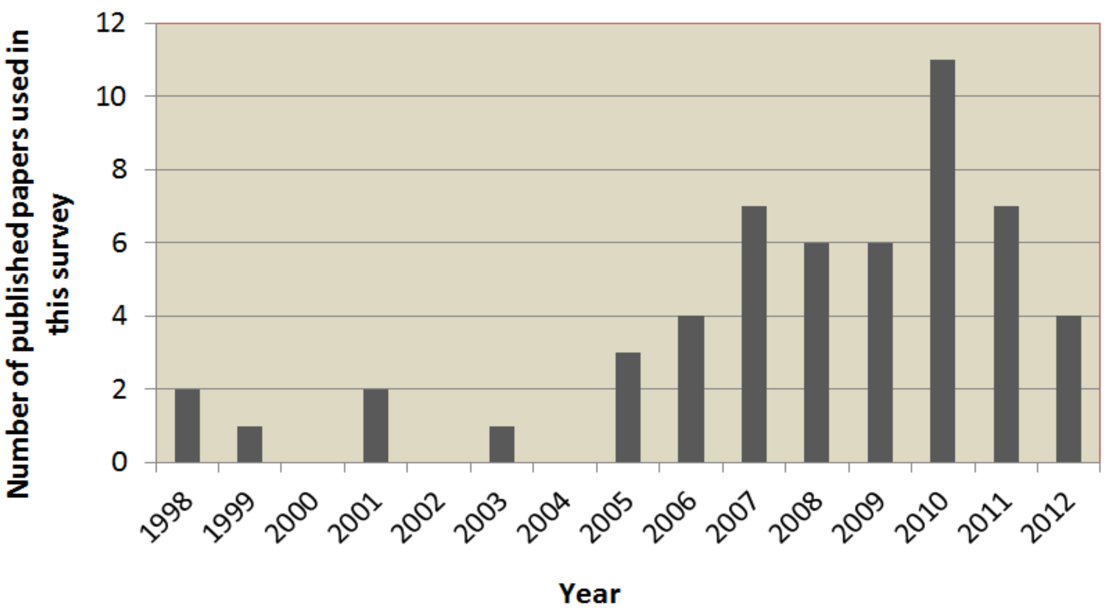

Figure 2. Evolution of the number of papers published on micromilling in recent years (adapted from [3]).

The paper is divided into three main parts which are dealing with process physic, micro cutting tools and micro machine tools, and within them subjects such as size effects, workpiece material requirements, surface quality, cutting tool material, geometry, wear and failure mechanisms, machine tools, sensors and other related technicalities are discussed.

MICROMACHINING PROCESS PHYSICS

SIZE EFFECTS

Size effects are certainly among the principal issues, if not the most relevant aspect, to be addressed in micromachining [2, 3, 5-8]. It is typically characterised by a dramatic and nonlinear increase in the specific energy (energy consumed per unit volume of material removed) as the undeformed chip thickness decreases. Experimental observation of this phenomenon in machining of ductile metal (SAE 1112 steel) has been reported in early work by Backer et al. [9]. They performed a special series of tests to determine relation between shear stress and chip thickness. The results from the experiment where later modified by Taniguchi [10] as shown in Figure 3.

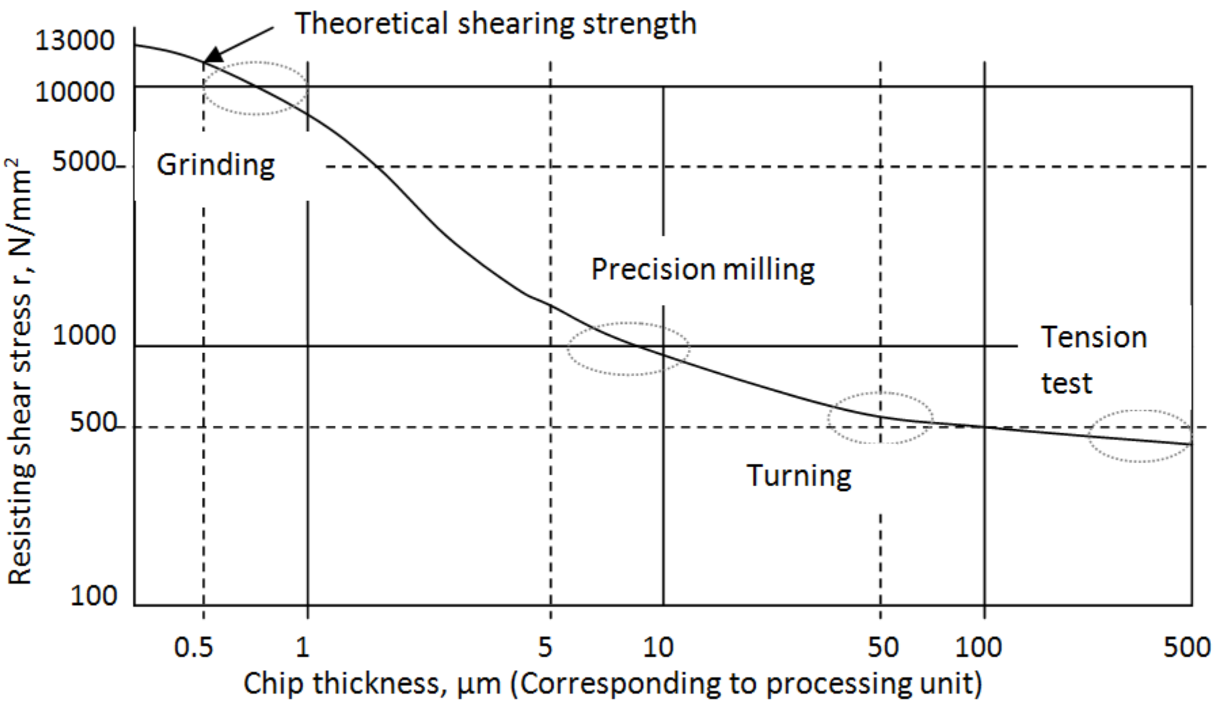

Figure 3. Relation between chip thickness and resisting shear stress modified by Taniguchi [10].

Although micromachining includes many characteristic of conventional (macro) machining process, the size effect modifies the mechanism of material removal and prevents the production parameters to be changed according to the rules of similarity. There are two different aspects of size effects of concern, when the thickness of material to be removed is of the same order of magnitude as the tool edge radius, or where the microstructure of workpiece material has significant influence on the cutting mechanism [6].

The size effect was attributed to tool edge radius effect, material micro-structure effect i.e. dislocation density/availability, crystallographic orientation, material strengthening effect due to strain, strain rate, strain gradient, subsurface plastic deformation, material separation effect and cutting speed. However, there is no clear agreement on the origin of the size effect [8].

WORKPIECE MATERIAL

In conventional machining workpiece is often considered to be homogeneous and isotropic. Such an assumption cannot be made when dealing with micromachining processes due to size effects caused by workpiece material microstructure. As evident from the Figure 1, key aspects to be considered in micromachining related to workpiece material are homogeneity, defects, grain size, hardness, elastic recovery, etc.

Backer et al. [9] and Shawn [5] discusses the origin of the size effect in metal cutting which consequence due to short range inhomogeneities present in all commercial engineering metals. When the volume of material deformed at one time is relatively large, there is a uniform density of imperfections and strain (and strain hardening) may be considered to be uniform. However, as the volume deformed approaches the small volume, the probability of encountering a stress-reducing defect (grain boundaries, missing and impurity atoms, etc.) decreases. In that case the specific energy required and mean flow stress rises and the material shows obvious signs of the basic inhomogeneous character of strain. As a result, active shear planes are evident in a free surface and can be observed at back free surface of chip.

When the ratio of average grain size to uncut chip thickness approaches the unit size effect becomes relevant. As a result, chip formation takes place by breaking up of the individual grains of a polycrystalline material [3, 7, 11]. Considering that in micromachining uncut chip thickness can be even smaller than the average grain size, most polycrystalline materials are thus treated as a collection of grains with random orientation and anisotropic properties [4, 6, 7, 12].

The crystallographic orientation affects the chip formation, shear strength and the subsurface crack generation [6, 13]. The variation in shear strength causes cutting force variation over different cutting direction which results with the material induced vibration, in addition to machine induced vibration, causing degraded surface quality. To et al. [14] obtained the effects of the crystallographic orientation and the depth of cut on the surface roughness by conducting the diamond turning of single-crystal aluminium roods (Figure 4). To avoid the crystallographic effects of grains, Furukawa et al. [15] suggested the use of about ten times larger depth of cut than the average grain size.

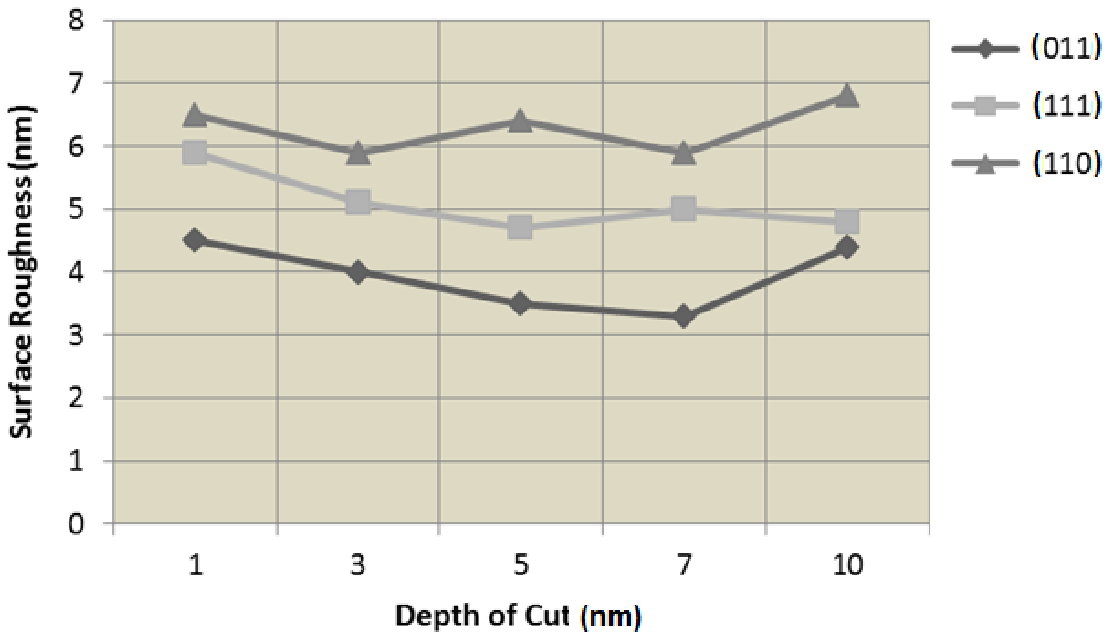

Figure 4. The effects of the crystallographic orientation and the depth of cut on the surface roughness; (011), (111), (110) – Miller's indices (adapted from [7]).

It is to be noted that changing crystallography (multi phases or multi grains) also affects the cutting mechanism [2-4, 6]. When the cutting tool engages from one metallurgical phase to another, the cutting conditions change, causing interrupted chip formation due to variations in the hardness of two adjacent grains. This results with variation in the cutting force and generation of additional vibration, accelerated tool wear and poor surface finish. Moreover, elastic recovery of particular grain plays important role in micromachining, especially when dealing with multiphase materials [4].

Majority of published work is dealing with work materials which are considered easy to cut, such as low hardness steels (carbon steels, high strength low alloy steels and high alloy steels which do not subject to hardening), aluminium and copper alloys, as illustrated in see Figure 5. Hardened steels, heat resistant alloys, ceramics, glasses and other hard to cut materials are less studied and seldom subject of investigation.

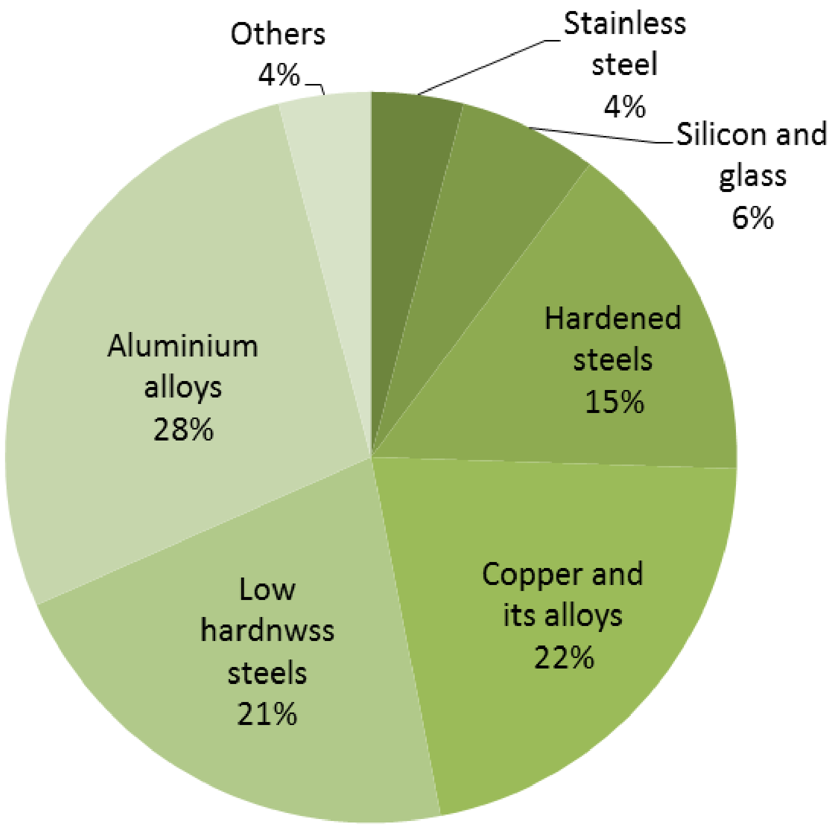

Figure 5. Typical workpiece materials used in micromachining (adapted from [3]).

MINIMUM CHIP THICKNESS

Considering conventional machining, it is assumed that cutting tool edge is perfectly sharp and that there is no contact between the tool's clearance face and machined surface. Chip is then formed mainly by sharing of the material in front of the tool tip. However, such an assumption cannot be made for micromachining where achievable tool edge radius is commonly on the same order as the chip thickness (cutting depth). Where in conventional machining shear takes place along shear plane, in micromachining shear stress rises continuously around the cutting edge [2, 7] and material seems to be pushed and deformed rather than sheared [16, 17]. Therefore, micromachining processes are greatly influenced by the ratio of the depth of cut to the cutting edge radius causing a significant influence to the cutting process by a small change in the depth of cut. This ratio defines the active material removal mechanism such as cutting, plowing, or sliding and thus the resulting surface quality.

The definition of minimum chip thickness is the minimum undeformed chip thickness below which chips may not form [2, 7]. Figure 6 illustrates the chip formation with respect to the cutting tool edge radius (R_e) and the uncut chip thickness (h). When the uncut chip thickness is smaller than the minimum chip thickness (h_m), as shown in Figure 6(a), only elastic deformation occurs and no workpiece material will be removed by the cutter. As the uncut chip thickness approaches the minimum chip thickness (Figure 6(b)), chips are formed by shearing of the workpiece, with some elastic deformation still occurring. As a result, actual depth of cut is less than the desired depth. However, when the uncut chip thickness is larger than the minimum chip thickness (Figure 6(c)), elastic deformation is significantly reduced and the entire depth of cut is removed as a chip.

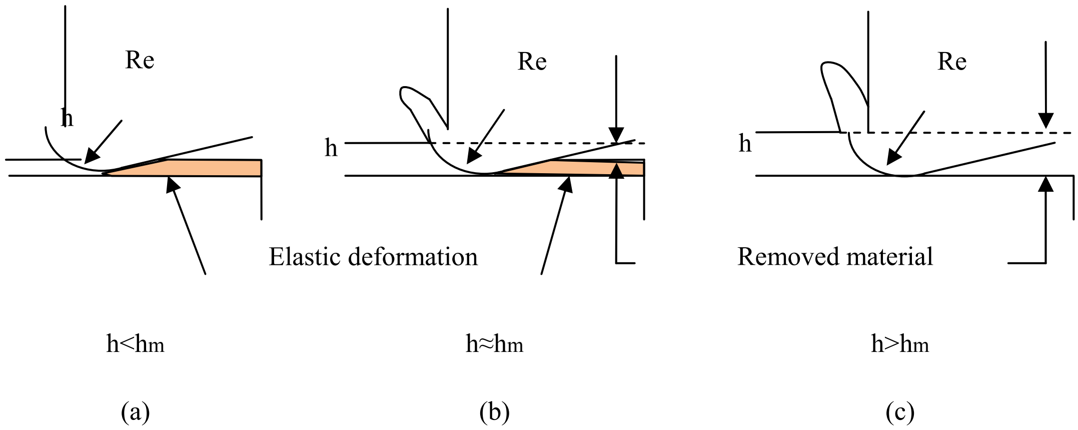

Figure 6. Schematic diagram of the effect of the minimum chip thickness (adapted from [2]).

Knowledge of the minimum chip thickness is essential in the selection of appropriate machining parameters to ensure a proper cutting and avoid plowing and sliding of the tool [2, 6, 7]. It is very difficult to directly measure the minimum chip thickness during the process, in spite of knowing the tool edge radius, so it is obtained by experimental results or trough numerical simulations. Minimum chip thickness depends primarily on the ratio of uncut chip thickness to cutting tool edge radius (cutting edge sharpness) and secondarily on the workpiece material properties [6] and the friction between the tool and workpiece material. Estimation of the minimum chip thickness is one of the present challenges in micromachining. Furthermore, minimum chip thickness cannot be expressed as precise and single value but rather as a range of values with unclear limits [17]. Depending on the material, minimum chip thickness was estimated to be between 5 % and 40 % of the tool edge radius [2, 6, 16, 17].

CUTTING FORCES

The majority of researchers who have investigated micromachining processes have used cutting force for monitoring or improving the quality of machined products. Excessive cutting force limits the accuracy and the depth of cut due to deflection of tool and work piece, defines the bending stress that determines the feed rate and introduce the built-up edge (B.U.E.) [2, 4]. Therefore, reducing the cutting force in micromachining operations significantly improves material removal productivity, decrease tool deflection and tool wear, delay tool failure, and narrow workpiece tolerance limits. As in conventional machining, micromachining cutting force consists mainly of normal and tangential components, usually called shearing/cutting and plowing/thrust force, respectively.

The cutting force is directly related to chip formation. Since cutting force also determines the tool deflection and bending stress as mentioned, the tool edge radius is often larger than the chip thickness to prevent plastic deformation or breakage of the tool [2]. This small depth of cut results with large negative rake angle as shown in Figure 7. In that case workpiece is mainly processed by cutting edge causing an increase in friction on the rake face of the tool and significant elastic recovery of the workpiece along the clearance face of the tool, thus increasing the specific energy. Therefore, high ratio of the normal to the tangential component is observed as uncut chip thickness decreases as illustrated in Figure **8**, which indicates a transition of material removal process from cutting to ploughing [6, 7].

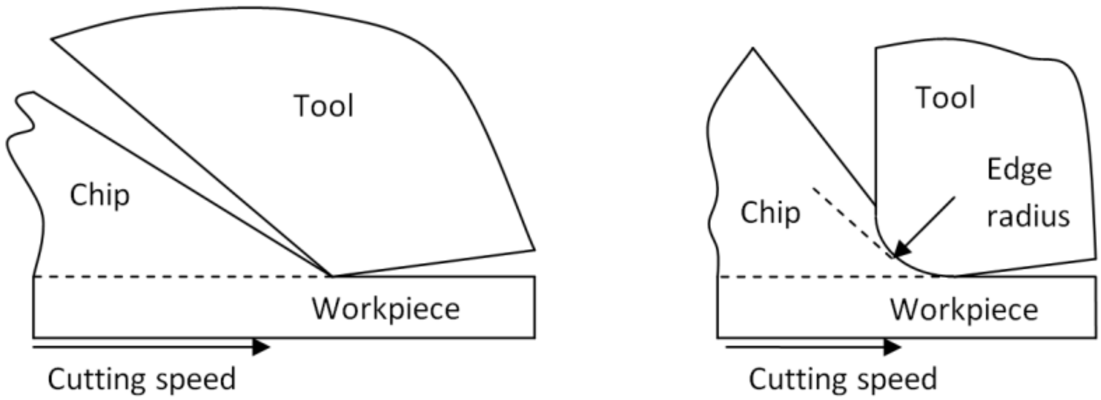

Figure 7. Schematic representation of the negative rake angle in orthogonal cutting (adapted from [17]).

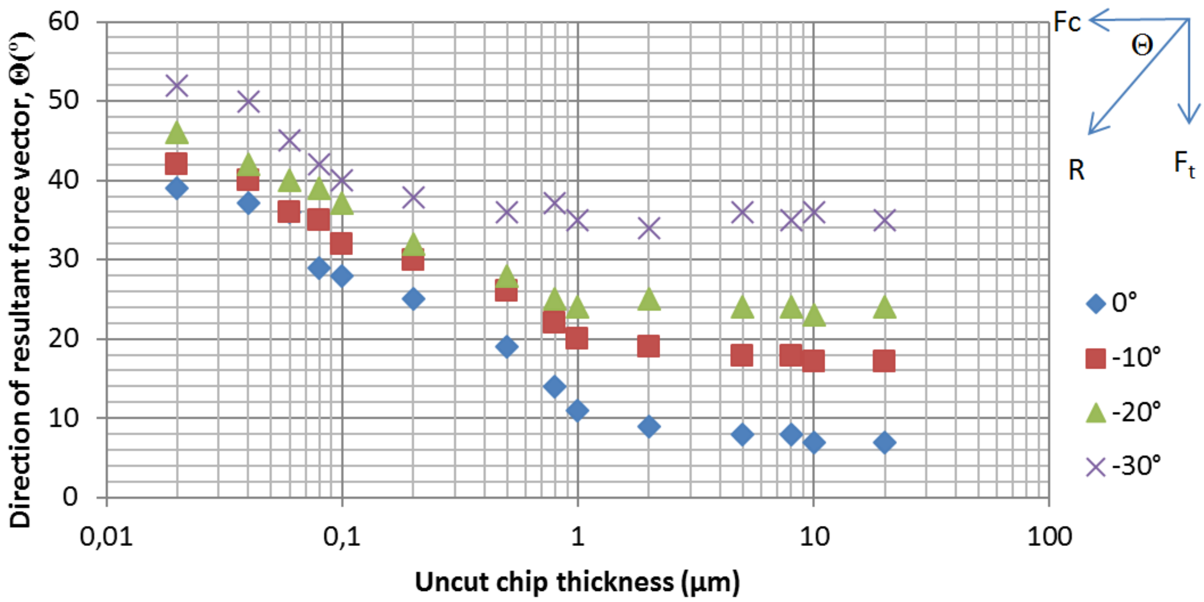

Figure 8. Resultant force vector versus uncut chip thickness at various rake angles (adapted from [7]).

Cutting force in micromachining is also significantly influenced by problems that are generally minor in macro-domain such as tool wear, unbalance (run-out) and instability (chatter) [2, 3, 6, 7, 18]. Accelerated tool wear results from increased friction between the tool and the workpiece because of the small uncut chip thickness and large negative rake angle. The smaller the uncut chip thickness, the greater the impacts on the tool wear and cutting force, i.e. cutting energy (Figure 9).

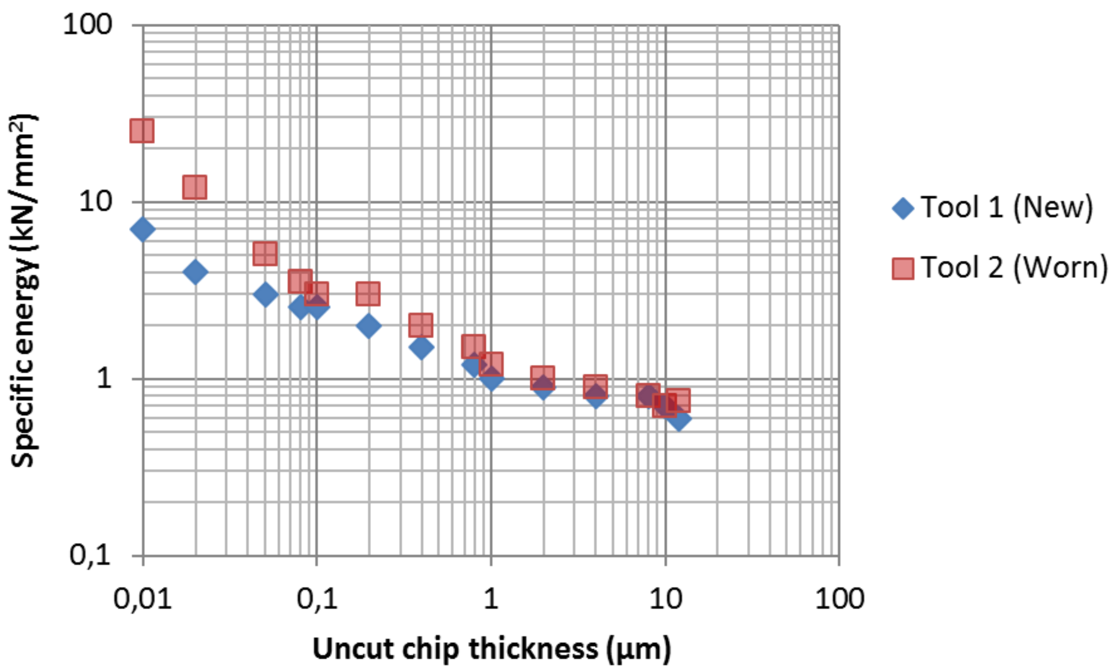

Figure 9. Specific energy versus uncut chip thickness for new and worn diamond tools (adapted from [7]).

Tool run-out is caused by tool deflection and a misalignment of the axis of symmetry between the tool and the tool holder or spindle. In macro-machining it often ignored, as the diameter of cutting tools is relatively large compared to the tool run-out and the speed is relatively slow compared to micro-machining. Tool run-out contribute to significant noise in force measurements, surface roughness and severe vibrations which causes burr formation.

Chatter introduces excessive vibrations that can lead to catastrophic failure and burr formation as a result of interaction between the dynamics of the machine tool and workpiece.

Additionally, laser assisted micromachining or vibration assisted micromachining can be applied when machining difficult to cut, hard materials, in order to reduce cutting force and extend tool life [2, 3, 6]. Micromachining forces and tool wear can be drastically reduced by focusing a laser beam ahead of the cutting path. This novel approach was reported by Ding et al. [19] and Kumar et al. [20]. Although the usage of laser assisted micromachining provides more consistent tool life behaviour, Kumar et al. reported larger burr heights and poorer surface finish and attribute it to impact of thermal softening. Ultrasonic vibration machining were introduced by Kumabe [21], and later improved by Moriwaki et al. [22, 23] through elliptical ultrasonic vibration (Figure 10), which showed improved cutting performance and surface quality. Vibration assisted micromachining also improves machining of ferrous metals with diamond tools by means of reduced tool wear [4]. Figure 11 shows a difference in chip formation due to no vibration, forced vibration and regenerative chatter.

BRITTLE AND DUCTILE MODE MACHINING

Although brittle materials (such as many optical glasses, ceramics, etc.) are normally machined using conventional processing techniques such as polishing, micromachining can bring many advantages due to increased flexibility in geometries produced, greater surface finish quality, and higher material removal rate, translating to higher production throughput [6, 7]. However, machining brittle material at high depth of cut has a tendency to generate excessive surface and subsurface cracking.

Shimada et al. [24] found out that, regardless of the material ductility, there exist the critical

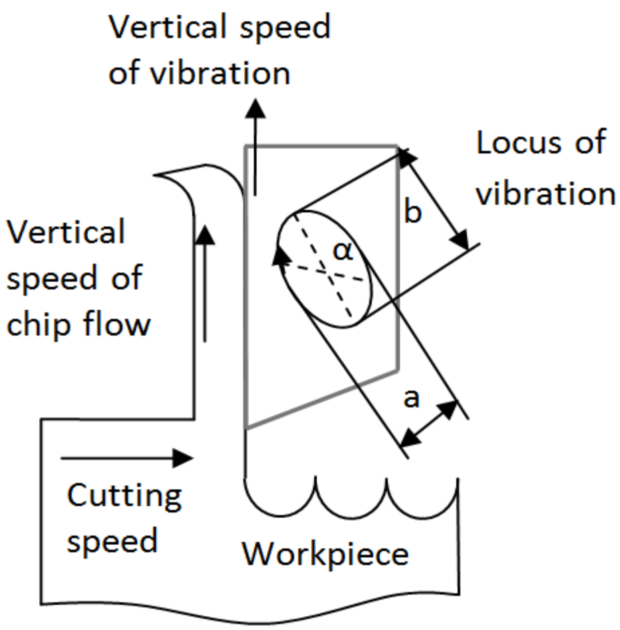

Figure 10. Principle of elliptical vibration cutting (adapted from [6]).

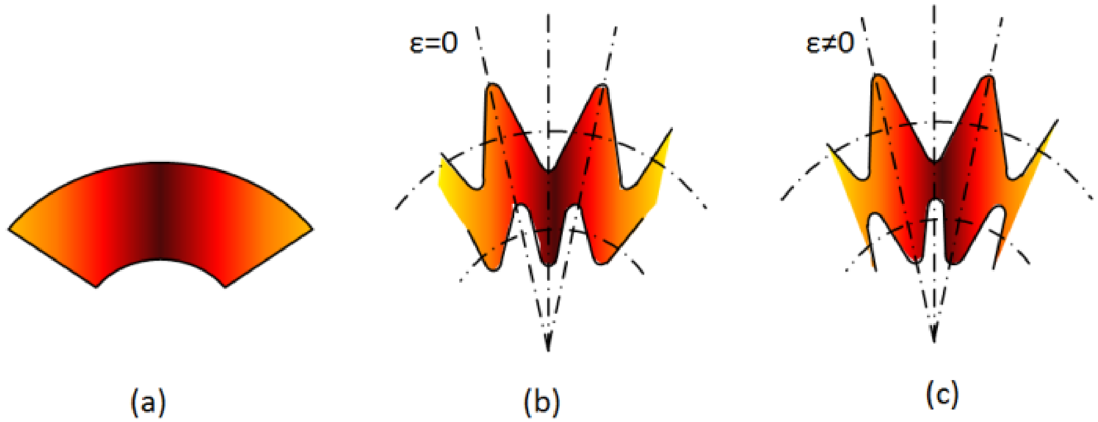

Figure 11. Chip generations due to: a) no vibration, b) forced vibration and c) regenerative chatter (adapted from [2]).

depth of cut, which causes translation from brittle to ductile material removal mechanism. Therefore, any brittle material can be machined in ductile mode if undeformed chip thickness is below the critical depth of cut, resulting with good surface finish and an uncracked surface.

The value of critical depth of cut depends on tool geometry and machining conditions. Excessive cutting velocity and higher negative rake angle increases the critical depth of cut [7], causing ductile mode machining difficult to obtain at higher feed rates. In addition, machining of some brittle materials in ductile mode is rather challenging due to extremely small depth of cut.

SURFACE QUALITY

Three dimensional assessments of the finished components are usually carried out using optical equipment (especially white light interferometry and atomic force microscopy) and scanning electron microscopy [3, 7, 18], and the surface quality is evaluated generally through surface roughness and burr formations.

While in the conventional machining processes a smaller uncut chip thickness generate smaller surface roughness, at micromachining there exist a critical depth of cut below which surface roughness starts to increase. This phenomenon shows a strong influence of size effects on surface generation, i.e. when unit removal size decreases, issues of tool edge geometry, cutting parameters and workpiece material properties becomes dominant factors with strong influences on resulting accuracy, surface quality and integrity of the machined component. Figure 12, obtained by [16], clearly shows the effects of size effects to the surface roughness, that is influence of the ratio of feed rate to tool edge radius (*a*/*r*) on the surface roughness. Therefore, optimal depth of cut depends highly on the degree of the size effects and for that depth of cut the best surface finish is produced.

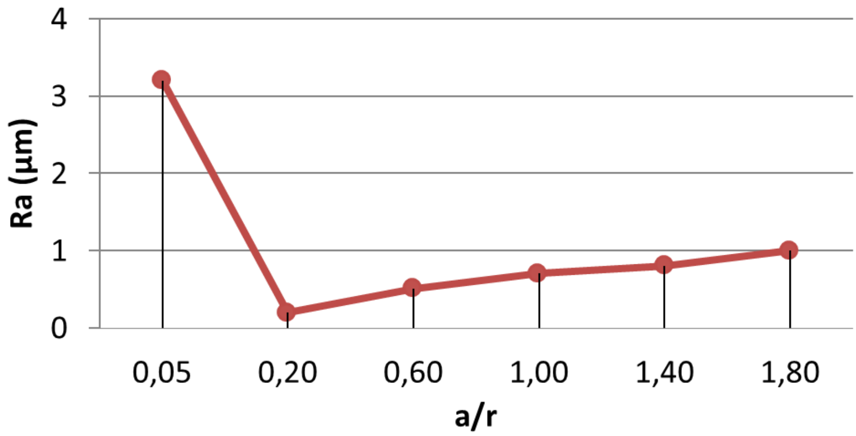

Figure 12. Experimental findings on surface roughness at the varying ratio of feed rate to tool edge radius (adapted from [16]).

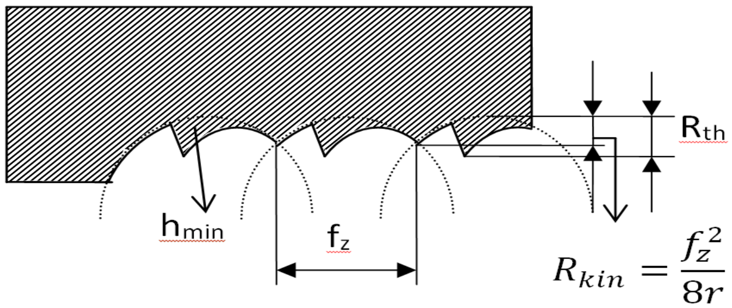

Figure 13. Theoretical surface profile based on spring back of elastically deformed material (adapted from [4]).

Many research papers [2-4, 6-8] associated optimal depth of cut with the minimum chip thickness, because below this threshold plowing and sliding effects tends to dominate machining mechanism producing discontinuous chips, bigger burr size, rough surface and elastic recovery of the workpiece material. As mentioned before, minimum chip thickness is a function of the ratio of uncut chip thickness to cutting tool edge radius and the workpiece material properties such as hardness, elastic recovery, etc., which are greatly affected by defects, impurities, grain size and crystallographic orientation, etc. Weule et al. [4] determined the achievable surface roughness of steel (SAE 1045) as a function of minimum chip thickness (and cutting edge radius). The achievable surface roughness can be predicted based on spring back of elastically deformed material as shown in Figure 13. Once the cutting depth reaches a minimum chip thickness material is removed by a shearing mechanism.

They also conducted experiments regarding relationship between machining parameters, material state and surface quality. Referring to Figure 14 it can be concluded that additionally to the ratio of feed rate to edge radius, cutting speed and material hardness are other relevant factors which have a significant influence on surface roughness. In order to generate smaller surface roughness, higher cutting velocity and harder workpiece materials

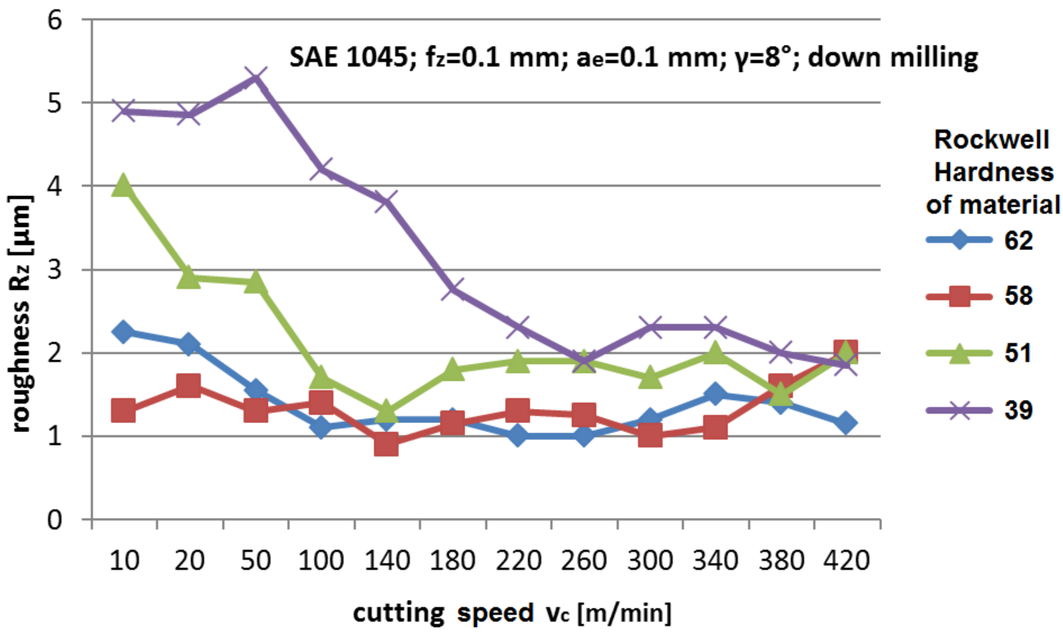

Figure 14. Influence of the cutting speed and material state on the surface roughness (adapted from [4]).

are preferable. Increased surface roughness at low cutting speeds was attributed to the formation of a built-up edge [3, 4]. Mian et al. [8] in their work confirmed significant influence of cutting speed to the surface roughness and observed that the same applies for the burr root thickness.

In addition to size effects, resulting accuracy and surface quality is also directly related to the cutting tools properties and machine tools where issues such as tool wear, tool deflection, tool run-out, chatter, etc. leads to additional surface deterioration [2, 3, 7, 8]. In order to decrease tool wear and thermal loads fluids are applied for lubrication and cooling. As fluid, either water-based emulsions or oils are used. They can be applied as a mist or flushed [18]. Flushed lubrication may be the better choice as they also improve chip evacuation process. Most unfavourable situation occurs when dealing with workpiece materials with high ductility. In that case long and continuously snarled chips are form which can easily interfere with tool engagement and burrs and contribute to poor surface quality [6, 18]. Moreover, different milling strategies can also affect surface quality [3, 18]. In case of machining aluminium alloy with a tungsten carbide cutter (Φ800 µm), lowest surface roughness was provided by the constant overlap spiral strategy, followed by the parallel spiral and parallel zigzag strategies [3].

Burr formation is probably the principal damage noticed on machined surfaces.

Burrs can be removed mechanically or by electro polishing. Disadvantage of mechanical approach is high manual effort or impracticability due to size of machined features [2], while electro polishing requests that no precipitations at grain boundaries or a different second phase are present [18]. Therefore, electro polishing is restricted to materials such as stainless steel, nickel and some copper base alloys. Furthermore, for monitoring purposes process must be stopped and the microstructure is evaluated by microscopy. Because there are also spots without burrs, where edges are eroded from beginning, prolonged exposure to electric field may cause rounded edges of product.

Similar to surface roughness, burr formation at micro scale is also affected with size effects. Sugawara [25] investigated the effect of the drill diameter on burr formation and concluded that burr size is reduced and cutting ability increased as drill size decreases. Generally, micromachining of ductile materials if often accompanied by burr formation, especially at the edges of microstructures [18].When the ratio of the depth of cut to the cutting edge radius is small, high biaxial compressive stress pushes material toward the free surface and generates large top burrs [26]. Also, the kinematics of the tool as it exits from the workpiece significantly influence burr formation due to plastic deformation (i.e. bending) of chips rather than shearing [27]. Schaller et al. [28] drastically reduced burr formation by coating the surface with cyanoacrylate polymeric material. After machining, the cyanoacrylate is removed with acetone in an ultrasonic bath.

Weule et al. [4] observed that, in contrast to surface roughness, burrs most frequently occurred when cutting hard materials. It is assumed that this is a result of faster tool wear, which increase cutting edge radius leading to burr formation. Additionally, tool coatings did not result in any substantial improvement on surface roughness [3, 8], while concerning burr size, best results are obtained when using tool coated with TiN, TiCN and CrTiAlN (in this order) [3].

Essentially, the relationship between surface roughness and cutting conditions is similar to that between burr size and cutting conditions. Both depend on the ratio of undeformed chip thickness to cutting edge radius, feed rate and cutting speed. However, the best process performance in terms of surface roughness and burr formation are not essentially obtained at the same cutting conditions [8].

MICRO CUTTING TOOLS

A material of Micro cutting tools (herein simply referred as tools) is the essential enabler for micromachining processes. Tool diameter and cutting edge radius determines achievable feature size and surface quality [7]. Cutting edge radius determines cutting tool sharpness and it influence on minimum chip thickness and determines effective rake angle of the tool as already discussed. If the diameter of micro-tools can decrease even further, the size of features on miniature components could be comparable to those produced with the lithographic techniques [2].

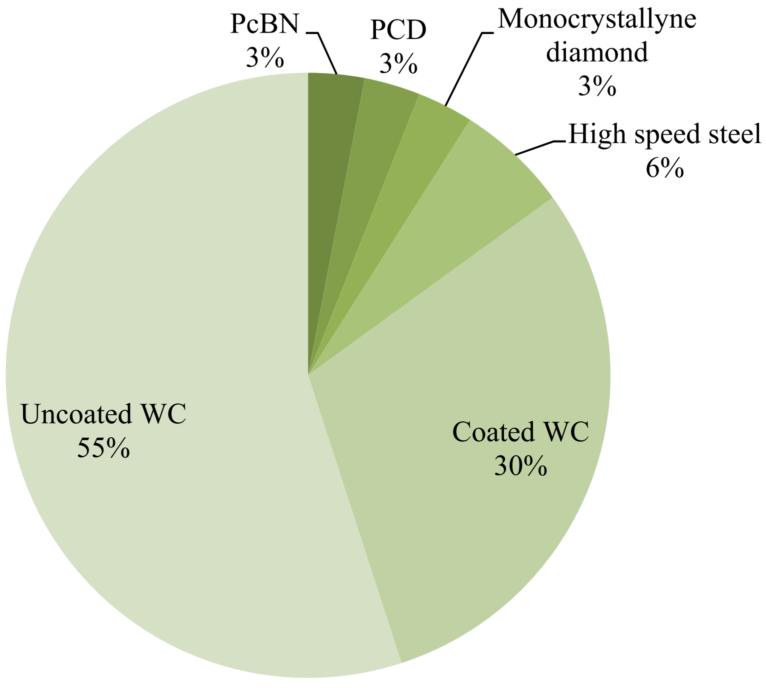

Figure 15. Principal tool materials used in micromachining (adapted from [3]).

As far as the tool material is concerned, either tungsten carbide or single crystal diamond are used. As can be seen in Figure 15, tungsten carbide is the most common choice due to its hardness, high toughness and relatively low price [2, 3, 18].

DIAMOND TOOLS

When dealing with non-ferrous and non-carbide materials, such as brass, aluminium, copper, nickel, etc., and brittle hard materials such as ceramics, silicon, glass, germanium, etc., single crystal diamond is preferred tool material due to its outstanding hardness, high thermal conductivity and elastic and shear moduli [2, 6, 7, 18]. Furthermore, diamond tools were used in most of the early micromachining research due to their homogeneous crystalline structure which makes it easy to generate a very sharp cutting edge through grinding, e.g. a cutting edge in tens of nanometres can be achieved [7]. Lower cutting edge radius enables lower depth of cut and ensures better surface quality.

However, diamond is limited to the cutting of non-ferrous materials because of the high chemical affinity between diamond and iron. When machining ferrous materials with diamond tools, carbon of the diamond can easily diffuse, causing severe tool wear. An exception occurs in the case of low cutting speeds, when low temperatures prevent diffusion [18], or in case of vibration assisted micromachining [4].

More recently, CVD (chemical vapour deposition) diamond coated tools have become available [29]. CVD diamond tools can be used to cut tungsten carbide with a cobalt percentage of 6 % or greater [7].

TUNGSTEN CARBIDE (WC) TOOLS

Tools that are used to machine ferrous materials are commonly made of tungsten (wolfram) carbide (WC) [2, 3, 6, 7, 18]. Tungsten carbide cutting tools are generally used due to their hardness and strength over a broad range of temperatures (Figure 16).

In general, published literature reports tool edge radii ranging from 1 μm to 3 μm [3]. However, in contrast to the homogeneous crystalline structure of diamond, tungsten carbide is a hard metal composite. As a consequence, tool cutting edge is always jagged causing burr formation on ductile materials like most metals [18].

Tungsten carbide is composed of a hard phase, mainly tungsten carbide powder, and a binder phase, typically cobalt [2], but nickel and iron are also possible [18]. Tungsten carbide powder is basically responsible for tool wear resistance and it consists of submicron particles with average size of 0,2 μm [18]. Binder content and average grain size determines the mechanical properties of the tool. Low binder content results with higher tool hardness and consequently higher wear resistance, where smaller grain size is responsible for higher fracture toughness. For interrupted cut or fluctuating load, higher binder content is recommended. Furthermore, to ensure isotropic mechanical properties, cross section of the tool must consist of a sufficient number of hard particles. Therefore, according to Gietzelt et al. [18], isotropic mechanical properties of tools with diameter below 30 μm are questionable.

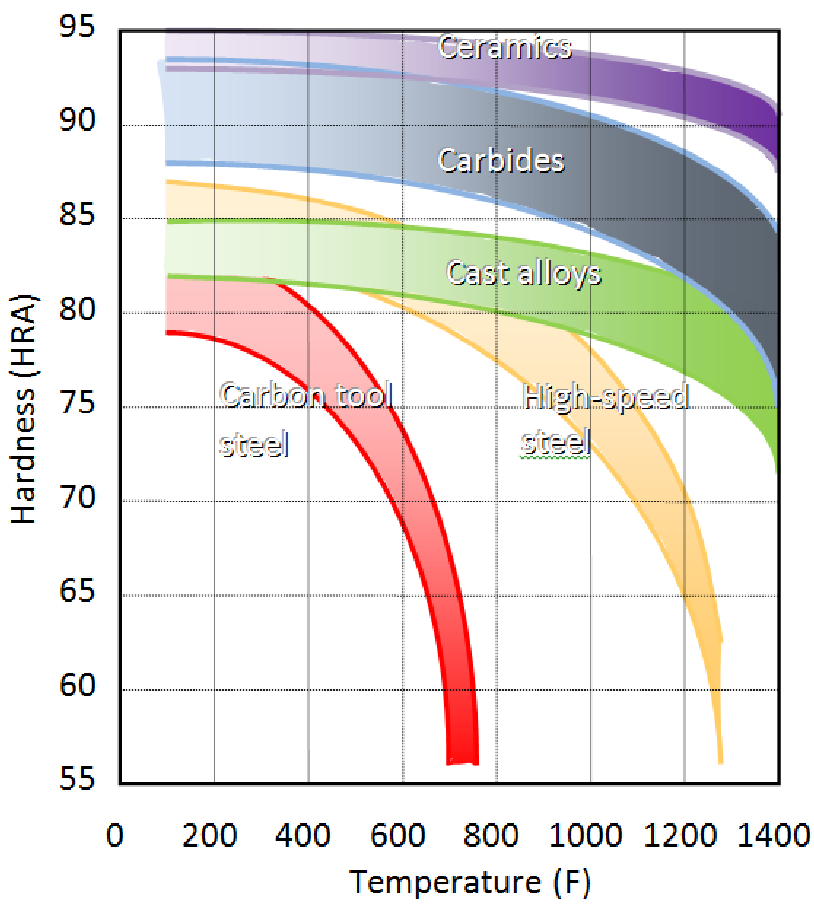

Figure 16. Hardness of cutting tool materials as a function of temperature (adapted from [2]).

COATINGS

Coating of tools with diameter below 0,3 mm become popular about five years ago with improvement in coating processes which enabled thinner and more uniform coating layers [18]. Main purpose of coating is to extent tool life by reducing a tool wear. In case of a thick coating, cutting edge radius is increased and consequently higher cutting forces are induced which undo the coating improvement regarding tool wear. Furthermore, formation of coating droplets must be avoided in order to prevent coating results in worse machining properties [30]. Additionally, chipping of coating layers were detected not only at the cutting edge but also in smooth substrate areas, as a result of poor adhesion of the coating.

Nowadays, TiAlN is the principal coating material applied to tungsten carbide cutters, but other coatings, such as TiN, TiCN, CrN, CrTiAlN, etc., can also be applied [3]. Majority of the coatings are quite uniform and below 1 µm in thickness, therefore rounding of the cutting edge can be neglected [18].

TOOL MANUFACTURING METHODS

Typically, mechanical micro grinding is used as a manufacturing process for production of micro tools. However, to achieve smaller diameters and more complex geometry, more accurate production methods may be required such as electrical discharge machining (EDM), wire electrical discharge grinding (WEDG), or focused ion beam (FIB) processes, etc. [2, 3, 6, 7, 31, 32].

Considering manufacturing and stability reasons, micro end mills made of single crystal diamond are no less than 50 µm in diameter [18] with achievable cutting edge radius in tens of nanometres [7]. In case of tools made of hard metals, end mills down to 20 μm in diameter [33-35] and drills down to 15 μm in diameter [36] are commercially available.

Egashira et al. [31] produced the smallest edge radius of 0,5 μm on carbide micro tool with diameter of 20 μm, using wire electrical discharge grinding process (WEDG). Moreover, the smallest tool diameter found was 3 μm tungsten carbide tool [32]. It was also produced by wire electrical discharge grinding process (WEDG) and used for slot milling of brass workpiece, but with unpredictable performance.

TOOL FAILURE

Tool failure is another major issue in micromachining, especially when dealing with hard and difficult to cut materials such as hardened steels, heat resistant alloys, ceramics, glasses, etc. In general, the life time of micro tools is unpredictable and depends strongly on the workpiece material [3, 18].

Smaller tools have decreased thermal expansion relative to their size, increased static stiffness from their compact structure, increased dynamic stability from their higher natural frequency, and the potential for decreased cost due to smaller quantities of material utilized [2, 7]. However, they are also more fragile and experience larger deflection which can manifest as tool run-out and chatter marks on the workpiece. Furthermore, catastrophic tool failure may occur as a result of chip clogging, failure by fatigue or failure caused by tool wear [3, 6]. Chip clogging is a result of poor chip evacuation process, and causes rapidly increase in cutting force and stress which lead to tool breakage. This mechanism is very unpredictable and happens extremely rapidly [37]. Failure by fatigue may occur as a result of tool deflection and high spindle speeds employed. Eventually, tool wear causes increase in cutting edge radius and burr formation leading to elevation of the cutting forces to levels high enough to cause failure of the tool shaft [3]. Hence, otherwise then visual inspection of the tool, tool condition could be predicted during machining based on monitoring of cutting force [6], burr formation [3] or acoustic emission [8]. Still, there is a lot of space for further work regarding this subject. Additionally, tool failure may occur as a consequence of cracks and impurities formed during manufacturing process and covered by the coating [18].

TOOL DESIGN

Under micromachining, micro tools experience a different loading situation from that seen in conventional machining. To reduce tool bending and deflection, avoid the chatter marks on the workpiece and ensure stable cutting process, conventional tool design had to be reconsidered. Uhlmann et al. [38] proposed a new parametric tool design for micro end mills considering dynamic load and strain analysis trough FEM analysis. The adapted tool design has a reduced fluted length to increase the tool shaft cross section and stiffness, rounded edge at the intersection of the constant tool shaft diameter and the conical part (Figure 9), where the bending moment is maximal, to prevent crack initiation and tapered shape with a reduced diameter at the tool peripheral edge (Figure 20) to avoid any contact with workpiece and to eliminate chatter marks on workpiece, which are result of tool deflection during machining process [6, 18, 38-40].

MACHINE TOOLS WITH MICROMACHINING CAPABILITY

The requirements of micro component manufacture over a range of applications are: high dimensional precision, typically better than 1 micron; accurate geometrical form, typically better than 50 nm departure from flatness or roundness; and good surface finish, in the range of 10 nm – 50 nm [41]. To accomplish those demands, the following characteristics are required for the machine tools: high static and dynamic stiffness, high thermal stability of

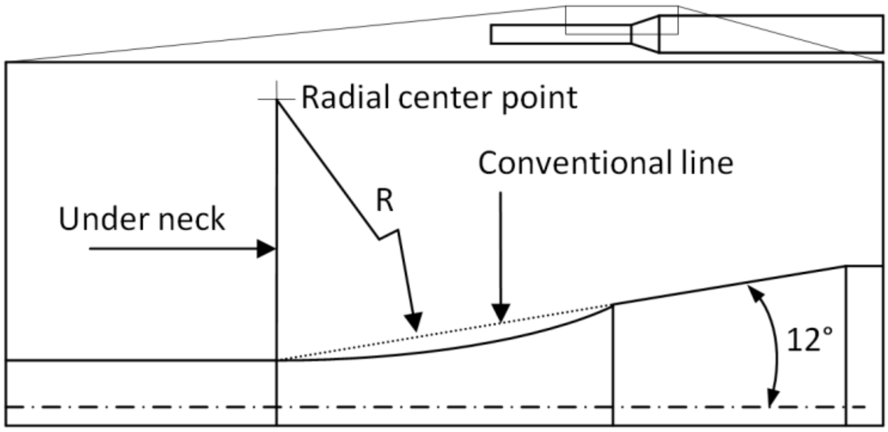

Figure 19. Cutting tool with rounded edge at the intersection of the constant tool shaft diameter and the conical part (adapted from [39]).

the frame materials, feed drives and control systems with high accuracy and short response time associated with large bandwidth and low following error for multi-axes interpolation, minimization and/or compensation of thermal effects and minimization and compensation of static and dynamic positioning errors [3]. Most of the experimental research for micromachining has been conducted on ultra-precision machine tools and machine centres or on miniaturized machine tools and micro factories built by researchers.

ULTRA-PRECISION MACHINE TOOLS AND MICROMACHINE CENTRES

Over the last two decades, knowledge has been accumulated for design of ultraprecision machine tools for micromachining, resulting in tough requirements such as thermal stability, precise spindle bearings and linear guides and high resolution of linear and rotary motions [6]. Currently available multi-axis controlled ultraprecision machining centres are based on conventional ultra-precision machines, operated under a temperature controlled environment [2, 6]. They are used to produce small workpieces with complex geometries and microscale patterns and texture, such as moulds and dies for CD pickup lenses, contact lenses, Fresnel lenses, etc.

Machine materials

The stability and damping behaviour of the machine are important to avoid vibrations and chatter marks on the work piece surface as well as additional stress of the micro tool due to vibrations. Thermal and damping properties are mostly determined with materials used to produce machine components, such as the machine base, column, worktable, slide, spindle cases and carriages. A constant room temperature within 1 K and absence of direct solar irradiation are advised [18].

Cast iron and granite have been widely used for fabricating machine bases and slideways [7]. Recently, as a cheaper replacement for granite, polymer concrete has become popular for ultra-precision machine tools where light weight with high damping capacity (much better than cast iron) and rigidity is required. Structural materials with a low thermal expansion coefficient and high dimensional stability have also found its application, including super-invar, synthetic granite, ceramics and Zerodur [6, 7, 18].

According to Gietzelt et al. [18], the shape and fixing position of the clamping to the machine also have a high impact on thermal drift due to the high thermal coefficient of expansion. For this reason, Invar and granite are most commonly used as a clamping material because of low thermal shift.

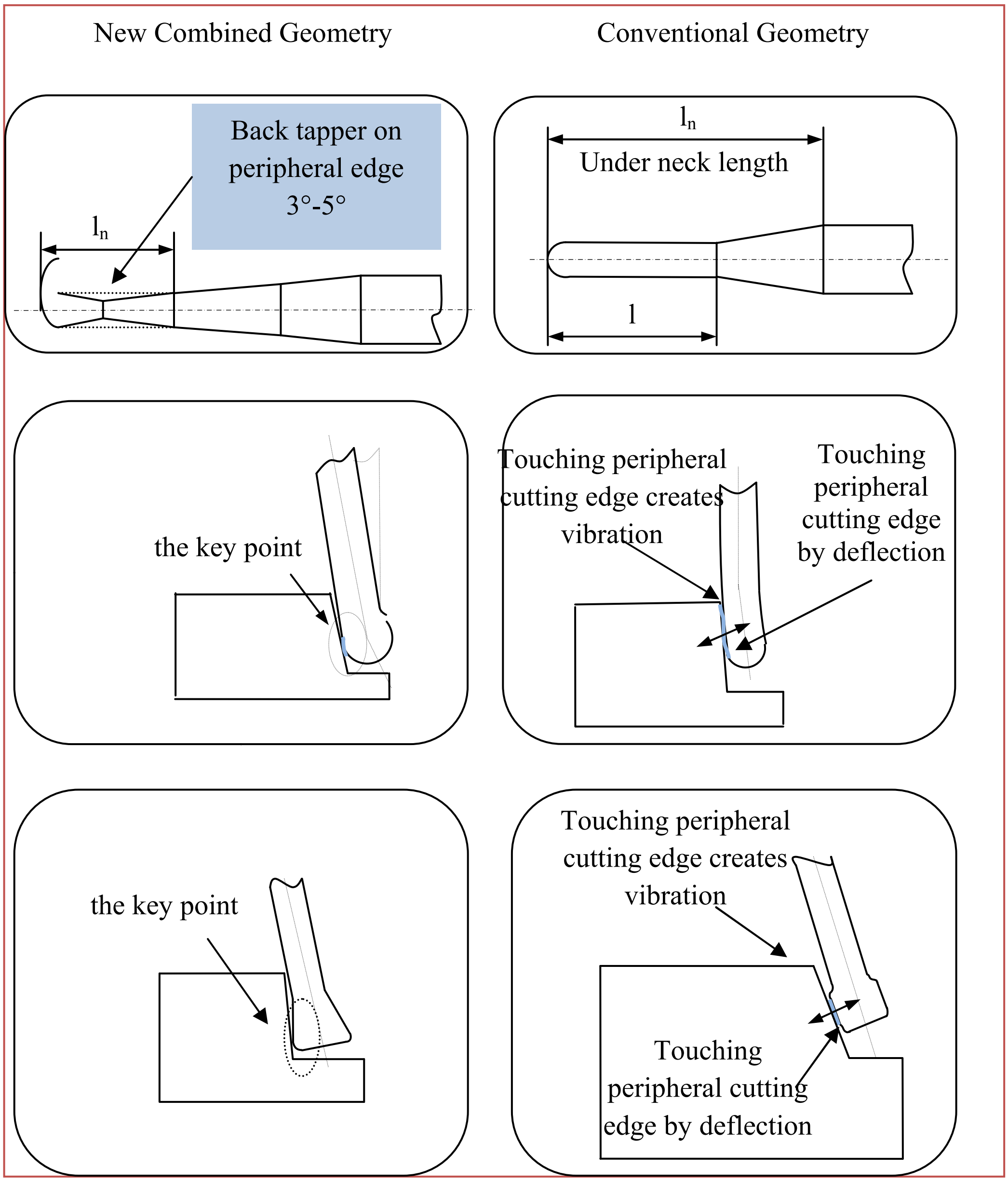

Figure 17. Cutting tool with tapered shape and reduced diameter at the tool peripheral edge (adapted from [40]).

Spindle bearings and linear guides

To maintain acceptable productivity, micromachining requires very high speed spindle speeds due to small tool diameters and thus the dynamic characteristics of the spindle dominate machining quality. Most conventional precision machine tools are equipped with bearings and guides based on direct mechanical contact, such as ball or needle roller bearings or guides [6]. These machines are capable of producing micromachining features, but cannot achieve optical surface quality. Nowadays, aerostatic and hydrostatic bearings or guides are most commonly used [3, 6, 7, 41, 42]. Due to absent of direct mechanical contact, they introduce very little or no friction and are capable of high rotational speed with high motion accuracy. Aerostatic bearing are normally better than other bearings [41], and widely used for spindles in machine tools with medium and small loading capacity. They usually have lower stiffness than oil hydrostatic bearing spindles, but they have lower thermal deformation and their stiffness can be increased by using magnets as a preload [3, 7] or with squeezed oil film dampers [41]. Hydrostatic bearing spindles are more suitable for large and heavily loaded machine tools and where very good damping properties are required.

Often to achieve higher speeds, ultra-precision machine tools are retrofitted with high-speed spindles that fit in the conventional tool holder interfaces [2] and mostly, three jaw chucks are used [18]. In that case a number of interfaces from tool to the spindle are adding up and a small deviations in the spindle may cause large run-out and result with the poor stiffness of micro-tools. Precision measurement of true running accuracy is needed to ensure constant engagement of the normally two cutting edges of a micro end mill. For minimization of the run-out it is favourable to use vector controlled spindles to ensure the same orientation of the chuck inside the spindle [18]. Run-out deviation for the main spindle should be inferior to 1 μm [3].

High resolution of linear and rotary motions

Linear direct drive motors and piezoelectric actuators are commonly used in ultra-precision machine tools [2, 3, 6, 7, 41, 42]. Compared to conventional drive mechanisms operated by friction drives, linear direct drive motors and piezoelectric actuators have no accumulative errors from friction and the motor-coupling, no loss of accuracy due to wear, and no backlash [2].

Friction drives have a long stroke and usually consist of a driving wheel, a flat or round bar and a supporting back-up roller. They offer low friction force, smooth motion, and good repeatability and reproducibility due to elastic deformation induced by preload [7].

Linear-motor direct drives (AC or DC), usually also have a long stroke and they offer better stiffness, acceleration, speed, motion smoothness, repeatability and accuracy. [43].

Piezoelectric actuators usually have a short stroke with high motion accuracy and wide response bandwidth. They have been employed in fine tool positioning so as to achieve high precision control of the cutting tool (e.g. a diamond cutting tool) [7].

A 5-axis ultraprecision micromachine centre, using aerostatic bearings and driven by linear direct drive motor, can achieve spindle rotation speed of $2{\cdot}10^5$ RPM [3, 41] with rotational resolution of 10^{-5} degrees, and the axes responsible for feed and depth of cut can achieve translational resolution of 1 nm and slideway straightness of about 10 nm/200mm [6].

Computer Numerical Control (CNC)

A numerical control is necessary to achieve smooth tool movements without changes in the feed rate, responsible for high accuracies of micro-structures. Following the invention of Computer Numerical Control (CNC) in the early 1970s, many companies started to develop their control systems for machine tools. The control system typically includes motors, amplifiers, switches and the controller. High speed multi-axis CNC controllers play an essential role in efficient and precision control of servo drives, error compensation (thermal and geometrical errors), optimized tool setting and direct entry of the equation of shapes [7, 41].

The NC unit of the machine must be able to process sufficient numbers of instructions per second. The dynamic behaviour, namely the acceleration of the axes, the velocity to the NC-control unit and the maximum number of instructions per seconds are important to maintain a programmed feed rate. In this context, also the definition of how accurately the machine has to meet the calculated tool path is important. If the tolerance is very low, the servo-loop can cause an extreme breakdown of the feed rate. This leads to squeezing of the cutting edges, increased tool wear or even tool rupture. In the last decade, the acceleration could be improved from about 1,2 m/s to more than 20 m/s (2G) by using hydrostatic drives [18].

Advanced PC-based control systems are commonly being used in the majority of commercially available ultra-precision machines as they can achieve nanometre or even sub-nanometre levels of control resolution for ultra-precision and micro-manufacturing purposes [7].

Position measurement and process monitoring

A major advantage of micromachining is its ability to fabricate increasingly smaller features reliably at very high tolerances. Sensor-based monitoring yields valuable information about the micromachining process that can serve the dual purpose of process control and quality monitoring, however, a high degree of confidence and reliability in characterizing the manufacturing process is required for any sensor to be utilized as a monitoring tool [6]. Figure 18 illustrates several different classes of sensors and their applicability to level of precision and type of control parameter.

Because of high resolution related to interferometers and ability to eliminate Abbe errors, laser encoders are suitable for ultra-precision position measurement [6, 7, 42]. They have a typical resolution of 20 nm, while some laser holographic-linear scales can achieve resolution of better than 10 nm [7]. Another alternative are high resolution optical encoders which can provide resolution close to that of laser encoders, but in a more industrially feasible and simple manner [7, 41].

Process monitoring systems can be used to characterize, control, and improve micromachining process. Monitoring may be applied to parameters or variables such as temperature, cutting force, chatter, vibration, etc. Compared with the conventional machining processes, micromachining processes are usually difficult to monitor because of the associated very small energy emissions and cutting forces [7]. Furthermore, some control parameters, such as tool wear, tool breakage, tool engagement, material anisotropy, subsurface damage, etc., often cannot be directly measured or evaluated. Hence, process monitoring through acoustic emission, force and vibration signals draw a great deal of attention.

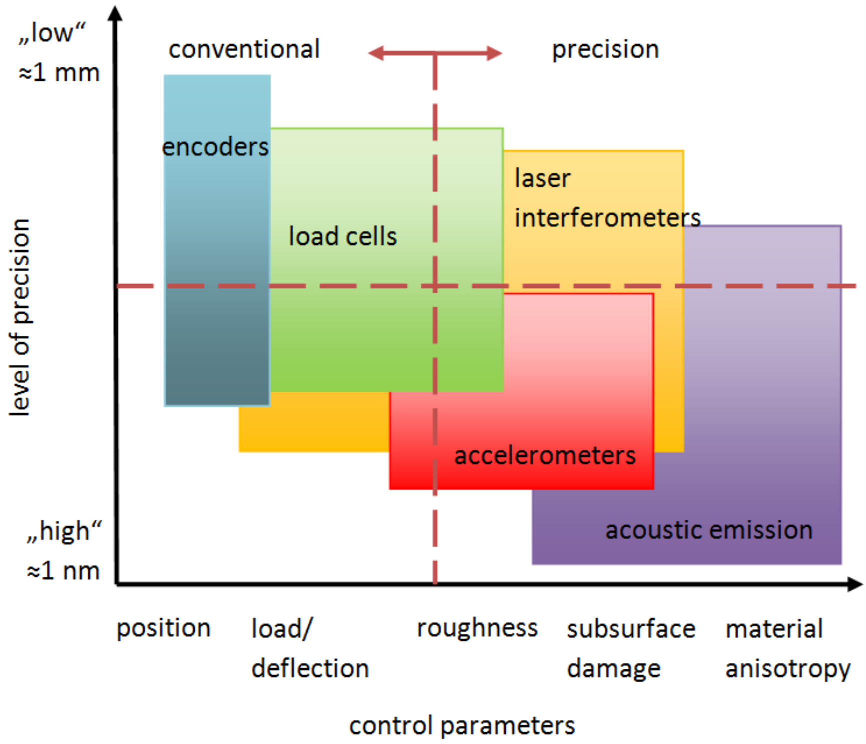

Figure 18. Sensor application versus level of precision and control parameters (adapted from [6]).

While process monitoring through acoustic emission is the most appropriate to characterize micromachining process in the nanometre range [8], force signals can also be successfully engaged [2, 3]. However, it is desirable to use multiple sensors to realize the smart and intelligent machine tool [7]. Process monitoring techniques are still subject of many research papers.

MINIATURIZED MACHINE TOOLS AND MICRO FACTORIES

In general, micromachining is performed on precision and ultra-precision machine tools with conventional dimensions [6]. Precision and ultra-precision machine tools have several advantages including high rigidity, damping and the ability to actuate precisely based on precision sensors and actuators. However, the large scale and precisely controlled machining environment may add very high costs for the fabrication of miniature components [2, 6]. Therefore, there has been strong interest by various research groups [44-47] for building miniaturized machine tools and micro factories capable to produce micro-size components and features. Micro factories are composed of different cells with different functionalities such as micro milling, micro drilling, micro press, etc. The advantage of such miniaturized machine tools and micro factories lies in increased flexibility, portability and economic benefits such as structural cost savings, shop floor space savings, energy reduction and performance benefits including reduction of thermal deformation, enhancement of static rigidity and dynamic stability as well [6]. Economic benefits also provide the ability to use more expensive construction materials that exhibit better engineering properties, while increased portability allows their deployment to any building or any location. For example, micro-factories may be suitable for the production of micro-components during military or space exploration applications, since the accessibility of large machine tools is very difficult [2]. One unique effort is to build a micro factory system where one or several minimized machine tools are small enough to be placed on the desktop.

As actuators either piezoelectric or linear direct drive (voice coil) actuators are used, in order to achieve sub-micrometre accuracies. They use high-speed air bearing spindles, as used in the majority of ultra-precision machines. However, there are challenges associated with the development of micro-machine tools. They require accurate sensors and actuators, which must be small enough to implant within the machines. The structural rigidity of micro-machine tools is less than those of precision machines. In addition, the micro machine tools can be excited by external disturbances; therefore, micro-factories require vibration isolation to achieve desired tolerances [2].

Majority of micro factory systems are still at the research stage, and only a few of them have so far found their way into industrial applications, but their application to high accuracy and fine surface quality are still constrained by low static/dynamic stiffness [2, 41].

CONCLUSION

The aim of this review article is to summarize existing knowledge and highlight current challenges, restrictions and advantages in the field of micro manufacturing. Although natural curiosity and industry demands are responsible for active research in this field for some time, particular issues and challenges still exists. Additional research motivation lies in bridging the knowledge gap between materials at the macro and micro scale.

The macro and micro machining processes share the same material removal principle and there are many similar issues between them, such as regenerative chatter, tool wear, monitoring strategies, etc. However, owning to the inevitable size effects, the direct knowledge transfer to the micro domain by pure scaling is not possible and many assumptions which are taken for granted in macro domain are not valid in micro domain. Hence, further research is required in order to fully understand micromachining process mechanics which is primarily influenced by grain size and different grain properties in case of multiphase materials and requires extensive research in chip removal processes and material properties.

Substantial advance in micromachining field can be evident from development of cutting tools and machining tools. Tungsten carbide material with micro grain size allows production of cutting tools with smaller cutting edge radius which enables the lower values of uncut chip thickness. Furthermore, redesigned tool geometry offer higher tool stiffness, and improved tool coatings (uniform and thin) provide tool wear reduction and longer tool life. However, micromachining of brittle and very ductile materials is still a challenge regarding reasonable surface quality. While ductile materials introduce bigger burr size, brittle materials cause low material removal rate and high tool wear. Burr formation is the most critical aspect regarding quality of the machined product and it is influenced by the material properties and the machining parameters and strategies. In order to assure more consistent tool life cutting forces encountered during micromachining can be reduced by employing novel approach such as laser or vibration assisted micromachining.

Although conventional machine centres are capable for micromachining processes, full advantage of micromachining benefits can be accomplished by employing machine tools specially designed for this purpose. Furthermore, from the last decade there exists a strong interest in building miniaturized machine tools and micro factories with micromachining capability. The advantages of such miniaturized machine tools and micro factories are flexibility, mobility and various economic. Regarding process monitoring techniques, acoustic emission stands out among the force and vibration signals monitoring. Current researches regarding acoustic emission are oriented at improving the prediction of tool failure, surface finish and burr formation.

Although mechanical micromachining processes still demands various improvement, mostly regarding higher material removal rates and selection of process parameters in order to achieve stable cutting process, compared with other microfabrication techniques (i.e. MEMS) its benefits lies in low cost production, small batch sizes and capability to produce accurate 3D free-form surfaces in a variety of metallic alloys, composites, polymers and ceramic materials.

This investigation of the existing knowledge in the field of micromachining surely leads to question of the possible developing directions. The trace need to be searched in mentioned challenges that are still waiting to be coped with. Nevertheless those challenges demand a foregoing development of necessary infrastructure in the form of advanced gauging which can result in better following of process parameters and in some new knowledge of their improvement.

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MIKROOBLIKOVANJE ODVAJANJEM ČESTICA

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SAŽETAK

Trend minijaturizacije proizvoda i njegovih komponenti danas je vidljiv u skoro svim proizvodnim granama. Mikrooblikovanje odvajanjem čestica pokazalo se kao proces koji daje zadovoljavajuće rezultate na tome polju, unatoč zahtjevima koje ono nameće. U ovome radu pod pojmom mikrooblikovanje odvajanjem čestica podrazumijevat će se proces u mikro području u kojem se materijal odstranjuje pomoću alata s geometrijski definiranom oštricom. Cilj rada je sumirati postojeće znanje, te istaknuti aktualne izazove, ograničenja i prednosti postupka mikrooblikovanja odvajanjem čestica.

KLJUČNE RIJEČI

minijaturizacija, non-MEMS, mikrooblikovanje odvajanjem čestica, efekti veličine

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A STUDY OF THE ROLE OF GOVERNMENT IN INCOME AND WEALTH DISTRIBUTION BY INTEGRATING THE WALRASIAN GENERAL EQUILIBRIUM AND NEOCLASSICAL GROWTH THEORIES

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ABSTRACT

This paper proposes a growth model of heterogeneous households with economic structure, wealth accumulation, endogenous labour supply, and tax rates. The paper is focused on effects of redistribution policies on income and wealth distribution, economic structure and economic growth. The paper integrates the Walrasian general equilibrium theory and neoclassical economic growth within a comprehensive framework. We overcome the controversial features in the two traditional theories by applying an alternative approach to households. We build an analytical framework for a disaggregated and microfounded general theory of economic growth with endogenous wealth accumulation. We simulate the model to identify equilibrium, stability and to plot the motion of the dynamic system with three groups. We also carry out comparative dynamic analysis with regard to the lump tax, human capital and propensity to use leisure time.

KEY WORDS

lump tax, tax rates, Walrasian general equilibrium theory, neoclassical growth theory, income and wealth distribution

CLASSIFICATION

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INTRODUCTION

It is well known that Walras' theory of pure exchange and production economies have provided the underpinning of contemporary general equilibrium theory. The Walrasian general equilibrium theory was initially proposed by Walras and in the 1950s further formalized by Arrow, Debreu and others [1-9]. According to Arrow [10], "From the time of Adam Smith's Wealth of Nations in 1776, one recurrent theme of economic analysis has been the remarkable degree of coherence among the vast numbers of individuals and seemingly separate decisions about the buying and selling of commodities. In everyday, normal experience, there is something of a balance between the amounts of goods and services that some individuals want to supply and the amount that other different individuals want to sell. Would be buyers ordinarily count correctly on being able to carry out their intensions, and would-be sellers do not ordinarily find themselves producing great amounts of goods that they cannot sell. This experience of balance is indeed so widespread that it raises no intellectual disquiet among laymen; they take it so much for granted that they are not disposed to understand the mechanism by which it occurs." The general equilibrium theory is important to understand economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. Nevertheless, this theory has not been successfully generalized and extended to growth theory of heterogeneous households with endogenous wealth. The purpose of this study is to introduce economic mechanisms of endogenous wealth accumulation with redistribution policy.

Walras introduced saving and capital accumulation in his general equilibrium theory. Nevertheless, his treatments of capital accumulation have many shortcomings, particularly, in the light of modern neoclassical growth model. As pointed out by Impicciatore et al. [11], "because of the absence of an explicit temporal indexation of the variables, the timeframe of Walras' theory is left to the reader's interpretation. In particular, it remains an open question whether the model is static (that is, a single-period model) or dynamic, and, in the latter case, if it pertains to the short run or long run." In fact, there is no profound microeconomic mechanism for wealth accumulation in Walras' original theory. Over years there are different attempts to further develop Walras' capital accumulation within Walras' framework of heterogeneous households (e.g., [12-17]). The common problem for these approaches is the lack of proper microeconomic foundation for wealth accumulation. To overcome this problem, Impicciatore et al. [11] propose a model in which it is assumed that consumers store capital goods in order to supply their services to the production sector in the next period under the condition that capital goods exiting in one period totally depreciate at the end of the period. The approach still relies on the strict assumption on household saving behaviour. This study introduces an alternative approach for modeling wealth accumulation with the traditional Walrasian general equilibrium framework of heterogeneous households.

There are some other studies in the literature of economic growth which introduce neoclassical growth theory into the general equilibrium analysis (e.g., [18]). As reviewed by Shoven and Whalley [19], "Most contemporary applied general models are numerical analogs of traditional two-sector general equilibrium models popularized by James Meade, Harry Johnson, Arnold Harberger, and others in the 1950s and 1960s. Earlier analytical work with these models has examined the distortionary effects of taxes, tariffs, and other policies, along with functional incidence questions." The history of analytical economics shows that it is not easy to properly model economic growth with wealth and income distribution. In fact, there are only a few formal dynamic models which explicitly deal with distribution issues among heterogeneous households in the neoclassical growth theory [22]. On the other hand, the

Arrow-Debreu general economic theory deals with economic equilibrium issues with heterogeneous households and firms. It is desirable to integrate the economic mechanisms of the two main approaches in economics into a single analytical framework. This study builds a model of integrating the two theories with an alternative approach to households behaviour by Zhang [23]. We develop a model to deal with interdependence between wealth and income distribution among heterogeneous households within the Uzawa two-sector growth modeling framework. This study synthesizes the ideas in the two-sector model with endogenous labour by Zhang [24] and the growth model with heterogeneous groups by Zhang [25]. In Zhang's two papers, no government distribution is introduced. This study introduces lump taxes (and subsidies) and taxes on production, consumption, wealth income and wages. The paper is organized as follows. Section 2 introduces the basic model with wealth and income distribution with distribution policy. Section 3 examines dynamic properties of the model and simulates the model with three types of households. Section 4 carries out comparative dynamic analysis with regard to redistribution policies, propensities to save, and propensities to use leisure time. Section 5 concludes the study.

THE BASIC MODEL

The economy consists of two sectors, like in the two-sector model by Uzawa [26]. Most aspects of the production sectors are neoclassical [21, 22, 27]. Different from the Solow one-sector growth model, the Uzawa two-sector growth model treats consumption and capital goods as different commodities, which are produced in two distinct sectors. The population is constant and homogeneous. There is only one malleable capital good. In the Uzawa model, capital goods can be used as an input in both sectors in the economy. Capital depreciates at a constant exponential rate δ_k, which is independent of the manner of use. Households own assets of the economy and distribute their incomes to consume and save. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. Each group has a fixed population, $\bar{N}_j$, (j = 1, ..., J). It should be noted that in the Walrasian general equilibrium theory, $\bar{N}_j = 1$. Let prices be measured in terms of capital goods and the price of the commodity be unity. We denote the wage rate of worker of type j and rate of interest by $w_j(t)$ and $r(t)$, respectively.

The total capital stock $K(t)$ is allocated between the two sectors. We use subscript index i and s to stand for capital goods and consumer goods sector, respectively. We use $N_j(t)$ and $K_j(t)$ to stand for the labor force and capital stocks employed by sector j. We use $T_j(t)$ and $\bar{T}_j(t)$ to stand for, respectively, the work time and leisure time of a typical worker in group j The total qualified labor supply $N(t)$ is defined by

$$N(t) = \sum_{j=1}^{J} h_j T_j(t) \bar{N}_j . \tag{1}$$

We introduce

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad n_j(t) \equiv \frac{N_j(t)}{N(t)}, \quad k(t) \equiv \frac{K(t)}{N(t)}, \quad j = i, s.$$

The assumption of labour force being fully employed implies

$$N_i(t) + N_s(t) = N(t). \tag{2}$$

THE CAPITAL GOODS SECTOR

It is well known that in modern literature of economic growth the Cobb-Douglas production function has been widely applied to different issues (see for instance [28-30]). We assume that production is to combine the labor force $N_i(t)$ and physical capital $K_i(t)$. We use τ_j to stand for the tax rate on sector $j's$ output, $j = i, s$. Let $\tau_i(t)$ represent the tax rate on the capital goods sector. The function $F_i(t)$ is specified as

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ A_i, \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1 \tag{3}$$

where A_i, α_i and β_i are parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rate are determined by markets. For any individual firm $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_k = \alpha_i \bar{\tau}_i(t) A_i K_i^{-\beta_i}(t) N_i^{\beta_i}(t), \ \ w_j(t) = \bar{\tau}_i(t) h_j \, w(t), \tag{4}$$

where

$$\bar{\tau}_i(t) \equiv 1 - \tau_i(t), \ \ w(t) \equiv \beta_i A_i K_i^{\alpha_i}(t) N_i^{-\alpha_i}(t).$$

CONSUMER GOODS SECTOR

We specify the production function of the consumer goods sector as follows

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \ \ A_s, \ \alpha_s, \beta_s > 0, \ \ \alpha_s + \beta_s = 1. \tag{5}$$

The marginal conditions are

$$r(t) + \delta_k = \alpha_s \bar{\tau}_s(t) p(t) A_s K_s^{-\beta_s}(t) N_s^{\beta_s}(t), \ \ w_j(t) = \beta_s \bar{\tau}_s(t) h_j \, p(t) A_s K_s^{\alpha_s}(t) N_s^{-\alpha_s}(t). \tag{6}$$

CONSUMER BEHAVIOURS AND WEALTH DYNAMICS

In this study, we use an alternative approach to modeling behaviour of households proposed by Zhang [23]. The preference over current and future consumption is reflected in the consumer's preference structure over leisure time, consumption and saving. Let $\bar{k}_j(t)$ stand for the per capita wealth of group j We have $\bar{k}_j(t) = \bar{K}_j(t)/\bar{N}_j$, where $\bar{K}_j(t)$ is the total wealth held by group j We use τ_j to stand for the lump sum transfer that group j's representative household receives from the government. Per capita current disposable income from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $T_j(t) w_j(t)$ is given by

$$y_j(t) = (1 - \tau_{rj}) r(t) \bar{k}_j(t) + (1 - \tau_{wj}) T_j(t) w_j(t) + \tau_j,$$

where τ_{rj} and τ_{wi} are respectively the tax rates on the income from wealth and on the wage income. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{7}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute among saving and consumption.

The representative household from group j would distribute the total available budget between savings $s_j(t)$ and consumption of goods $c_j(t)$. Let the tax rate on group $j's$ consumption be denoted by τ_{cj}. The budget constraint is given by

$$\left(1+\tau_{cj}\right)p(t)c_j(t)+s_j(t)=\hat{y}_j(t). \tag{8}$$

Denote $\overline{T}_j(t)$ the leisure time at time t and the (fixed) available time for work and leisure by T_0. The time constraint is expressed by

$$T_j(t)+\overline{T}_j(t)=T_0. \tag{9}$$

Substituting (11) into (10) implies

$$\left(1-\tau_{wj}\right)w_j(t)\overline{T}_j(t)+\left(1+\tau_{cj}\right)p(t)c_j(t)+s_j(t)=\bar{y}_j(t), \tag{10}$$

where

$$\bar{y}_j(t)\equiv\left(\left(1-\tau_{rj}\right)r(t)+1\right)\bar{k}_j(t)+\left(1-\tau_{wj}\right)T_0\,w_j(t)+\tau_j.$$

In this model, at each point of time, consumers have three variables to decide. We assume that utility level $U_j(t)$ that the consumers obtain is dependent on the leisure time, $T_j(t)$, the consumption level of consumption goods $c_j(t)$ and savings $s_j(t)$ as follows

$$U_j(t)=\overline{T}_j^{\sigma_{0j}}(t)c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}(t),\ \ \sigma_{0j},\,\xi_{0j},\,\lambda_{0j}>0,$$

where σ_{0j} is the propensity to use leisure time, ξ_{0j} is the propensity to consume consumption goods, and λ_{0j} propensity to own wealth. Some growth models with endogenous wealth accumulation consider heterogeneous households. Nevertheless, the heterogeneity in these studies is by the differences in the initial endowments of wealth among different types of households rather than in preferences (see for instance [31-35]). Different households are essentially homogeneous in the sense that all the households have the same preference utility function in the approach. In our approach we consider different types of households have different utility functions.

Maximizing the utility subject to (10) yields

$$w_j(t)\overline{T}_j(t)=\sigma_j\,\bar{y}_j(t),\ \ p(t)c_j(t)=\xi_j\,\bar{y}_j(t),\ \ s_j(t)=\lambda_j\,\bar{y}_j(t), \tag{11}$$

where

$$\sigma_j\equiv\frac{\rho_j\,\sigma_{0j}}{1-\tau_{wj}},\ \ \xi_j\equiv\frac{\rho_j\,\xi_{0j}}{1+\tau_{cj}},\ \ \lambda_j\equiv\rho_j\,\lambda_{0j},\ \ \rho_j\equiv\frac{1}{\sigma_{0j}+\xi_{0j}+\lambda_{0j}}.$$

We now find dynamics of capital accumulation. According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t)=s_j(t)-\bar{k}_j(t)=\lambda_j\,\bar{y}_j(t)-\bar{k}_j(t). \tag{12}$$

This equation simply states that the change in wealth is equal to the saving minus dissaving.

DEMAND AND SUPPLY

The output of the consumer goods sector is consumed by the households. That is

$$\sum_{j=1}^{J}c_j(t)\overline{N}_j=F_s(t) \tag{13}$$

As output of the capital goods sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{14}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t)\bar{N}_j, \quad K(t) = \sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j.$$

CAPITAL BEING FULLY UTILIZED

Total capital stock $K(t)$ is allocated to the two sectors and households. As full employment of labor and capital is assumed, we have

$$K_i(t) + K_s(t) = K(t). \tag{15}$$

THE GOVERNMENT'S BUDGET

The government spends all the tax income on redistribution. We have

$$\tau_i(t)F_i(t) + \tau_s(t)p(t)F_s(t) + \Gamma_r(t) + \Gamma_w(t) + \Gamma_c(t) = \sum_{j=1}^{J} \tau_j \bar{N}_j, \tag{16}$$

where

$$\Gamma_r(t) \equiv \sum_{j=1}^{J} \tau_{rj}\, r(t)\bar{k}_j(t)\bar{N}_j, \quad \Gamma_w(t) \equiv \sum_{j=1}^{J} \tau_{wj}\, T_j(t)w_j(t)\bar{N}_j, \quad \Gamma_c(t) \equiv \sum_{j=1}^{J} \tau_{cj}\, p(t)c_j(t)\bar{N}_j.$$

THE BEHAVIOUR OF THE GOVERNMENT

The government chooses the following tax and subsidy rates $\tau_i(t)$, $\tau_s(t)$, $\tau_{rj}(t)$, $\tau_{wj}(t)$, $\tau_{cj}(t)$, $\tau_j(t)$, $j = 1, ..., J$. There is only one budget constraint. For simplicity of discussion, we assume that the tax rates on the two sectors are interrelated as follows

$$\tau_s(t) = \tau_0\, \tau_i(t), \tag{17}$$

where τ_0 is a constant. The two sectors' tax rates are proportional. We further assume that the tax rate on the capital sector is determined by (16). From (16) and (17)we have

$$\tau_i(t) = \frac{\Gamma(t)}{F_i(t) + \tau_0\, p(t)F_s(t)}, \tag{18}$$

where

$$\Gamma(t) \equiv \sum_{j=1}^{J} \tau_j \bar{N}_j - \Gamma_r(t) - \Gamma_w(t) - \Gamma_c(t).$$

We complete the model. As far as economic structure and growth theory with endogenous capital are concerned, our model is general in the sense that the model is built on the basis of economic mechanisms of the Walras-Arrow-Debreu general economic theory, the Solow growth model and the Uzawa two sector model. For instance, if the economic system has only two sectors, then the Arrow-Debreu equilibrium theory (which treats capital exogenous) can be considered as a special case of our model with heterogeneous households with endogenous leisure time and wealth. It is straightforward to see that the Solow-one sector and the Uzawa two sector model are special cases of our model. As our model also includes labor supply and tax policies, it is closely related with some other growth models in the literature of, for instance, public economics. We now examine behaviour of the economic system.

THE DYNAMICS AND ITS PROPERTIES

The dynamic system consists of any (finite) number of households. As behavioural patterns vary among different types, the dynamic system is of high dimension. The following lemma shows that the dimension of the dynamical system is equal to the number of types of households. We also provide a computational procedure for calculating all the variables at any point of time. Before stating the lemma, we introduce a new variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w_j(t)/h_j}.$$

LEMMA

The motion of the economic system is determined by J differential equations with $z(t)$, $\tau_i(t)$ and $\{\bar{k}_j(t)\}$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_3(t), \cdots, \bar{k}_J(t))$, as the variables

$$\dot{z}(t) = \Lambda_1(z(t), \tau_i(t), \{\bar{k}_j(t)\}),$$
$$\dot{\tau}_i(t) = \Lambda_2(z(t), \tau_i(t), \{\bar{k}_j(t)\}),$$
$$\dot{\bar{k}}_j(t) = \Lambda_j(z(t), \tau_i(t), \{\bar{k}_j(t)\}), \quad j = 3, \ldots, J, \tag{19}$$

in which $\Lambda_j(t)$ are unique functions of $z(t)$, $\tau_i(t)$, and $\{\bar{k}_j(t)\}$ defined in the appendix. At any point of time the other variables are unique functions of $z(t)$, $\tau_i(t)$, and $\{\bar{k}_j(t)\}$ determined by the following procedure: $\bar{k}_1(t)$ and $\bar{k}_2(t)$ by (A21) $\rightarrow$ $r(t)$ and $w_j(t)$ by (A3) $\rightarrow$ $\bar{y}_j(t)$ by (A4) $\rightarrow$ $N(t)$ by (A13) $\rightarrow$ $K_i(t)$ and $K_s(t)$ by (A15) $\rightarrow$ $N_i(t)$ and $N_s(t)$ by (A1) $\rightarrow$ $F_i(t)$ by (3) $\rightarrow$ $F_s(t)$ by (5) $\rightarrow$ $p(t)$ by (A8) $\rightarrow$ $\bar{T}(t)_j$, $c_j(t)$, and $s_j(t)$ by (11) $\rightarrow$ $T_j(t) = T_0(t) - \bar{T}_j(t)$ $\rightarrow$ $K(t) = K_i(t) + K_s(t)$.

The lemma gives a computational procedure for plotting the motion of the economic system with any number of types of households. It is well known that calibration of general equilibrium involves solving high-dimensional nonlinear equations. With regard to the Arrow-Debreu concept of general equilibrium the final stage of analysis is to find a price vector at which excess demand is zero [36]. There are numerical approaches for calculating equilibria (e.g., [37-38]). We can apply these traditional methods to find how the prices and other variables are related to the variables in the differential equations. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model with the following parameters:

$$A_i = 1.3,\ A_s = 1,\ \alpha_i = 0.29,\ \alpha_s = 0.32,\ T_0 = 1,\ \tau_0 = 0.8,\ \delta_k = 0.05,$$
$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 50 \\ 300 \\ 200 \end{pmatrix}, \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0.6 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.12 \\ 0.16 \\ 0.18 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.75 \\ 0.7 \end{pmatrix}, \begin{pmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{30} \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.18 \\ 0.15 \end{pmatrix},$$
$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.03 \\ 0.15 \end{pmatrix}, \begin{pmatrix} \tau_{r1} \\ \tau_{r2} \\ \tau_{r3} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \tau_{w1} \\ \tau_{w2} \\ \tau_{w3} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \tau_{c1} \\ \tau_{c2} \\ \tau_{c3} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.01 \\ 0.01 \end{pmatrix}. \tag{20}$$

The population of group 2 is largest, while the population of group 3 is the next. The human capital level of group 1 is highest, while the human capital level of group 3 is lowest. The capital goods sector and consumer goods sector's total productivities are respectively 1,3 and 1.

We specify the values of the parameters, α_j in the Cobb-Douglas productions approximately equal to 0,3 (for instance [39, 40]). The depreciation rate of physical capital is specified at 0,05. Group 1 propensity to save is 0,78 and group 3 propensity to save is 0,7 The value of group 2 propensity is between the two groups. The tax rates on different groups are mild. The rich group pays lump tax, $\tau_1 = -0{,}01$. Groups 2 and 3 receive subsidies, respectively, $\tau_3 = 0{,}02$. We specify the initial conditions as follows

$$z(0) = 0.048, \quad \tau_i(0) = 0.047, \quad \bar{k}_3(0) = 2.3.$$

The motion of the variables is plotted in Figure 1. The output level of the capital goods sector is enhanced and the output level of the consumer goods sector is slightly lowered over time. The tax rates on the capital and consumer goods sectors fall slightly. The rate of interest rises slightly. The price of consumer goods and the wage rates of the three groups vary slightly. The total supply and labor force employed by the consumer goods sector are reduced, and the labor force employed by the capital goods sector is augmented slightly. The total capital and the capital input of the consumer goods sector are reduce slightly, and the capital input of the consumer goods sector is increased. The national output is lowered. Group 1 and group 3 wealth levels are increased, group 2 wealth is diminished. Group 1 and group 3 reduce work hours, and group 2 increases work hours. Group 1 and group 3 consumption levels are increased, group 2 consumption level is diminished. It should be noted that there are empirical studies which find negative relationships between wealth and labor supply (for instance [41-43]). In our model with the specified parameter values, the negative relationship is obvious for the groups.

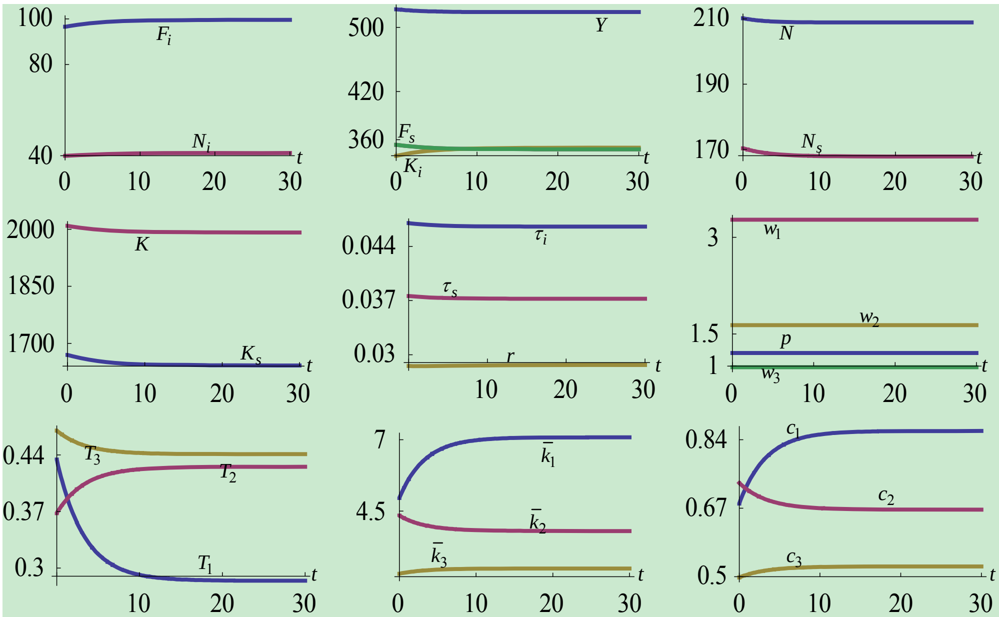

Figure 1. The motion of the economic system.

It is straightforward to confirm that the variables become stationary. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (21).

$$\tau_i = 0.047, \ \tau_s = 0.037, \ F_i = 99.6, \ F_s = 348.09, \ w_1 = 3.27, \ w_2 = 1.64, \ w_3 = 0.98,$$
$$r = 0.029, \ p = 1.21, \ N = 208.85, \ N_i = 41.20, \ N_s = 167.75, \ K = 1991.98, \ K_i = 349.98,$$
$$K_s = 1642, \ \bar{k}_1 = 7.09, \ \bar{k}_2 = 3.80, \ \bar{k}_3 = 2.49, \ T_1 = 0.29, \ T_2 = 0.43, \ T_3 = 0.44, \ c_1 = 0.86,$$
$$c_2 = 0.67, \ c_3 = 0.53. \tag{21}$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.30,\ -0.30,\ -0.24\}.$$

The eigenvalues are real and negative. The unique equilibrium is locally stable.

COMPARATIVE DYNAMIC ANALYSIS

We already simulated the motion of the national economy under (20). We are now concerned with how the economic system reactions to some exogenous change. As the lemma gives the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$ in percentage due to changes in the parameter value.

GROUP 1 PAYS MORE LUMP TAX

First, we examine the case that group 1 pays more lump tax to the government in the following way: τ_1: $-0{,}01 \rightarrow 0{,}015$. The simulation results are given in Figure 2. The immediate effects on group 1 are that the group reduces the consumption and wealth levels and works less hours, even though the change in the lump tax has little effects on these variables. Group 2 increases the consumption and wealth levels and works longer hours, even though these variables are slightly affected in the long term. Group 3 consumption and wealth levels and work hours are slightly affected. The tax rates on the capital and consumer goods are increased in association with the lessened lump tax on the rich group. The wage rates are slightly augmented. The rate of interest falls initially and rises in the long term. The total labor supply, total capital and national output are increased initially but become to the original stationary values in the long term. The economic structure shifts initially but maintains unaffected in the long term. The price is slightly affected. We see that the long-term effect of increasing group 1 lump tax is to reduce the tax rates on the production sectors and has almost no impact on the other variables (except the small change the rate of interest). It should be noted that from Figure 1 we see that group 1 consumption and wealth experience large changes during very short period, even though it does not take long for the variables to approach their stationary values. This character makes the group's wealth and consumption levels have strong reactions to exogenous changes as illustrated in Figure 2.

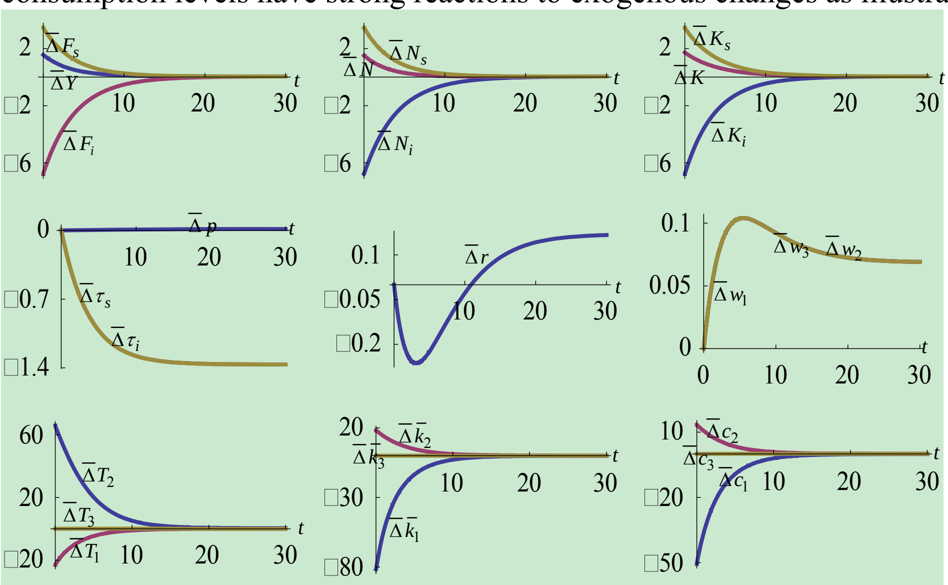

Figure 2. A rise in Group 1 lump tax.

GROUP 3 HUMAN CAPITAL BEING ENHANCED

Group 3 human capital is changed as follows: h_3: 0,6 $\rightarrow$ 0,7. We plot the simulation results in Figure 3. Group 3 wage rate is increased, while the other two groups' wage rates are only slightly affected. Group 3 work time is increased, as the opportunity cost of staying at home is increased. The other two groups' work hours are slightly reduced. As group 3 works more effectively, the national output, the output levels and two input factors of the two sectors are all increased. Hence, an improvement in the group's human capital enhances the national and sectorial economic performance. The tax rates on the two sectors are slightly increased. Group 3 wealth and consumption levels are increased. The other two groups' wealth and consumption levels are slightly affected in the long term. It should be noted that relations between wealth and income distribution and growth have caused attention of economists long time ago. For instance, Kaldor [44] argues that as income inequality is enlarged, growth should be encouraged as savings are promoted. This positive relation between income inequality and growth is also observed in studies, [45-47]. There are other studies which find negative relations between income inequality and economic growth. Solow [48] makes a hypothesis on a negative relationship between income inequality and growth. Some formal models which predicate negative relations are referred to, for instance [49-51]. Some empirical studies by, for instance, Persson and Tabellini [52] also confirm negative relations. From our simulation, we see that relations between inequality and economic growth are complicated in the sense that these relations are determined by many factors. For instance, as group increases the level of human capital, the income and wealth gaps between group 1 and group 3 are reduced in association with positive economic growth. On the other hand, if group 1 reduces the level of human capital, the income and wealth gaps between group 1 and group 3 are reduced in association with negative economic growth. It can be seen that different empirical studies expectably may give different answers.

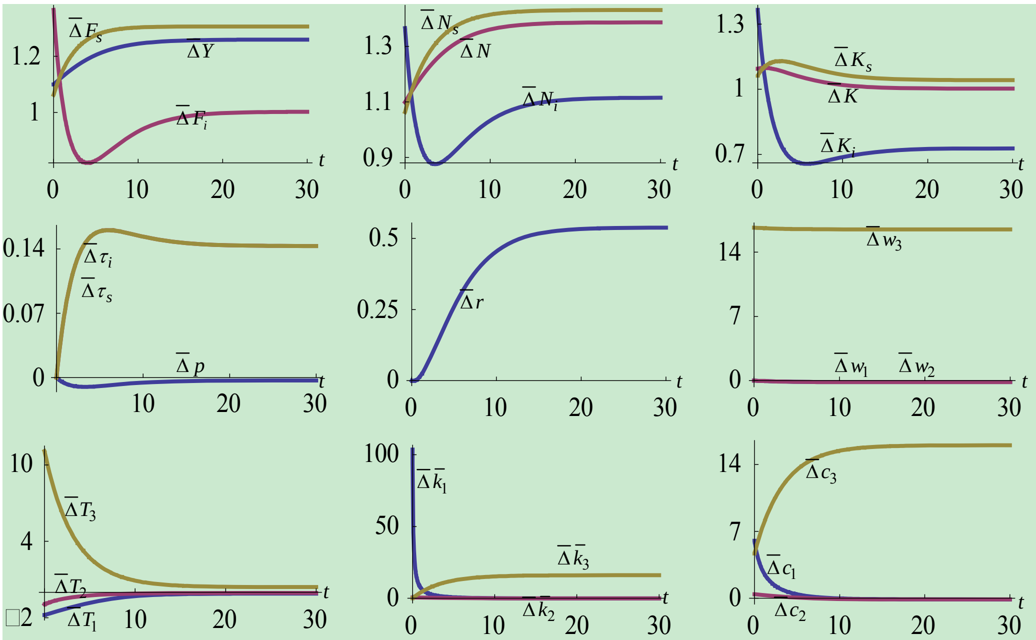

Figure 3. Group 3 human capital being enhanced.

GROUP 3 PROPENSITY TO USE LEISURE TIME BEING AUGMENTED

We now study what will happen to the economic system if group 3 propensity to use leisure time is increased as follows: σ_{03}: 0,15 $\rightarrow$ 0,18. The simulation results are plotted in Figure 4. As group 3 propensity to use leisure time is increased, the group's work hours are reduced. The households from the group stay at home longer. The total labor supply is reduced. The reduction in the total labor supply partly explains the rise in the wage rates of the three groups. Each of the two sectors employs less labor. The price of consumer goods is increased slightly. Group 3 wealth and consumption are reduced as the household stays longer at home. In the long term the other two groups' time distribution, and wealth and consumption levels. The tax rates on the two sectors are reduced. The total capital and capital stocks employed by each sector are reduced. The national output level and output levels of the two sectors are all reduced. The rate of interest rises initially and subsequently falls.

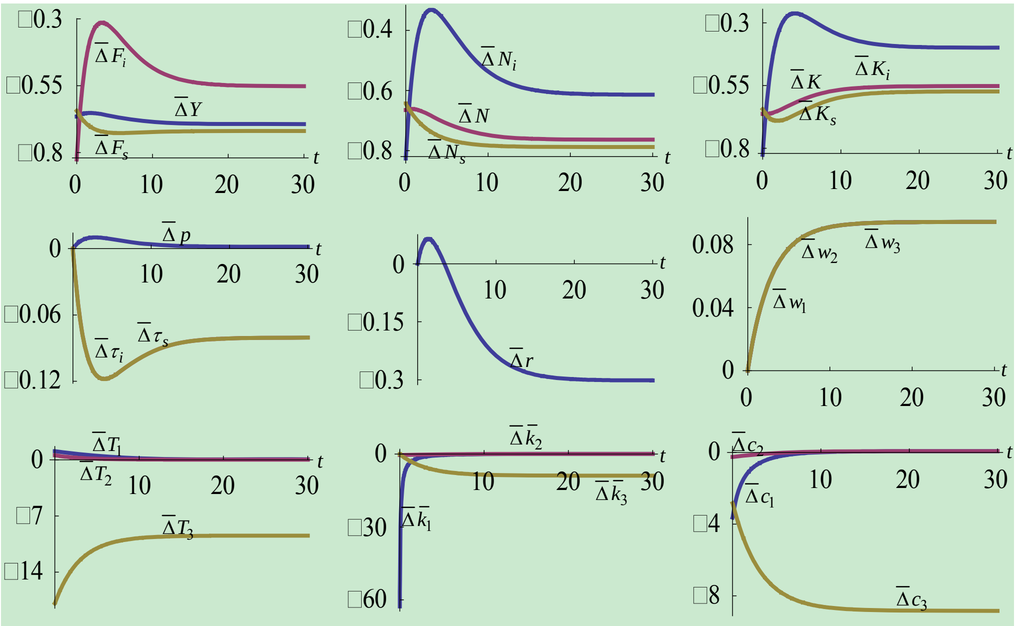

Figure 4. A rise in Group 3 propensity to save.

CONCLUDING REMARKS

This paper proposed a growth model of heterogeneous households with economic structure. The framework is influenced by the Walrasian general equilibrium and neoclassical growth theories. We were mainly concerned with the role of government in income and wealth distribution in an economy with endogenous wealth accumulation. The economic system consists of one capital goods sector, one consumer goods sector, and any number of households. Different from the traditional Uzawa model where the population is homogeneous, the population is classified into different groups. The model shows how wealth accumulation, income and wealth distribution, time distribution and division of labor interact under perfect competition and government intervention over time. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system with three groups. We identified the existence of a unique stable equilibrium point. We also carried out comparative dynamic analysis. We discussed implications of our simulation results for empirical studies in the literature of relations between work time and wealth and the literature of relations among wealth and income distribution and economic growth. Because our model is structurally general, it may be generalized and extended.

APPENDIX: PROVING THE LEMMA

By (4) and (6), we obtain

$$z \equiv \frac{r + \delta_k}{w_j / h_j} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s}, \tag{A1}$$

where $\overline{\beta}_j \equiv \beta_j / \alpha_j$. From (A1) and (2), we obtain

$$\overline{\beta}_i K_i + \overline{\beta}_s K_s = \frac{N}{z}. \tag{A2}$$

Insert (A1) in (4)

$$r = \alpha_r \overline{\tau}_i z^{\beta_i} - \delta_k, \quad w_j = \alpha_j \overline{\tau}_i z^{-\alpha_i}, \tag{A3}$$

where

$$\alpha_r = \alpha_i A_i \overline{\beta}_i^{\beta_i}, \quad \alpha_j = h_j \beta_i A_i \overline{\beta}_i^{-\alpha_i}.$$

Hence, we determine the rate of interest and the wage rates as functions of τ_i and z From (A3) and the definitions of $\overline{y}_j$, we have

$$\overline{y}_j = g_j \overline{k}_j + \overline{g}_j, \tag{A4}$$

where

$$g_j(z, \tau_i) \equiv \alpha_r \overline{\tau}_i \overline{\tau}_{rj} z^{\beta_i} + \delta_j, \quad \overline{g}_j(z, \tau_i) \equiv T_0 \alpha_j \overline{\tau}_{wj} \overline{\tau}_i z^{-\alpha_i} + \tau_j, \quad \delta_j \equiv 1 - \overline{\tau}_{rj} \delta_k,$$
$$\overline{\tau}_{rj} \equiv 1 - \tau_{rj}, \quad \overline{\tau}_{wj} \equiv 1 - \tau_{wj}.$$

Insert $p c_j = \xi_j \overline{y}_j$ in (13)

$$\sum_{j=1}^{J} \xi_j \overline{N}_j \overline{y}_j = p F_s. \tag{A5}$$

Substituting (A4) in (A5) yields

$$\sum_{j=1}^{J} \tilde{g}_j \overline{k}_j = p F_s - \tau, \tag{A6}$$

where

$$\tilde{g}_j(z, \tau_i) \equiv \xi_j \overline{N}_j g_j, \quad \tau(z, \tau_i) \equiv \sum_{j=1}^{J} \xi_j \overline{N}_j \overline{g}_j.$$

From (4) and (6), we solve

$$r + \delta_k = \alpha_i \overline{\tau}_i A_i K_i^{-\beta_i} N_i^{\beta_i} = \alpha_s \overline{\tau}_s p A_s K_s^{-\beta_s} N_s^{\beta_s}. \tag{A7}$$

Inserting (A1) in (A7), we have

$$p = \frac{\alpha_i A_i \overline{\beta}_i^{\beta_i} \overline{\tau}_i z^{\beta_i - \beta_s}}{\alpha_s A_s \overline{\tau}_s \overline{\beta}_s^{\beta_s}}. \tag{A8}$$

From (6), we have

$$p F_s = \frac{w_1 N_s}{h_1 \beta_s \overline{\tau}_s}. \tag{A9}$$

From (A9) and (A1), we have

$$p F_s = \frac{\overline{\beta}_s w_1 z K_s}{h_1 \beta_s \overline{\tau}_s}. \tag{A10}$$

Insert (A10) in (A6)

$$\sum_{j=1}^{J} \tilde{g}_j \overline{k}_j = g_0 K_s - \tau, \tag{A11}$$

where

$$g_0(z, \tau_i) \equiv \frac{\overline{\beta}_s w_1 z}{h_1 \beta_s \overline{\tau}_s}.$$

Using (1) and (9), we get

$$N = T_0 \sum_{j=1}^{J} h_j \bar{N}_j - \sum_{j=1}^{J} \frac{h_j \sigma_j \bar{y}_j \bar{N}_j}{w_j}, \tag{A12}$$

in which we also use $w_j \bar{T}_j = \sigma_j \bar{y}_j$. Substitute (A4) into (A12)

$$N = \tilde{\varphi}_0 - \sum_{j=1}^{J} \tilde{\varphi}_j \bar{k}_j, \tag{A13}$$

where

$$\tilde{\varphi}_0(\tau_i, z) \equiv \sum_{j=1}^{J} \left(T_0 - \frac{\sigma_j \bar{g}_j}{w_j} \right) h_j \bar{N}_j, \quad \tilde{\varphi}_j(\tau_i, z) \equiv \frac{h_j \sigma_j \bar{N}_j g_j}{w_j}.$$

From (15), we have

$$K_i + K_s = K = \sum_{j=1}^{J} \bar{k}_j \bar{N}_j. \tag{A14}$$

Solve (A2) and (A14) with K_i and K_s as the variables

$$K_i = \beta \bar{\beta}_s \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \frac{\beta N}{z}, \quad K_s = \frac{\beta N}{z} - \beta \bar{\beta}_i \sum_{j=1}^{J} \bar{k}_j \bar{N}_j, \tag{A15}$$

where $\beta \equiv 1/(\bar{\beta}_s - \bar{\beta}_i)$. From (A3), we determine r and w_j as functions of τ_i and z. Insert K_s from (A15) in (A11)

$$\sum_{j=1}^{J} \left(\tilde{g}_j + \beta \bar{\beta}_i g_0 \bar{N}_j \right) \bar{k}_j = \frac{\beta g_0 N}{z} - \tau. \tag{A16}$$

Insert (A13) in (A16)

$$\phi_1 \bar{k}_1 + \phi_2 \bar{k}_2 = \phi_0, \tag{A17}$$

where

$$\phi_0\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \frac{\beta g_0}{z} \tilde{\varphi}_0 - \tau - \sum_{j=3}^{J} \phi_j \bar{k}_j, \quad \phi_j(z, \tau_i) \equiv \tilde{g}_j + \beta \bar{\beta}_i g_0 \bar{N}_j + \frac{\tilde{\varphi}_j \beta g_0}{z}.$$

in which $\{\bar{k}_j\} \equiv (\bar{k}_3, \ldots \bar{k}_J)$. From (14) we have

$$\sum_{j=1}^{J} s_j \bar{N}_j - (1 - \delta_k) \sum_{j=1}^{J} \bar{k}_j \bar{N}_j = \frac{w N_i}{\beta_i}, \tag{A18}$$

where we also use $F_i = w N_i / \beta_i$. Insert $s_j = \lambda_j \bar{y}_j$ and $N_i = \bar{\beta}_i K_i z$ from (A1) in (A18)

$$\sum_{j=1}^{J} \lambda_j \bar{N}_j \bar{y}_j - (1 - \delta_k) \sum_{j=1}^{J} \bar{k}_j \bar{N}_j = \frac{\beta \bar{\beta}_s \bar{\beta}_i w z}{\beta_i} \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \frac{w \bar{\beta}_i \beta N}{\beta_i}. \tag{A19}$$

Insert (A4) and (A13) in (A19)

$$\varphi_1 \bar{k}_1 + \varphi_2 \bar{k}_2 = \varphi_0, \tag{A20}$$

where

$$\varphi_j(z, \tau_i) \equiv \left(\frac{w \bar{\beta}_i \beta \tilde{\varphi}_j}{\bar{N}_j \beta_i} - \lambda_j g_j + 1 - \delta_k + \frac{\beta \bar{\beta}_s \bar{\beta}_i w z}{\beta_i} \right) \bar{N}_j,$$

$$\varphi_0\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv - \sum_{j=3}^{J} \varphi_j \bar{k}_j + \sum_{j=1}^{J} \lambda_j \bar{N}_j \bar{g}_j + \frac{w \bar{\beta}_i \beta \tilde{\varphi}_0}{\beta_i}.$$

Solving the linear equations (A17) and (A20) with $\bar{k}_1$ and $\bar{k}_2$ as the variables, we have

$$\bar{k}_j = \Omega_j\left(z, \tau_i, \{\bar{k}_j\}\right), \quad j = 1, 2. \tag{A21}$$

Here, we do not give the expressions of the functions in (A21) as it is straightforward and the expressions are tedious. It is straightforward to confirm that all the variables can be expressed as functions of z, τ_i, and $\{\bar{k}_j\}$ by the following procedure: $\bar{k}_1$ and $\bar{k}_2$ by (A21) $\rightarrow$ r and w_j by (A3) $\rightarrow$ $\bar{y}_j$ by (A4) $\rightarrow$ N by (A13) $\rightarrow$ K_i and K_s by (A15) $\rightarrow$ N_i and N_s by (A1) $\rightarrow$ F_i by (3) $\rightarrow$ F_s by (5) $\rightarrow$ p by (A8) $\rightarrow$ $\bar{T}_j$, c_j, and s_j by (11) $\rightarrow$ $T_j = T_0 - \bar{T}_j$ $\rightarrow$ $K = K_i + K_s$ by (15). From this procedure, (A21), and (12), we have

$$\dot{\bar{k}}_j = \bar{\Omega}_j\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \lambda_j \bar{y}_j - \Omega_j, \quad j = 1, 2, \tag{A22}$$

$$\dot{\bar{k}}_j = \Lambda_j\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \lambda_j \bar{y}_j - \bar{k}_j, \quad j = 3, \ldots, J. \tag{A23}$$

Taking derivatives of equation (A21) with respect to t and combining with (A23) implies

$$\dot{\bar{k}}_j = \frac{\partial \Omega_j}{\partial z}\dot{z} + \frac{\partial \Omega_j}{\partial \tau_i}\dot{\tau}_i + \sum_{j=3}^{J} \Lambda_j \frac{\partial \Omega_j}{\partial \bar{k}_j}, \quad j = 1, 2. \tag{A24}$$

Equaling the right-hand sizes of equations (A24) and (A22), we get

$$\frac{\partial \Omega_j}{\partial z}\dot{z} + \frac{\partial \Omega_j}{\partial \tau_i}\dot{\tau}_i = \bar{\Omega}_j - \sum_{j=3}^{J} \Lambda_j \frac{\partial \Omega_j}{\partial \bar{k}_j}, \quad j = 1, 2. \tag{A25}$$

Solving the linear equations (A25) with $\dot{z}$ and $\dot{\tau}_i$ as the variables, we have

$$\begin{aligned} \dot{z} &= \Lambda_1\left(z, \tau_i, \{\bar{k}_j\}\right), \\ \dot{\tau}_i &= \Lambda_2\left(z, \tau_i, \{\bar{k}_j\}\right). \end{aligned} \tag{A26}$$

Here, we do not give the expressions of the functions in (A26) as it is straightforward and the expressions are tedious. In summary, we proved the lemma.

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A STUDY OF THE ROLE OF GOVERNMENT IN INCOME AND WEALTH DISTRIBUTION BY INTEGRATING THE WALRASIAN GENERAL EQUILIBRIUM AND NEOCLASSICAL GROWTH THEORIES

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SAŽETAK

U radu se postavlja model rasta heterogenih kućanstava koji uključuje ekonomsku strukturu, akumulaciju bogatstva, endogenu ponudu radne snage i porezne stope. Fokus rada je na učincima mjera redistribucije na distribuciju prihoda i bogatstva, na ekonomsku strukturu i na ekonomski rast. Cjelovito su objedinjene Walrasova teorija opće ravnoteže i neoklasični ekonomski rast. Nadiđene su kontroverzne karakteristike dviju tradicionalnih teorija primjenom alternativnog pristupa kućanstvima. Postavljen je analitički okvir za disagregiranu i mikroutemeljenu opću teoriju ekonomskog rasta s endogenom akumulacijom bogatstva. Simulacijom modela identificirani su ravnoteža i stabilnost te iscrtana gibanja dinamičkog sustava s tri grupacije. Također je provedena komparativna dinamička analiza s osvrtom na paušalni porez, ljudski kapital i sklonost korištenju slobodnog vremena.

KLJUČNE RIJEČI

paušalni porez, porezne stope, Walrasianska teorija opće ravnoteže, neoklasične teorije rasta, dohodak i raspodjela bogatstva

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MODEL FOR ENERGY ALLOCATION IN HEATING SYSTEMS WITH PARTIAL DISTRIBUTION OF HEAT ALLOCATORS

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ABSTRACT

More than 150 000 households in Croatia have centralized heating system with heated water. Most of heat energy customers are connected to one common heat meter located in the heating substation of the building. These are the buildings built before 2001 in which the piping did not provide for individual metering of heat energy for each apartment. To make cost allocation fairer for customers on a common heat meter, there is a possibility of installing heat cost allocators. For technical functionality of the system, heat cost allocators must be installed on heating fixtures in at least 50 % of the apartments connected to the same metering point. In this paper we will analyze formula for allocation of energy consumption between customers that have cost allocators installed, primarily compared with energy distributed to customers without one. We will propose new method of energy distribution and compare its results and properties with those of formula in use.

KEY WORDS

heating systems, distribution, allocators

CLASSIFICATION

JEL: D12, O21, Q41

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INTRODUCTION

More than 150 000 households in Croatia have centralized heating system with heated water. Most of heat energy customers are connected to one common heat meter located in the heating substation of the building. These are the buildings built before 2001 in which the piping did not provide for individual metering of heat energy for each apartment. To make cost allocation fairer for customers on a common heat meter, there is a possibility of installing heat cost allocators. For technical functionality of the system, heat cost allocators must be installed on heating fixtures in at least 50 % of the apartments connected to the same metering point. Method for energy distribution was legislated by Ministry of Economy, Labor and Entrepreneurship [1] in year 2008. After first allocators were installed, there was some media reports that customers who installed it were not satisfied with distribution of heat energy consumed; namely they objected that energy (per meter squared) alocated to customers who did not install allocators was lower than those allocated to some customers with allocators. In year 2011, Ministry of Economy, Labor and Entrepreneurship legislated new Regulations for cost allocation of heat energy [2, 3], and this method is currently in use.

First object of this paper is to explain and analyze results of this method. In second chapter, we will introduce alternate method for cost allocation of heat energy, and compare its properties and results with those of current method.

FORMULA FOR ENERGY ALLOCATION

Parameters used for energy allocation are:

- total amount of energy read on a common heat meter, denoted with E_{ZJ} (notification, although little odd, is in correspondence with official notification of Ministry of Economy, Labor and Entrepreneurship regulation),
- area of households connected to a common heat meter, denoted with P_i. Index $i \in \{1, ..., k\}$ ranges over all households connected to a common heat meter. Without loss of generality, we will assume that first m indexed households have installed cost allocators,
- total number of impulses read on all cost allocators in one households, denoted with BI_i, $i \in \{1, ..., m\}$. Energy value of impulses read on allocators is unique for each system. Therefore, those values should be used only relatively one to another.

Those are input data of method used for calculating energy distribution among households connected on a common heat meter. Method uses following calculated values:

- total area of all households connected on a common heat meter, denoted with P_{SSUC},

$$P_{\mathrm{SSUC}} = \sum_{i=1}^{k} P_i ,$$

- total area of all households connected on a common heat meter with installed cost allocators, denoted with P_{SSR},

$$P_{\mathrm{SSR}} = \sum_{i=1}^{m} P_i ,$$

- total area of all households connected on a common heat meter without installed cost allocators, denoted with P_{SSBR},

$$P_{\mathrm{SSBR}} = \sum_{i=m+1}^{k} P_i ,$$

- factor U_{ST} legislated with *Regulations for cost allocation of heat energy* [1-3]. This factor is initially set to 25, and should be changed yearly in dependence of last year heat consumption on common heat meter.

Since heat consumption in households without heat allocators are not measured (weather in actual energy consumption, or number of index points) there is no way to determine part of energy consumed by those households, and part of energy which is lost in transport (within building) or which is used for heating common spaces, such as corridors. Even more, since indexes measured by heat allocators do not have energy value, there is no way to determine even energy consumption of households with heat allocators. That is, if we are determined to use current method of measurement with heat allocators. Therefore, *Regulations for cost allocation of heat energy* set factor U_{ST} as an administrative measure of energy consumption of households without heat allocators in the following way:

$$E_{\mathrm{SSBR}} = E_{\mathrm{ZJ}} \frac{P_{\mathrm{SSBR}}}{P_{\mathrm{SSUC}}} \left(1 + \frac{U_{\mathrm{ST}}}{100}\right), \tag{1}$$

$$E_{\mathrm{SSR}} = E_{\mathrm{ZJ}} - E_{\mathrm{SSBR}}. \tag{2}$$

In equation (1) E_{SSBR} stands for total energy assigned to all households without heat allocators, and it is calculated as part of total consumed energy (E_{ZJ}) proportional to share of area of all households without heat allocators (P_{SSBR}) in total area (P_{SSUC}), and increased for a factor 1,25 (if U_{ST} is set to 25). This way, formula tries to guess how much households without heat allocators consume more energy (relative to its area) than those with heat allocators. After that, equation (2) allocate remaining energy (up to amount of total energy consumed) to households with heat allocators.

After the totals of energy for all households without heat allocators (1) and for those with heat allocators (2) are determined, further formulas determine amount of energy allocated to each household. For those without heat allocators, it is share of E_{SSBR} proportional to its area:

$$E_i = E_{\mathrm{SSBR}} \cdot \frac{P_i}{P_{\mathrm{SSBR}}}, \quad i \in \{m+1, ..., k\}. \tag{3}$$

For calculation of energy allocated to households with heat allocators *Regulations for cost allocation of heat energy* introduce one more corrective factor, named U_{POV}. This factor can take value between 0 and 50, and it is used to allocate to households part of consumed energy which is lost in internal transport, or used for heating of common areas. Now we have:

$$E_i = E_{\mathrm{SSR}} \cdot \left[\left(1 - \frac{U_{POV}}{100}\right)\frac{BI_i}{BI} + \frac{U_{\mathrm{POV}}}{100}\frac{P_i}{P_{\mathrm{SSB}}}\right], \quad i \in \{1, ..., m\}. \tag{4}$$

BI in equation (4) stands for sum of all impulses BI_i read on heat allocators. Expression in brackets is convex combination of two values, P_i/P_{SSR} (which is constant) and BI_i/BI (which depends on energy consumption).

Formula for energy allocation can be described in following way: first, there is administrative division of total amount of energy into two parts – one allocated to all households without heat allocators installed, and second part, allocated to all households with heat allocators installed. After that, within each part further allocation is made, with regard to area of apartments and number of measured number of impulses.

ANALYSIS OF FORMULA FOR ENERGY ALLOCATION

First thing we need to do, is determine a goal of formula analysis. Since formula for energy allocation can be viewed as a function which assigns *n*-placed vector (values of allocated energy to each of *n* households) to an ($n + k + 3$)-placed vector of arguments (areas of *n* households, number of impulses measured on *k* heat allocators, two administrative factors, and total amount of energy consumed), it is important to determine a way of valuating formula.

From the perspective of consumer, there are two comparisons from which he or she values benefits of installing heat allocators: does reduced consumption after installation of heat allocators results with a lower energy allocation; and how allocation of energy for a household with installed heat allocators compare to energy allocation for a households without one. Regrettably, formula fails in certain situations on both of those accounts – it is possible that higher energy value will be allocated to a household after installation of heat allocators in spite lower energy consumption than before; it is also possible that higher energy value will be allocated to a households with heat allocators than to those without one, even if first one is consuming less energy than latter. Let us see how and why it happens.

Questions from last paragraph actually cannot be answered from real data. Consumption of energy from one month to another is not the same even in one household, let alone in two of them. Even more, there is no (direct) way of determining amount of energy consumed in a household without heat allocators. Real life data simply does not provide enough amount of control over energy consumption.

On the other hand, simulations provide total control over input parameters. In first series of simulations we will set energy consumption level on maximum for all apartments without heat allocators (which is the most common way of behaving), and in household with heat allocators we will set energy consumption level on some percentage of maximal consumption. Each time, we will single out one household, and analyze formula based allocation of energy for that apartment while its energy consumption rises from none to maximal. In those simulations we will have few assumptions which do not hold in real life, but they provide much clearer picture of formula effects. We will assume that all households consume level of energy proportional to its area. This means that all apartments have the same energy efficiency, which is not so in real life, but that assumption will help us understand ramifications of formula alone, without interference of energy efficiency factor.

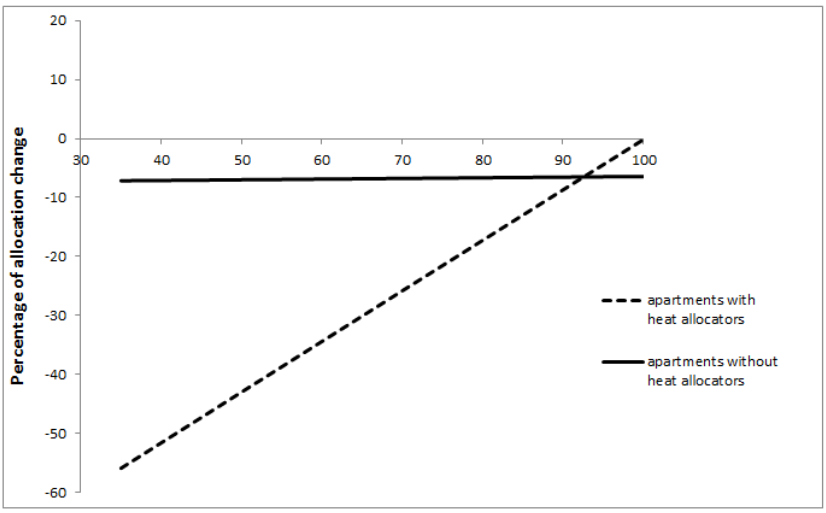

Figure 1. Comparison of energy allocation on 80 % consumption level.

In Figure 1 we can see results of such simulations. Simulations are made on a system with 100 apartments, 80 with heat allocators and 20 without them. Apartments without allocators consumed maximal amount of energy (per square meter), and those with allocators installed consumed 80 % of maximal energy (per square meter). One apartment was singled out, and graph in Figure 2 shows its results. On *x*-axes is percentage of energy (of maximal energy) consumed in that apartment, while on *y*-axis we can read percentage difference between its apartment energy allocation if there are no allocators installed and current situation. Red line marks difference in energy allocation for a apartment without allocators installed, and blue one marks difference for an apartment with heat allocators.

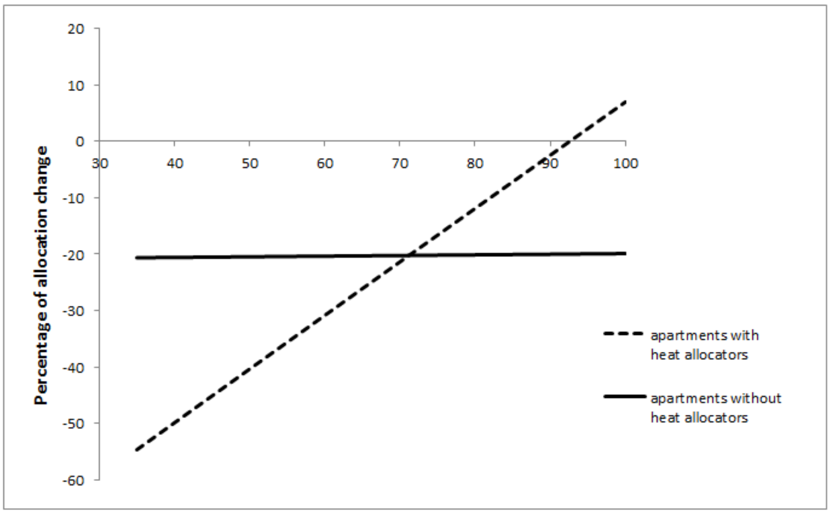

Figure 2. Comparison of energy allocation on 50 % consumption level.

As we can see in graph, apartment without heat allocators has approximately 7 % less energy allocation compared to situation prior to installment of allocators in building, almost regardless to its consumption; energy allocation is almost the same, in case of 30 % consumption, just as in case of 100 % consumption. On the other hand, energy allocation for an apartment with heat allocators depends on level of consumption. But, while 60 % saving of energy results with approximately same decrease of energy allocation, consumption of maximal amount of energy brings allocation greater than those of an apartment without heat allocator – at this level of consumption red line in graph gives higher values than the blue one. This feature becomes even more evident in situations with greater overall savings of apartments with heat allocators. In next two figures we can see how energy is allocated in situations when apartments with heat allocators reduce consumption of energy by 50 % and 70 %.

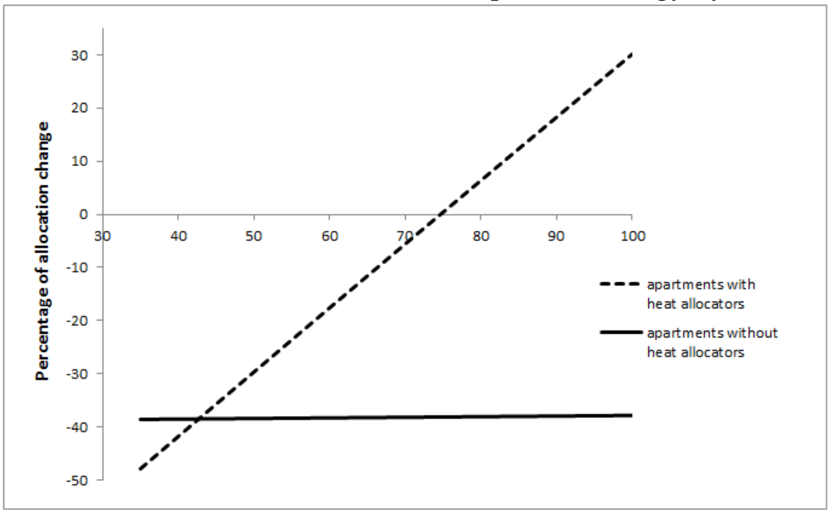

Figure 3. Comparison of energy allocation on 30 % consumption level.

As we can see in Figure 2, when average consumption of energy by households with heat allocators equals 50 % of available energy, when one of those apartments crosses level of consumption of 70 %, it will be allocated more energy than apartments without allocators, even if they are using all available energy. To make things worse, if that apartment uses all available energy, it will be allocated more energy than prior to installation of heat allocators, which in graph can be seen as blue line crosses to positive numbers.

Graph in Figure 3 stresses the problem even more. With average 70 % lesser consumption by households with heat allocators, even consumption of half of available energy brings greater allocation to apartments with heat allocators, than those to apartments without one. If energy savings are less than 20 %, household will be allocated greater amount of energy than prior to installation of heat allocators. In both cases, households without heat allocators, despite maximal consumption of available energy, are allocated with smaller amounts of energy than prior to installation of heat allocators in building.

Described problem is a result of formula construction. Since, formula first determine energy totals allocated to apartments with and without heat allocators, regardless to actual consumption, and then divides those totals between apartments, it is possible for a part allocated to apartment with heat allocators to be greater than any other part. In an extreme case, it can happen that one apartment would be allocated complete energy total reserved for apartments with heat allocators – one apartment can have one impulse spending, while all others have zero. In that case, all energy consumption comes from apartments without allocators, but greatest share of energy allocation goes to an apartment with allocator, in spite its minimal consumption. Simulation of that extreme scenario results with allocation for that apartment 16 times greater than prior to installation of heat allocators.

Next step is an analysis of formula behavior in situation with random consumption. For this purpose energy consumption on households with heat allocators will be randomly generated through uniform distribution of percentage of consumption, rating from 0 % to 100 %. Goal of this simulation is to analyze correlation between energy consumption and energy allocation, so we will set factor U_{POV} to zero. In that case energy allocated by formula should only depend on consumption, since "common energy" factor in equation (4) is set to zero. Result is shown in next figure:

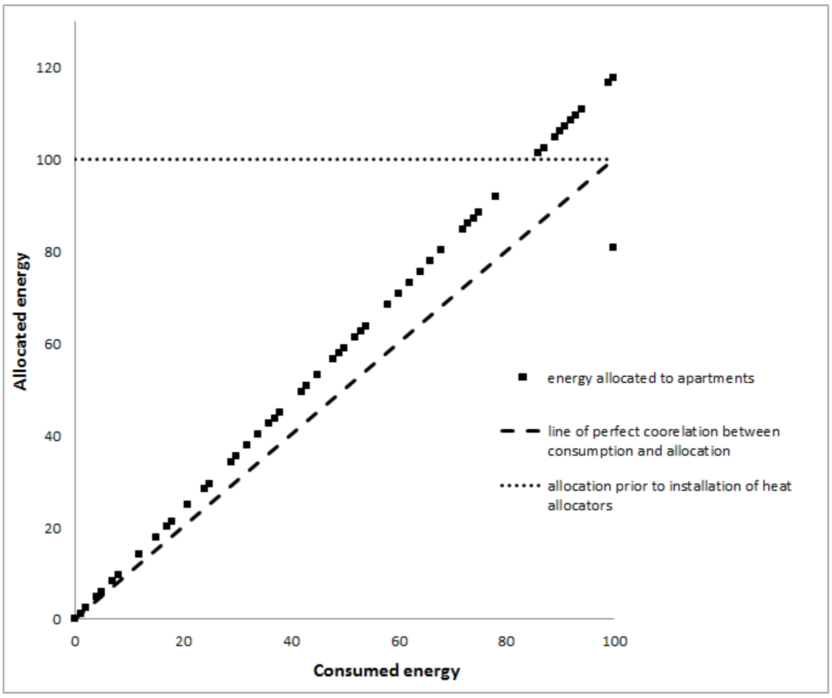

Figure 4. Comparison of energy allocation and energy consumption.

As we can see in Figure 4, energy allocation for all apartments (except for one type) are greater than actual consumption of energy, and for a number of them (those apartments with heat allocator that do not consume the least amount of available energy), even greater than value allocated prior to installation of heat allocators. Only type of apartment that benefits form formula is that without heat allocators, marked with square beneath line of perfect correlation.

CONSTRUCTION OF NEW FORMULA FOR ENERGY ALLOCATION

It is a fair question if anomalies described in last section could be avoided. Could we construct better formula which operates under same assumptions as current one, but produces no irregularities? In this section we will show that this construction is possible. New formula will work under same assumptions as current one, but it will always allocate less energy (per square meter) to an apartment with heat allocators installed than to one without allocators. Also, it will assure that energy allocated to an apartment with heat allocators is less than allocation prior to heat allocator installation. Both of those demands are valid; it is reasonable to assume that energy consumption in apartments without heat allocators is maximal, so no apartment with heat allocators, even with maximal consumption should not be allocated greater amount of energy. There is, also question of energy efficiency, but since there is no information of energy efficiency level of apartments in current model, it is only fair to treat apartments without heat allocators at least the same way as an apartment with heat allocator and lowest level of energy efficiency.

Basic idea of new formula for energy allocation is first to determine ratio between energy allocated to households with and without allocators, and after that to make correction toward energy totals, while keeping established ratios. To do that we will need following labels:

- $BI_i^p = BI_i / P_i$, which we will call "weight index", and which gives us measure of number of impulsses relative to area of an apartment, where i ranges from 1 to m,

- $BI_{\max}^{p} = \max_{i}\{BI_{i}^{p}\}$, or “maximal weight ind”, which gives us maximal value of weight index among apartments with heat allocators instaled,
- $U_i = BI_i^p / BI_{\max}^p$, index needed for determening energy allocation for an apartment with heat allocators. It gives us ratio of its apartment weight index compared to maximal weight index among all such apartments.

We start from division of energy into two parts, one for households with allocators, and other for households without one, as follows:

$$\sum_{i=1}^{m} E_i + \sum_{i=m+1}^{k} E_i = \sum_{i=1}^{m}\left\{E_{ZJ}\cdot\frac{P_i}{P_{\text{SSUC}}}\left[\frac{U_{\text{POV}}}{100}+\left(1-\frac{U_{POV}}{100}\right)\cdot U_i\right]\right\} + \sum_{i=m+1}^{k} E_{ZJ}\cdot\frac{P_i}{P_{\text{SSUC}}}\cdot\left(1+\frac{U_{\text{ST}}}{100}\right). \quad (5)$$

In expression (5) we find basic relations between energy allocated to households. Both allocations have same expression, $E_{ZJ}\cdot P_i/P_{SSUC}$, which is a part of energy proportional to apartments area, multiplied by different factor.

In the case of apartments without heat allocators, that factor equals to $1 + U_{SZ}/100$. Since factor U_{ST} can be changed and proscribed at desired level, it gives us possibility to determine a level for which factor for allocating energy will be greater than 1.

On the other hand, in the case of apartments with heat allocators, basic energy value is multiplied with $U_{POV}/100 + (1 - U_{POV}/100)\cdot U_i$, which is convex combination of two values, value 1 and value U_i. Since U_i is ratio of nonnegative values where nominator is surely less or equal to denominator, U_i has value less than 1. Therefore, convex combination of 1 and U_i cannot be greater than 1.

This leads us to conclusion that area proportional value of energy for an apartment with heat allocators will be multiplied by factor less than 1, while the same area proportional value for an apartment without heat allocators will be multiplied by factor greater than 1. So, expression (5) establishes relation between energy allocated to apartments with and without heat allocators in which energy (relative to its area) allocated to an apartment with allocators is less than energy allocated to an apartment without heat allocators (relative to its area). This satisfies elementary sense of justice – a household which saves energy should not pay more than one that does not.

Problem with expression (5) is that while it preserves desired relations between households, there is no guarantee that its value equals to energy E_{ZJ}, so in present form could not be used for allocation of total energy, E_{ZJ}. What expression (5) needs is linear adjustment, which will preserve relations among its components. Therefore, we denote value of the expression (5) with E_1,

$$E_1 = \sum_{i=1}^{m}\left\{E_{ZJ}\cdot\frac{P_i}{P_{\text{SSUC}}}\left[\frac{U_{\text{POV}}}{100}+\left(1-\frac{U_{POV}}{100}\right)\cdot U_i\right]\right\} + \sum_{i=m+1}^{k} E_{ZJ}\cdot\frac{P_i}{P_{\text{SSUC}}}\cdot\left(1+\frac{U_{\text{ST}}}{100}\right).$$

and calculate factor U_{ZJ} in following way:

$$U_{ZJ} = \frac{E_{ZJ}}{E_1}.$$

Now, we have factor, which by multiplying summands in expression (5) gives us allocations with sum E_{ZJ}, but with desired mutual relations. Therefore, for apartments with heat allocators installed, we calculate energy allocation by formula:

$$E_1 = E_{ZJ}\cdot\frac{P_i}{P_{\text{SSUC}}}\cdot\left(1+\frac{U_{\text{ST}}}{100}\right)\cdot U_{ZJ}, \quad i = m+1, ..., k. \quad (6)$$

For apartments with heat allocators installed, we use following formula:

$$E_1 = E_{\mathrm{ZJ}} \cdot \frac{P_i}{P_{\mathrm{SSUC}}} \cdot \left[\frac{U_{\mathrm{POV}}}{100} + \left(1 - \frac{U_{\mathrm{POV}}}{100}\right) \cdot U_i\right] \cdot U_{\mathrm{ZJ}}, \quad i = 1, \ldots, m. \tag{7}$$

If we want to describe formula construction, we would call this construction "bottom-up". Instead of determining totals first, new formula establishes relation between allocations for each apartment, and then extends or shrinks totals so they would fit total consumed energy value. In following figures we can see results of simulations made under same conditions as those in Figures 1, 2 and 3.

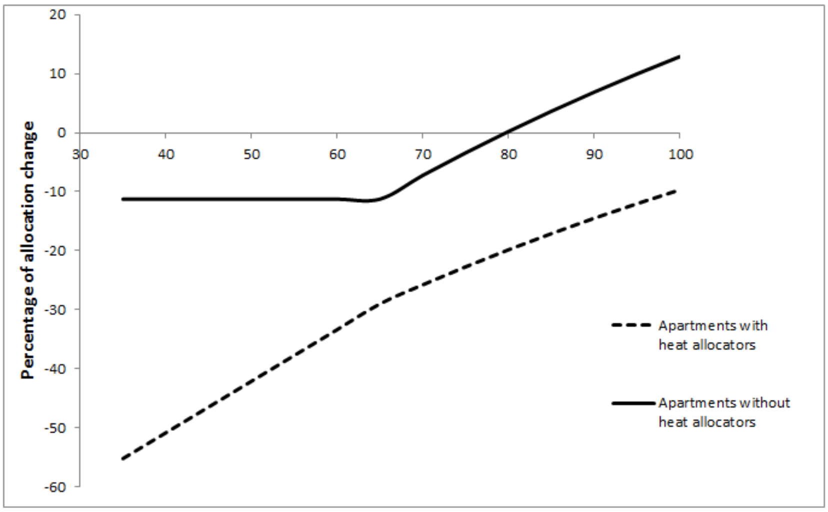

Figure 5. Comparison of energy allocation by proposed formula on 80 % consumption level.

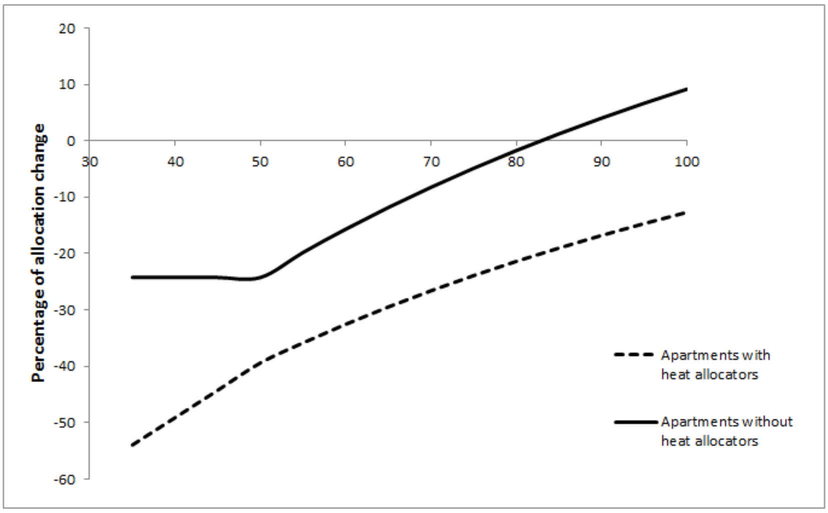

Figure 6. Comparison of energy allocation by proposed formula on 50 % consumption level.

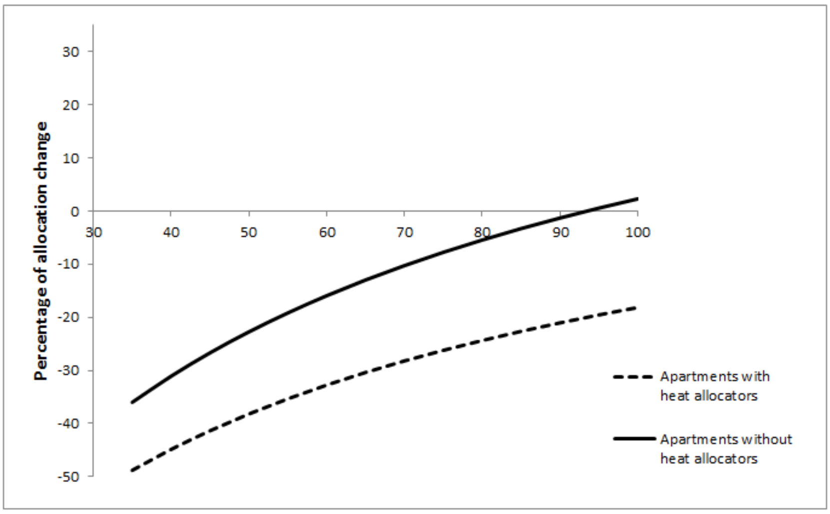

Figure 7. Comparison of energy allocation by proposed formula on 30 % consumption level.

As we can see in Figures 5, 6 and 7, energy allocated to apartment with heat allocators installed is always, as expected, lower than energy allocated to apartments without one. Even more, gap between two values can be moderated by choosing different values for factor U_{ST}. Furthermore, we can see that energy allocated to apartment with heat allocators does not cross x-axis in any simulation, that is, energy allocated to it is smaller than prior to installation of heat allocators. This is consequence of allocating more energy to apartments without allocators than it is proportional to theirs area share. This results with lower amount of energy total which is allocated to apartments with heat allocators. Nevertheless, this property of energy allocation would not hold in real life scenarios, since all energy consumption in simulations are proportional to an apartment's area. There is hidden assumption that all those apartments have same energy efficiency, which is not necessarily so.

When comparing results of simulations with random energy consumption (see Figure 4) of current formula with proposed one, we can also notice improvement.

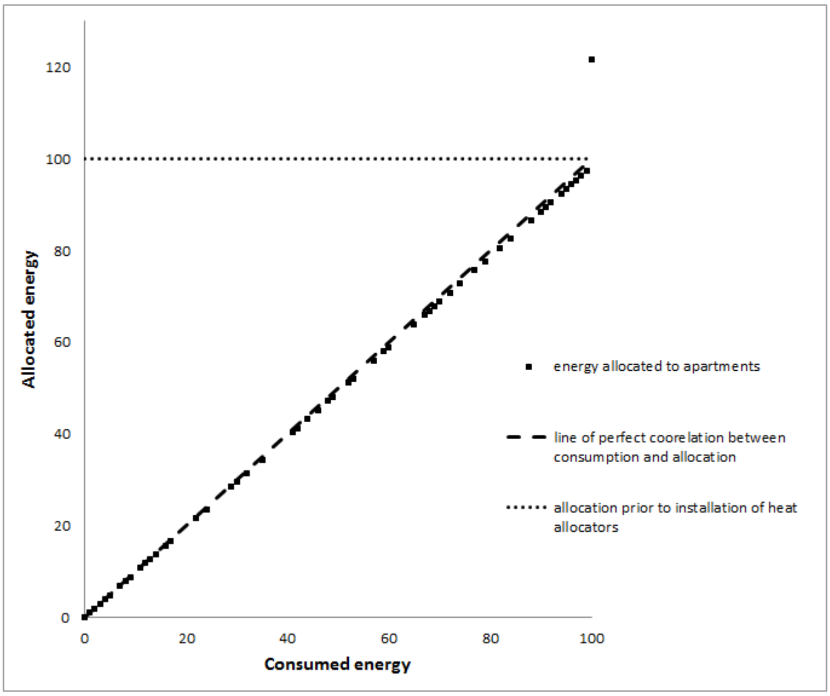

Figure 8. Comparison of energy allocation and energy consumption according to proposed formula.

Correlation between energy consumption and energy allocation is this time almost perfect. This is consequence of setting maximal impulse consumption BI^{p}_{max} as a basis for energy allocation. Apartments with heat allocators are allocated energy proportional to its relation to BI^{p}_{max}, while apartments without one are allocated greater amount (for a factor U_{ST}) of energy than apartment with maximal impulse consumption. Since, there is fair chance that one of the apartments with heat allocators uses (almost) all available energy, correlation in simulations becomes almost perfect. Isolated point above line that represents allocation prior to installation of heat allocators represents energy allocated to apartments without allocators. Again, gap between those two values can be moderated with value of U_{ST}.

Understanding why proposed formula works well in a given situation can lead us to conclusion when and why its results are not same level. As said, existence of at least one apartment with consumption remarkably higher than average consumption of apartments with heat allocators, enables formula to establish desired relations between apartments with heat allocators and apartments without one, since apartment with highest relative impulse consumption is set as one with consumption of all available energy (for that apartment). If apartments with heat allocators were all consuming exactly the same amount of energy (relative to its area), formula would lose its important benchmark. In that case, proposed formula would be unable to distinguish different levels of average consumption – it would treat them as same (since impulses are not normalized to some energy level); only difference would come from indirect influence of different energy totals. Let us see how proposed formula is working under those conditions. In following simulations all apartments with heat allocators were set to equal (relative) energy consumption, and "solidarity" factor U_{POV} was set to zero, so we would emphasize influence of consumption part of formula.

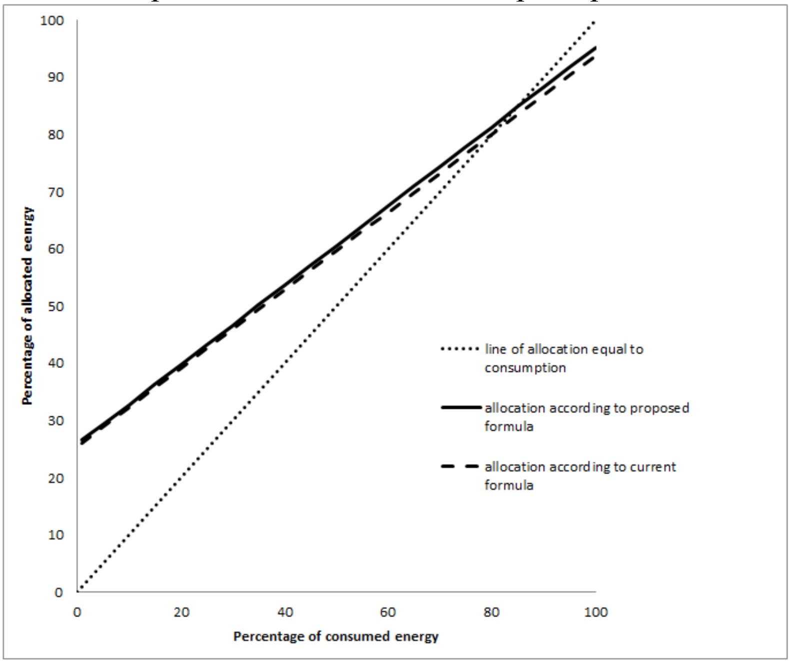

Figure 9. Comparison of current and proposed formula effects in "worst case scenario".

As we can see in Figure 9, if we take into account highly unlikely consumption distribution, in which all apartments with heat allocators consume exactly same percentage of available energy, proposed formula gives results significantly worse than those presented in Figure 8. Again, we stress unlikelihood of that scenario, since neither all apartments can have same energy efficiency, even if all its residents decide to use same percentage of available energy.

Even then, as shown in Figure 9, results of proposed formula equal to those of formula in current use, so even then usage of proposed formula would not give worse results, but the same as now. But in vast majority in situations, in which at least one apartment have higher consumption than average, proposed formula gives better result.

TESTING THE FORMULA ON REAL DATA

Through the article we were dealing with simulations, which cannot give us complete picture of how formula works. For instance, one of underlying assumption in all simulations was that all apartments consume energy proportional to its area; that is, all apartments have same energy efficiency. That, of course, does not have to be so. That is why it is useful to test formula, both current one and proposed one, on real data, to see how they behave. First we will present data obtained in city of Rijeka, Croatia, in apartment building located on Ivana Lenca 28, during January, February and March of 2012.

As we can see on left side of Figures 10, 11 and 12, significant number of apartments (to be precise, 21 in January, 18 in February and 22 in March out of 73 apartments) is allocated greater amount of energy than one allocated to apartments without heat allocators. One of them (dot in a upper right corner) is allocated four times as much energy per square meter in March! Current formula also gives one more surprising result; there are apartments with highest relative energy consumption, which are allocated approximately 70 kWh/m^2 in February and March of 2012. But in February we find that apartment to consume 30 impulses per meter squared, while consumption in March equals to 10 impulses per meter squared. On the other hand, proposed formula marks difference between those consumptions, with higher allocation for higher consumption.

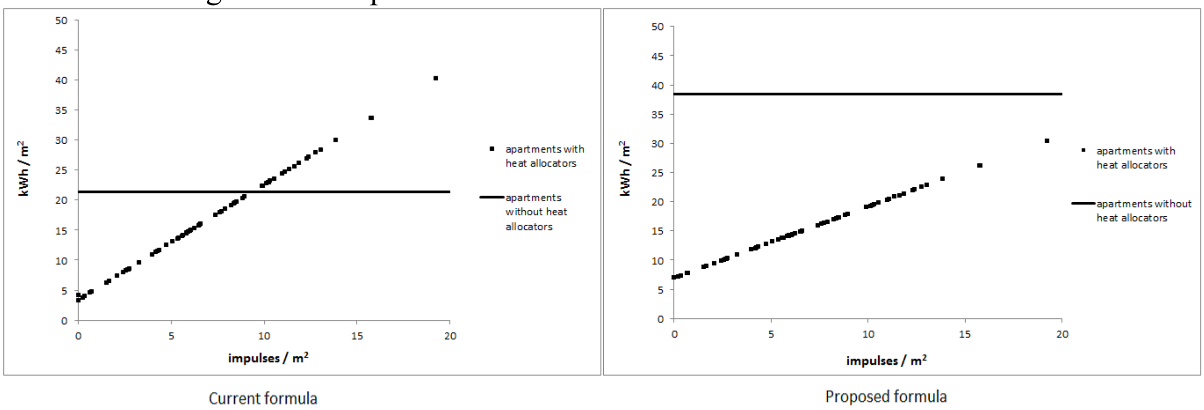

Figure 10. Comparison of allocated energies for an apartment building in city of Rijeka, based on measuring taken in January of 2012.

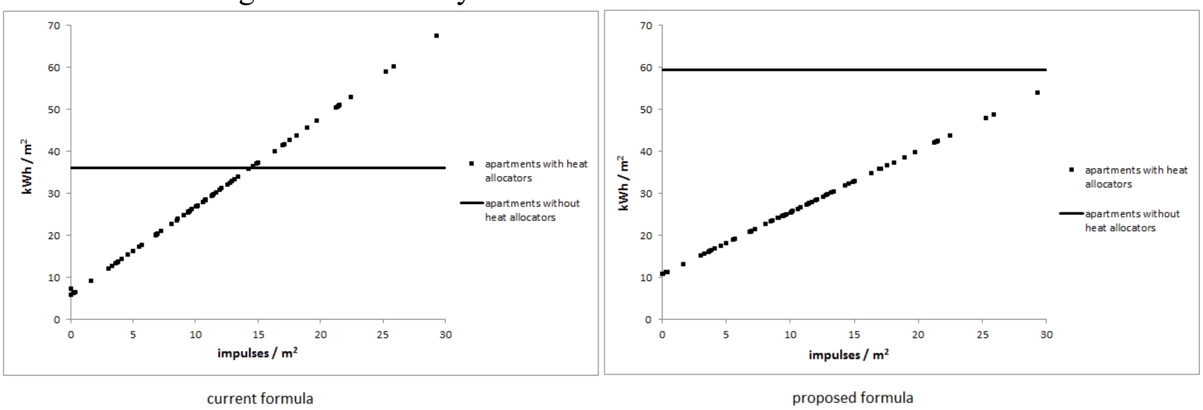

Figure 11. Comparison of allocated energies for an apartment building in city of Rijeka, based on measuring taken in February of 2012.

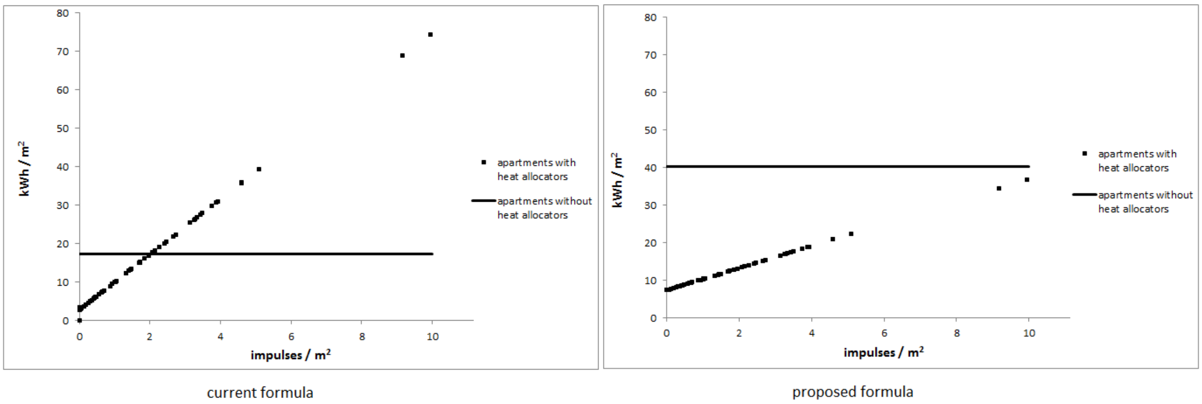

Figure 12. Comparison of allocated energies for an apartment building in city of Rijeka, based on measuring taken in March of 2012.

If we use proposed formula for same data, we find all apartments with heat allocators have less energy allocated than apartments without one. Furthermore, since greater number of apartments is allocated maximum energy (all apartments without allocators, as opposed to an apartment singled out in first distribution), spread of allocation is narrower. In this case, apartments with area of 94 % of total area have heat allocators installed. For both calculations $U_{\mathrm{POV}} = 20$ is used, while U_{ST} is set to 25 in current formula, and to 10 in proposed formula.

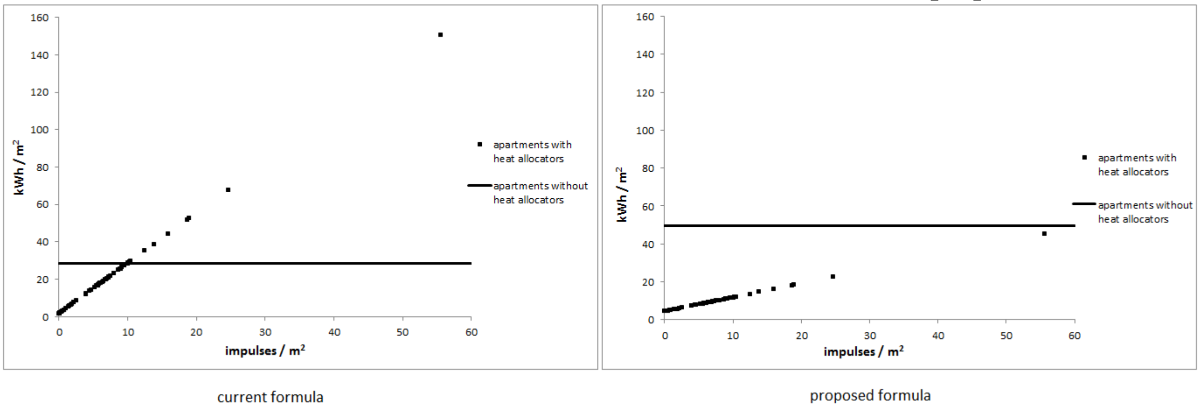

Figure 13. Comparison of allocated energies for an apartment building in city of Slavonski brod, based on measuring taken in December of 2012.

Same behavior we find in results of formulas calculated over data collected on measure position “Centar 4” in city of Slavonski brod in December of 2012. Parameters used in calculation are the same as those in calculations for Rijeka example. In this case, apartments with heat allocators account to 67 % of total area. Once again we can see that energy allocation for a number of apartments with heat allocators is higher than allocation to apartments without one. One of them is allocated four times more energy. Proposed formula, once again tone down extremes, with allocation to all apartments with heat allocators lower than those without one. It is noticeable that highest energy allocation according to current formula equals to 150 kWh/m^2, while with proposed formula it is 50 kWh/m^2.

CONCLUSIONS

One of the main goals of energy allocation model in multiple-occupancy residential buildings is developing “the formulas for conversion from measurement to allocation which would adjust the allocation for unmeasured parameters in accordance with the concept of equal payment for equal thermal comfort amenity” as stated in Guidelines of American Society of Heating, Refrigerating and Air-conditioning Engineers Inc [4; p.8]. Guidelines propose several methods of evaluation for share of energy consumed by non-monitored apartments. Guiding principle should be allocation of energy as close as possible to actual consumption. Unfortunately, results of simulations and data testing in this article shows that formula currently in use in Croatia does not seem to do this very well.

Both simulations and data testing showed that current formula through allocation favors non-monitored apartments, at the expense of part of monitored apartments with higher relative consumption. Proposed formula, on the other hand, sets different method of calculation; by starting with relation between allocation for monitored and non-monitored apartment, it builds "bottom-up" energy totals, which is in the end corrected to an actual energy total consumption. This method is recognized in ASHRAE Guideline, which states: "The estimates of monitored energy use, losses, auxiliary energy, and non-HVAC loads described in the preceding two paragraphs are unlikely to add exactly to the estimated total primary HVAC energy use. The two estimates should be adjusted to sum to total primary HVAC energy use, using for guidance such objective criteria as expected seasonal trends in monitored use, losses, and non-HVAC loads."

It is our belief that proposed formula is giving dynamic way of estimation of energy consumed by monitored and non-monitored apartments, which are then adjusted (by factor E_1) with total consumption. Result of such dynamic formula, which does not uses prescribed factors for consumed energy fractions, are allocations that work well in different types of consumption – with ones with low level of consumption, just as well as with ones with high level of consumption; something that proscribed energy allocation cannot deal with. We hope that this model will eventually help in construction of better models for energy allocation in complex systems with partial distribution of monitoring energy consumption.

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MODEL ALOCIRANJA ENERGIJE U SUSTAVIMA GRIJANJA S PARCIJALNOM DISTRIBUCIJOM RAZDJELNIKA TOPLINE

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SAŽETAK

Više od 150 000 kućanstava u Republici Hrvatskoj se grije putem centralnih sustava opskrbe toplinskom energijom. Većina tih potrošača priključena je na zajedničko brojilo utrošene energije, koje je locirano u toplinskoj stanici u zgradi. Najvećim dijelom radi se o zgradama građenima prije 2001. godine, tako da toplinske instalacije nisu predvidjele ugrađivanje kalorimetra kojim bi se mjerio utrošak toplinske energije u svakom stanu zasebno. Kako bi se uvela pravednija raspodjela troškova grijanja, koja bi bila u proporciji sa konzumiranim dijelom toplinske energije, uvedena je mogućnost ugradnje toplinskih razdjelnika, koji se prema zakonskim odredbama moraju ugraditi stanovima čija ukupna površina premašuje 50% ukupnih grijanih površina priključenih na zajedničko mjerilo. U ovom se radu analizira formula koja alocira dijelove energije stanovima sa ugrađenim toplinskim razdjelnicima, prvenstveno u usporedbi sa energijom alociranom stanovima bez ugrađenih razdjelnika. Također, u radu je izložena konstrukcija nov formule za alokaciju energije u opisanim sustavima, te su njezini rezultati uspoređeni sa rezultatima formule u upotrebi.

KLJUČNE RIJEČI

sustavi grijanja, distribucija energije, alokacija energije

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IMPACT OF STRATEGIC PLANNING ON MANAGEMENT OF PUBLIC ORGANIZATIONS IN BOSNIA AND HERZEGOVINA

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ABSTRACT

Modern public organizations should be familiar with the internal and external factors that affect their business. Striking a balance between these factors is the main prerequisite for building a successful business model in the current conditions of rapid change and increased competition. The aim of this study is to determine the impact of strategic planning on management of public organizations. An empirical research was conducted in public organizations in Bosnia and Herzegovina. The results of the research indicate that the use of strategic planning in public organizations enables more rational, efficient and effective management of organizational resources. Strategic planning defines certain aspects of the performance measurement, which reduces the possibility that managers allocate resources on the basis of their subjective preferences or feelings, ambitions or as some kind of response to certain political pressures. This contributes to a transparent, rational, more efficient and effective management of the organization in providing quality public services.

KEY WORDS

strategic management, strategic planning, public organization, management

CLASSIFICATION

JEL: H1

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INTRODUCTION

Strategic planning is a way and approach to the work of public organizations which has been successfully applied in a private sector for a long time. In this way, public organizations can make timely decisions, with an aim to manage limited resources in a more rational manner, to increase and improve services and achieve a greater satisfaction of citizens and business entities.

Public organizations serve social interest, i.e. provide public services with an aim to meet the interests and needs of citizens and business entities. Until recently public organizations were poorly organized and their operations have been developed in an unstructured manner, without any serious plans. That resulted in inefficiency and ineffectiveness, which ultimately led to a decline in the quality of public services and increased dissatisfaction of all stakeholders. That very *modus operandi* of public organizations has long-term consequences, because public organizations create environment for the development of economy and create ambiance for social entrepreneurship, as a special aspect of strategic entrepreneurship. Accordingly, public organizations have a great social responsibility and implications on the economic development of a certain country.

Strategic planning is a concept that has already shown remarkable results in market operations. Today, it is almost impossible to find a serious profit organization which does not apply the methods and approaches of strategic planning in their business. Organizations that have a clearly defined concept of strategic planning are more likely to achieve their goals. Organizations that do not apply strategic planning in their business have little chance of survival in the market. A formulated strategy takes into account the external factors that have a significant impact on the organization, analyses the internal strengths and weaknesses of the organization, defines the organization's goals and ways to achieve the set goals, the strategy implementation method, as well as the means for the measurement and evaluation of the implementation of goals. It is very difficult for an organization which does not apply strategic planning to measure a degree of implementation of the goals as well as work efficiency. Such organizations run their business *in an unstructured manner,* without serious plans and that often leads to high costs and produces very little results.

In Bosnia and Herzegovina, as well as in other transition countries, the importance of strategic planning in public organizations is still not sufficiently studied. A great number of public organizations in Bosnia and Herzegovina do not have sufficient knowledge, culture and routine to strategically plan their activities. The consequences of such a situation are extremely negative. This is the reason why these problems should be consistently researched, identified and properly addressed, because they imply and multiply many adverse consequences for B&H society and economy.

LITERATURE REVIEW

Dooris, Kelley and coach believe that strategic planning is still a relatively new concept in management. These authors have identified the period between the 1950s and 1970s, as the time when strategic planning emerged. They find that the last several decades have been a boom period for strategic planning [1; p.6]. When strategic planning gained popularity, researchers have begun paying more attention to the definition of strategic planning. Bryson defines strategic planning as "a disciplined effort to produce fundamental decisions and actions that shape and guide what an organization is, what it does, and why it does it" [2; p.6]. Mintzberg says that the concept of formalization is a key to understand planning. He defines strategic planning as "formalized procedure to produce articulated result, in the form of an integrated system of decision" [3; p.12]. Wilkinson and Monkhouse define strategic planning as “a method used to position an organization, through prioritizing its use of resources according to identified goals, in an effort to guide its direction and development over a period of time” [4; p.16]. Šehić defines strategic planning as “making present decisions in the light of their future implementation” [5; p.27]. Kotler defines strategic planning as „the process of developing and maintaining a strategic fit between the organization’s goals and capabilities and its changing marketing [6; p.44]. Strategic planning in public organizations has a task to analyze external factors influencing their operations, perceive internal strengths and weaknesses and, on that basis, set organizational goals and takes all necessary measures in order to achieve planned goals.

Many authors [7] believe that application of strategic planning can bring many benefits to an organization, such as: development of mission and vision of the organization, adjustment to the surroundings and achievement of set goals. Strategic planning in public organizations includes variety of activities, such as: setting organizational goals, defining tasks, establishing internal and external tasks and task forces, identifying key issues, developing strategies for each particular issue, planning control and adopting of procedures, planning adoption and producing fundamental decisions, taking actions, constant control and communication of results [7]. Bryson lists five benefits of strategic planning in the public and non-profit organizations: 1) promotion of strategic thinking and action, 2) improvement of a decision making process 3) improvement of organization, 4) improvement of the overall organization of work and results within an organization and 5) strategic planning can directly benefit all employees within the organization [7; p.5].

Strategic planning focuses attention on the important issues and challenges in organizational forms and helps key decision-makers to find the ways to address them. Strategic planning can, therefore, help organizations to define their strategic goals and to make today’s decisions in the light of their future consequences. Organizations that use strategic planning pay more attention to major organizational issues and are prepared to respond to internal and external demands and pressures, efficiently dealing with the consequences of all changes. Strategic planning can directly benefit employees of an organisation, enabling those who create policies and make key decisions to work more effectively and fulfil their obligations. It can also help them build teamwork and expertise. [7].

The process of implementation of the strategy involves defining specific requirements which an organization needs to fulfil in order to achieve the expected effects. We can say that the strategy entails the use and development of certain resources, abilities and skills, in order to enforce the implementation process. Organizational structure of a company defines the manner, structure, distribution and model of the management of tangible and intangible assets of a company [8; p.40].

There is a large degree of agreement in theoretical works that managers have a very important role in the planning process. The literature states that managers are very important part of the planning process, i.e. that people are the most important factor for successful implementation of strategic planning [9-12]. Streib [12] acknowledges that it is difficult to define the components of a successful strategic effort, but he identifies four management functions that he finds critical to the success of any strategic planning effort: leadership, human resources, managerial skills, and external support. The importance of people in the strategic planning process is evident in the fact that three of the four critical functions specifically address people and their role in planning. Eadie states the importance of people to the planning process by writing “the human factor looms large in strategy implementation, as well as in formulation and selection of strategies”. [11; p.448]. Hosmer describes strategic planning as an organizational task. She writes “Strategic management is an organizational task and requires an integrated effort by all members of the organization for successful completion" [13; p.55]. Bloom (1986) states that "the failure to involve interested parties in the planning process can reduce the chances for implementation" [10; p.254].

Lorange and Vancil [14] draw attention to the specific role of corporate planners and suggest that planning must be done by line managers because it is likely to be successful if it is not the work of just one person, but a people-interactive process involving all employees. The authors acknowledge the need for corporate planners, but view the corporate planner as an organizer who facilitates the process of planning. Bryson and Roering [15] acknowledge a similar role that they refer to as a manager. In their study of strategic planning in government, they identify the need that a manager is present everywhere in the process, in order to implement strategic planning.

Leadership has been identified as critical to the planning process because it is important for balancing the internal and external forces that affect the organization [9, 16]. Additionally, an active leader builds managerial support for the planning process which results in greater support for implementation of the plan [10]. Hosmer [13] clearly identifies the critical importance of leadership to the planning process. Leadership is important; it is not an outdated concept. There is a need for leaders. Much of the literature agrees that strong leadership committed to strategic planning is important for successful implementation of the plan. If the leader makes strategic planning a priority, which means that the organisation considers strategic planning as priority. The challenge, however, comes from the idea that the people in the organization are more likely to be accountable for the plan if they are involved in the development of the plan. This, intuitively, is very easy to comprehend. In practice, however, it is more difficult because the strong leader, the only person planning the process, may also be the leader who does not like to relinquish control of the process. With participation and involvement of other employees in the planning process, the leader necessarily loses some control. As a result, the strong leadership that the literature calls for and the participatory process for all employees can make implementing the strategic plan a difficult challenge. In addition to this challenge, the literature acknowledges that people play an important role in the planning process for strategic planning to be effective. Organizations must ensure participation and support of leaders and employees who will implement the plan.

Streib [12] after identifying the importance of leadership to the strategic planning process questions whether the public sector possesses the level of leadership necessary to succeed. Difficulties arise in maintaining a shared vision among elected and appointed officials, who change frequently due to elections and staff changes. Streib and Poister [17] discuss public sector limitation in terms of strategic capacity and question whether public organizations are able to compile the information necessary for the completion of a strategic plan. While continuity of leadership certainly can help an organization maintain a consistent vision which would, in turn, help the strategic planning process, one could argue that the author's questioning of leadership and strategic capacity within public organizations is too general and fails to acknowledge individual levels of leadership and strategic capacity. It is safe to assume that just as there are strong and weak leaders in the private sector, there are also strong and weak leaders of public organizations.

METHODOLOGY

RESEARCH APPROACH

The subject of this study is to analyze the connection between strategic planning and responsible management of public organizations in Bosnia and Herzegovina. Public organizations include public administration bodies, public institutions and public companies. Based on the subject, the objective of the research is defined and that is to determine how and in what way strategic planning leads to a more responsible management of public organizations in Bosnia and Herzegovina. It will be examined to what extent the application of strategic management in public organizations in Bosnia and Herzegovina enables the creation of an efficient strategic planning process, with a clear and distinctive strategy that the organization follows, a more effective development and implementation of business plans, planning and performance measurement. This will contribute to a transparent, rational, efficient and effective management of the organization whilst providing better quality public services.

In public organizations' practice, it is important to ensure objectivity of managers, which directly affects the success of organizational performance. To further support this practice, it will be investigated to what extent the application of strategic planning in public organizations affects their managers' objectivity in allocating resources.

Based on the defined problem, the subject and the research questions, the research hypothesis has been formulated: strategic planning in public organizations defines certain aspects of performance measurement, reduces the possibility that managers allocate resources guided by their subjective assessments and feelings, personal ambition or as some kind of response to certain political pressures.

In order to achieve the stated objectives of the study and examine the above hypothesis, a survey was conducted on a selected sample of public organizations.

RESEARCH DESIGN

Research design defines the characteristics of this research that follow. The survey research for this study has exploratory and explanatory character, stemming from the collection of data on strategic planning in public organizations and the analysis of the impact of strategic planning on a number of indicators of public organizations' performance. The method of online interviewing was selected and conducted in a cross-sectional survey, thus enabling the collection of data from a greater number of public organizations in a given time and space.

The questionnaires were sent to the respondents - top managers of public organizations (directors, managers, administrators, ministers, mayors), selected for the sample. The questionnaire was sent directly to the respondents via e-mail, using a special application to collect responses (www.SurveyConsole.com).

SURVEY INSTRUMENTATION

Strategic planning was analyzed in the sample, with a separate analyzes of the "function of institutional planning" in the sample (4 items, A1-A4), "existence of a strategic base/framework" in the sample (4 items, B1-B4), as well as "strategic planning development team optimization" in the sample (4 units, F1-F4). Development of the functions of institutional planning in the sample investigated using 4 items, wherein the respondents expressed the extent to which they agree with the statements (items G1-G4). Cronabach's alpha was calculated to test the reliability of the used scales.

SURVEY SAMPLE

The study population was as follows: in Bosnia and Herzegovina there are 200 organizations belonging to public administration, 500 organizations that are organized as public institutions and 100 public companies in which the state holds a majority stake (the state is the owner). It can be concluded that in Bosnia and Herzegovina there are total of 800 public organizations, of which 200 (25 %) are public institutions, 500 (62,5 %) are public administration bodies and 100 (12,5 %) are public companies. This study sample included 200 public organizations. The sample was proportionally stratified with proportions of groups in the sample matching the population proportions. In order to control the structure of the sample, three strata were used: public institutions, public administration organizations and public enterprises. To select the sampling units within each stratum, a random number generator was used. The stratified sample model of public organizations is based on a legal form as the only criterion of stratification, and is applied with roughly equal allocation of sample units per strata. In this way, the representation of the three types of public organizations in this study is ensured. In this study, the sample of 200 public organizations includes n1 = 124 public administration organizations, n2 = 47 public institutions and n3 = 18 public companies. The sample consists of 24,87 % public institutions, 65,61 % organizations of public administration and 9,62 % public companies. Chi-square test examined whether the difference in the structure of public organizations in the sample is significantly different from the structure of public organizations in the population. Chi-square test showed that the difference was not statistically significant (chi-square = 0,866, p-value = 0,648).

RESULTS

FUNCTION OF INSTITUTIONAL PLANNING IN THE SAMPLE (A1-A4)

Table 1 shows the average values of units from A1 to A4. Each unit can be assigned a numerical value on a scale of 1 to 5. The data in the table show that unit A1 has the highest average value (3,97). It is followed by unit A4 with the average value of 3,82. The third is unit A3 (3,68), and the last, fourth, is unit A2 (3,64).

EXISTENCE OF A STRATEGIC BASE/FRAMEWORK IN THE SAMPLE (B1-B4)

Table 2 shows the average values of units from B1 to B4. Each unit can be assigned a numerical value on a scale of 1 to 5. The data in the table show that unit B1 has the highest average value (4,14). It is followed by unit B3 with the average value of 4,08. The third is unit B4 (3,56), and the last, fourth, is unit B2 (3,43).

STRATEGIC PLANNING DEVELOPMENT TEAM OPTIMIZATION IN THE SAMPLE (F1-F4)

Table 3 shows the average values of units from F1 to F4. Each unit can be assigned a numerical value on a scale of 1 to 5. The data in the table show that unit F2 has the highest average value (3,44). It is followed by units F1 and F4 with the average value of 3,55, and the last, fourth, is unit F3 with the average value of 3,31.

Table 1. Average values of units A1-A4. Source: author’s research in November 2013.

Function of institutional planning (A1-A4)	***N***	**Min**	**Max**	**Average**	**St. Dev.**
A1. In your organization, top management takes care of strategic planning.	191	1	5	3,97	1,06
A2. Strategic planning in your organization is a priority activity.	185	1	5	3,64	1,08
A3. The organization follows a defined set of procedures in strategic planning.	184	1	5	3,68	1,02
A4. The organization ensures that all managers participate in the planning process.	186	1	5	3,82	1,09

Table 2. Average values of units B1-B4. Source: author's research in November 2013.

Existence of a strategic base/ framework (B1-B4)	***N***	**Min**	**Max**	**Average**	**St. Dev.**
B1. The organization has a clearly written mission statement.	183	1	5	4,14	1,03
B2. All employees and all levels of management in the organization understand the mission.	181	1	5	3,43	1,12
B3. The organization has written short-term (1 year) and long term (3-5 years) goals.	181	1	5	4,08	0,99
B4. The organization, as needed, defines the goals by location or geographical area (municipalities, regions, ...).	180	1	5	3,56	1,10

Table 3. Average values of units F1-F4. Source: author's research in November 2013.

Strategic planning development team optimization (F1-F4)	***N***	**Min**	**Max**	**Average**	**St. Dev.**
F1. The organization makes sure to use the talents and time of each individual board member.	187	1	5	3,35	1,11
F2. The organization ensures adequate involvement of each strategic plan development team member.	190	1	5	3,44	1,10
F3. The organization takes care of the development of new members of the strategic plan development team.	185	1	5	3,31	1,16
F4. The strategic plan development team strives to get all employees involved in the strategic planning process.	184	1	5	3,35	1,18

SUBJECTIVITY AND OBJECTIVITY OF THE MANAGEMENT IN THE SAMPLE (G1-G4)

Table 4 shows the average value of units from G1 to G4. Each unit can be assigned a numerical value on a scale of 1 to 5. The data in the table show that units G1 and G2 have the highest average value (4.06). They are followed by unit G2 with the average value of 3,84, and the fourth is unit G4 with the average value of 3,50.

Table 4. Average values of units G1-G4. Source: author's research in November 2013.

Subjectivity and objectivity of management, in the sample (G1-G4)	***N***	**Min**	**Max**	**Average**	**St. Dev.**
G1. The organization manager ensures fulfilment of organizational goals.	189	1	5	4,06	0,88
G2. The organization manager takes into account the interests of the community.	183	1	5	4,06	0,97
G3. The organization manager makes decisions on the basis of an objective assessment.	184	1	5	3,84	1,06
G4. The organization manager does not make decisions under the influence of external factors.	181	1	5	3,50	0,96

DISCUSSION

This section of the study will examine the first hypothesis (H1) which assumes that strategic planning in public organizations defines certain aspects of performance measurement, thereby reducing the possibility that managers allocate resources on the basis of their subjective assessments and feelings, personal ambitions or as some kind of response to certain political pressures. In doing so, strategic planning in public organizations shall be measured by the following constructs: "function of institutional planning" (units A1-A4, aggregate unit AM), "existence of a strategic base/framework" (units B1-B4, aggregate unit BM) and "strategic planning development team optimization" (units F1-F4, aggregate unit FM).

To examine the hypotheses, the correlation and regression method will be used, whereby the following analysis will be conducted:

• 4 multiple linear regression analyses, using aggregate units as independent variables: (1) dependent variable: unit G1i independent variables: aggregate units AM, BM and FM, (2) dependent variable: unit G2i independent variables: aggregate units AM, BM and FM, (3) dependent variable: unit G3i independent variables: aggregate units AM, BM and FM, and (4) dependent variable: unit G4i independent variables: aggregate units AM, BM and FM.

• 4 Stepwise multiple linear regression analyses using individual units as independent variables: (1) dependent variable: unit G1i independent variables: units A1-A4, B1-B4 and F1-F4, (2) dependent variable: unit G2i independent variables: units A1-A4 , B1-B4 and F1-F4, (3) dependent variable: unit G3i independent variables: units A1-A4, B1-B4 and F1-F4 and (4) dependent variable: unit G4i independent variables: units A1-A4, B1-B4 and F1-F4 .

Table 5 shows the regression model where the unit G1 "organization manager ensures fulfilment of organizational goals" is the dependent variable, and the independent variables are the average values of the units: "institutional planning function (AM)", "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM). The coefficient of determination is 0,555, meaning that 55,5 % of the variations of the dependent variable can be explained by the variations of the independent variables. The regression coefficients with all the independent variables are positive and statistically significant. It can be concluded that "development of institutional planning function" (AM), "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM) significantly influence whether an organization manager ensures fulfilment of organizational goals.

In order to investigate which individual units have statistically significant influence on the dependent variable G1 "the organization manager ensures the fulfilment of organizational goals" a stepwise regression analysis method was used. The independent variables were the units measuring the „function of institutional planning" (A1-A4), "existence of a strategic base/framework" (B1-B4) and "strategic planning development team optimization" (F1-F4). The stepwise regression analysis method was used with an aim of finding such a reduced set of independent variables, of the total number of candidate independent variables, which

Table 5. The regression model – the dependent variable is unit G1: "the organization manager ensures fulfilment of organizational goals"; the independent variables are the average values of the units: "institutional planning function"(AM), "existence of a strategic base/ framework"(BM) and "strategic planning development team optimization" (FM). Source: author's research in November 2013.

Independent variables	**Regression model using all independent variables: dependent variable is G1**			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	1,330	0,214	6,208	0,000**
AM	0,164	0,093	1,762	0,080*
BM	0,238	0,084	2,845	0,005**
FM	0,359	0,075	4,795	0,000**
R2 adjusted for the number of independent variables	0,555			

**statistically significant at 1 % probability
*statistically significant at 10 % probability

Table 6. The regression model, using all variables, and a reduced regression model (stepwise regression) – the dependent variable is unit G1: “the organization manager ensures fulfilment of organizational goals”; the independent variables: units A1-A4, B1-B4 and F1-F4. Source: author’s research in November 2013.

Independent variables	Reduced model (stepwise regression) dependent variable is G1			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	1,471	0,186	7,903	0,000**
A2. Strategic planning in your organization is a priority activity.	0,129	0,048	2,706	0,008**
A4. The organization ensures that all managers participate in the planning process.	0,159	0,067	2,369	0,019*
B2. All employees and all levels of management in the organization understand the mission.	0,309	0,061	5,098	0,000**
B4. The organization, as needed, defines the goals by location or geographical area (municipalities, regions ...).	0,151	0,057	2,660	0,009**
F1. The organization makes sure to use the talents and time of each individual board member.	1,471	0,186	7,903	0,000**
R2 adjusted for the number of independent variables	0,571			

**statistically significant at 1 % probability,
*statistically significant at 5 % probability

maximizes the validity of the regression model. The coefficient of the regression model, using all the independent variables, is 0,571, meaning that 57,1 % of the variations of the dependent variable can be explained by the variations of the independent variables. The units that statistically significantly influence the dependent variable are shown in the table. It can be concluded that the units: “strategic planning as a priority activity in an organization” (unit A1), “all managers participate in the planning process” (unit A2), “understanding of the organization’s mission by all employees and all levels of management” (unit B2), “defining goals by place or geographical area” (unit B4) and “use of talents and time of each individual board member” (unit F1) significantly affect whether an organization manager ensures the fulfilment of organizational goals.

Table 7 shows the regression model where the dependent variable is unit G2: “the organization manager takes into account the interests of the community” and the independent variables are the average values of the units: “institutional planning function” (AM), “existence of a strategic base/framework” (BM) and “strategic planning development team optimization” (FM). The coefficient of determination is 0.601, meaning that 60.1% of the variations of the dependent variable can be explained by the variations of the independent variables. The regression coefficients, with all the independent variables, are positive and statistically significant. It can be concluded that the units: “development of institutional planning function” (AM), “existence of a strategic base/framework” (BM) and “strategic planning development team optimization” (FM) statistically significantly affect whether an organization manager takes into account the interests of the community.

In order to investigate which individual units have statistically significant influence on the dependent variable G2: “the organization manager takes into account the interests of the community”, a stepwise regression analysis method was used, taking as independent variables the units measuring the “function of institutional planning” (A1-A4), “existence of a strategic base/framework” (B1-B4) and “strategic planning development team optimization”

Table 7. The regression model – the dependent variable is unit G2: "the organization manager takes into account the interests of the community"; the independent variables are the average values of the units: "institutional planning function" (AM), "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM). Source: author's research in November 2013.

Independent variables	Regression model using all independent variables: dependent variable is G2			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	0,917	0,219	4,181	0,000**
AM	0,411	0,096	4,306	0,000**
BM	0,213	0,086	2,494	0,014*
FM	0,234	0,077	3,053	0,003**
R2 adjusted for the number of independent variables	0,601			

**statistically significant at 1 % probability,
*statistically significant at 5% probability

(F1-F4). The stepwise regression analysis method is used with an aim of finding such a reduced set of independent variables, of the total number of candidate independent variables, which maximizes the validity of the regression model. The coefficient of the regression model, using all the independent variables, is 0,612, meaning that 61,2 % of the variations of the dependent variable can be explained by the variations of the independent variables. The units that significantly influence the dependent variable are shown in the table. It can be concluded that "strategic planning as a priority activity in an organization" (unit A1), "use of a defined set of procedures in strategic planning" (unit A3), "written short and long term goals" (unit B3), "use of time and talents of each individual board member" (unit F1) but also "of each team member" (unit F2) statistically significantly influence whether an organization manager takes into account the interests of the community.

Table 8. The regression model using all variables and a reduced regression model (stepwise regression) – the dependent variable is unit G2: "the organization manager takes into account the interests of the community"; the independent variables are units: A1-A4, B1-B4 and F1-F4. author's research in November 2013.

Independent variables	Reduced model (Stepwise regression): dependent variable is G2			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	1,037	0,206	5,031	0,000***
A1. The top management takes care of strategic planning in your organization.	0,173	0,075	2,311	0,022**
A3. The organization follows a defined set of procedures in strategic planning.	0,146	0,082	1,769	0,079*
B3. The organization has written short-term (one year) and long term (3-5 years) goals.	0,124	0,059	2,101	0,037**
F1. The organization makes sure to use the talents and time of each individual board member.	0,124	0,049	2,531	0,012**
F2. The organization ensures adequate involvement of each strategic plan development team member.	0,282	0,063	4,450	0,000***
R2 adjusted for the number of independent variables	0,612			

***statistically significant at 1 % probability,
**statistically significant at 5 % probability,
*statistically significant at 10 % probability

Table 9 shows the regression model where the dependent variable is the unit “organization manager makes decisions on the basis of an objective assessment”, and the independent variables are the average values of the units: “institutional planning function” (AM), “existence of a strategic base/framework” (BM) and “strategic planning development team optimization” (FM). The coefficient of determination is 0,522, meaning that 52,2 % of the variations of the dependent variable can be explained by the variations of the independent variables. The regression coefficients, with all the independent variables, are positive and statistically significant. It can be concluded that the functions “development of institutional planning”(AM), “existence of a strategic base/framework" (BM) and „strategic planning development team optimization” (FM) significantly affect whether an organization manager makes decisions on the basis of an objective assessment.

In order to investigate which individual units have statistically significant influence on the dependent variable G3: “the organization manager makes decisions on the basis of an objective assessment”, the stepwise regression analysis method was used, taking as independent variables the units measuring the “function of institutional planning” (A1-A4), “existence of a strategic base/framework” (B1-B4) and “strategic planning development team optimization” (F1-F4). The stepwise regression analysis method is used with the aim of finding such a reduced set of independent variables, of the total number of candidate independent variables, which maximizes the validity of the regression model. The coefficient of the regression model, using all independent variables, is 0,517, meaning that 51,7 % of the variations of the dependent variable can be explained by the variations of the independent variables. The units that statistically significantly influence the dependent variable are shown in the table. It can be concluded that the units: “strategic planning as a priority activity in an organization” (unit A1), “defining goals by place or geographical area”(unit B4) and “use of talents and time of each individual board member” (unit F1) statistically significantly influence whether an organization manager makes decisions on the basis of an objective assessment.

Table 11 shows the regression model where the dependent variable is unit G4: “the organization manager does not make decisions under the influence of external factors”, and the independent variables are the average values of the units: “institutional planning function” (AM), “existence of a strategic base/framework” (BM) and “strategic planning development team optimization” (FM). The coefficient of determination is 0,222, meaning that 22,2 % of the variations of the dependent variable can be explained by the variations of the independent variables. The regression coefficients, with all the independent variables, are positive and

Table 9. The regression model – the dependent variable is unit G3: “the organization manager makes decisions on the basis of an objective assessment”; the independent variables are the average values of the units: “institutional planning function” (AM), “existence of a strategic base/framework”(BM) and “strategic planning development team optimization” (FM). Source: author’s research in November 2013.

Independent variables	**Regression model using all the independent variables: dependent variable is G3**			
	B	**Std. Error**	**t**	**Sig.**
(Constant)	0,771	0,260	2,966	0,003**
AM	0,235	0,113	2,078	0,039*
BM	0,225	0,101	2,220	0,028*
FM	0,397	0,091	4,378	0,000**
R2 adjusted for the number of independent variables	0,522			

**statistically significant at 1 % probability,
*statistically significant at 5 % probability

Table 10. The regression model using all variables and a reduced regression model (stepwise regression) – the dependent variable is unit G3: "the organization manager makes decisions on the basis of an objective assessment"; the independent variables are units A1-A4, B1-B4 and F1-F4. Source: author's research in November 2013.

Independent variables	Reduced model (stepwise regression): dependent variable is G3			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	0,872	0,247	3,529	0,001*
A1. The top management takes care of strategic planning in your organization.	0,248	0,072	3,470	0,001*
B4. The organization, as needed, defines the goals by location or geographical area (municipalities, regions ...).	0,171	0,057	2,968	0,003*
F1. The organization makes sure to use the talents and time of each individual board member.	0,412	0,070	5,895	0,000*
R2 adjusted for the number of independent variables	0,517			

*statistically significant at 1 % probability

Table 11. The regression model – the dependent variable is unit G4: "the organization manager does not make decisions under the influence of external factors"; the independent variables are the average values of the units: "institutional planning function" (AM), "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM). Source: author's research in November 2013.

Independent variables	Regression model using all independent variables: dependent variable is G4			
	B	**Std. Error**	**t**	**Sig.**
(Constant)	1,793	0,304		0,000*
AM	0,117	0,132	0,117	0,376
BM	0,044	0,118	0,039	0,710
FM	0,328	0,106	0,357	0,002*
R2 adjusted for the number of independent variables	0,222			

*statistically significant at 1 % probability

statistically significant. It can be concluded that development of "institutional planning function" (AM), "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM) statistically significantly influence whether an organization manager makes decisions under the influence of external factors.

In order to investigate which individual units have statistically significant influence on the dependent variable G4: "the organization manager does not make decisions under the influence of external factors" a stepwise regression analysis method was used, taking as independent variables the units which measure "function of institutional planning" (A1-A4), "existence of a strategic base/framework" (B1-B4) and "strategic planning development team optimization" (F1-F4). The stepwise regression analysis method is used with the aim of finding such a reduced set of independent variables, of the total number of candidate independent variables, which maximizes the validity of the regression model. The coefficient of the regression model, using all independent variables, is 0,282, meaning that 28,2 % of the variations of the dependent variable can be explained by the variations of the independent variables. The units that significantly influence the dependent variable are shown in the table. It can be concluded that a "clearly written mission statement of the organization" (unit B1),

Table 12. The regression model using all variables and a reduced regression model (stepwise regression) – the dependent variable is unit G4: "the organization manager does not make decisions under the influence of external factors", the independent variables are units A1-A4, B1-B4 and F1-F4. Source: author's research in November 2013.

Independent variables	Reduced model (stepwise regression): Dependent variable G4			
	B	**Std. Error**	***t***	**Sig.**
(Constant)	1,950	0,287	6,803	0,000**
B1. The organization has a clearly written mission statement.	–0,278	0,099	–2,806	0,006**
B2. All employees and all levels of management in the organization understand the mission.	0,255	0,085	2,980	0,003**
B3. The organization has written short-term (one year) and long term (3-5 years) goals.	0,184	0,089	2,058	0,041*
F1. The organization makes sure to use the talents and time of each individual board member.	0,324	0,075	4,345	0,000**
R2 adjusted for the number of independent variables	0,282			

**statistically significant at 1 % probability,
*statistically significant at 5 % probability

"understanding of the mission by all employees and all levels of management" (unit B2), "written form of short-term and long-term goals" (unit B3) and "use of talents and time of each individual board member" (unit F1) significantly affect whether an organization manager makes decisions under the influence of external factors.

To test the hypothesis (H1) of this study, the results of the conducted correlation and regression analyzes shall be summarized.

Table 13 shows the determination coefficients and regression coefficients of the individual independent variables of the regression models, with dependent variables of units G1-G4, the average values of the independent variables of units: "institutional planning function"(AM), "existence of a strategic base/framework" (BM) and "strategic planning development team optimization" (FM). It can be noted that all the aggregate independent variables are statistically significant in all the regression models. Also, the determination coefficients, adjusted for the number of independent variables, were high for the regression models with dependent variables of units G1, G2 and G3, with an exception of the determination coefficient of the regression model with the dependent variable G4. This result is consistent with the results of the correlation analysis presented in the preceding table. The regression analysis suggests that there is a statistically significant relationship between strategic planning and subjectivity/ objectivity in managing organizations.

Table 14 shows the determination and regression coefficients of individual regression models, using as dependent variables the units G1-G4, and as independent variables the units: "function of institutional planning" (A1-A4), "existence of a strategic base/framework" (B1-B4) and "strategic planning development team optimization" (F1-F4). This analysis was conducted to determine which specific aspects of the functions "institutional planning", "existence of a strategic base/framework" and "strategic planning development team optimization" have the greatest impact on the objectivity of managers in public organizations. It turned out that only the units F3 and F4 do not have statistically significant influence on dependent variables. The stepwise regression analysis leads to a conclusion that there is a statistically significant relationship between strategic planning and subjectivity/objectivity in managing organizations.

Table 13. The determination coefficient and the regression coefficients of individual regression models; the dependent variables: units G1-G4, the independent variables: average values of the units "institutional planning function" (AM), "existence of a strategic base/ framework" (BM) and "strategic planning development team optimization" (FM). Source: author's research in November 2013.

Independent variables	**Regression models: dependent variables**			
	G1. The organization manager ensures the fulfilment of organizational goals.	**G2. The organization manager takes into account the interests of the community.**	**G3. The organization manager makes decisions on the basis of an objective assessment.**	**G4. The organization managerdoes not make decisions under the influence of external factors.**
(Constant)	1,330***	0,917***	0,771***	1,793***
Function of institutional planning AM	0,164*	0,411***	0,235**	0,117
Existence of a strategic base/ framework BM	0,238***	0,213**	0,225**	0,044
Strategic planning development team optimization	0,359***	0,234***	0,397***	0,328***
R2 adjusted for the number of independent variables	0,555	0,601	0,522	0,222

***statistically significant at 1 % probability,
**statistically significant at 5 % probability

Based on the presented analyses, it can be concluded that the research hypothesis (H1) that "strategic planning in public organizations defines certain aspects of performance measurement, thereby reducing the possibility that managers allocate resources on the basis of their subjective assessments and feelings, personal ambitions or as some kind of response to certain political pressures", can be accepted.

CONCLUSION

The aim of this study was to analyze the importance of strategic planning to responsible management of public organizations. The conducted study confirmed that application of strategic planning in public organizations in B&H helps managers to manage public organizations in a more responsible manner. Based on the results of the empirical study, we can conclude that the research hypothesis (H1) that "strategic planning in public organizations defines certain aspects of performance measurement, thereby reducing the possibility that managers allocate resources on the basis of their subjective assessments and feelings, personal ambitions or as some kind of response to certain political pressures", can be accepted.

These results suggest that state/public organizations should pay more attention to the implementation of strategic management, i.e. strategic planning, strategy implementation and control of activities of public organizations, as that will increase the work efficiency and improve the quality of public services delivery.

Table 14. (Next page) The determination and regression coefficients of individual regression models: the dependent variables are units G1-G4, the independent variables are units: "institutional planning function" (A1-A4), "existence of a strategic base/framework" (B1-B4) and "strategic planning development team optimization" (F1-F4). Source: author's research in November 2013.

Independent variables	**Reduced model (stepwise regression): dependent variables**			
	G1. The organization manager ensures the fulfilment of organizational goals.	**G2. The organization manager takes into account the interests of the community.**	**G3. The organization manager makes decisions on the basis of an objective assessment.**	**G4. The organization manager does not make decisions under the influence of external factors.**
(Constant)	1,471***	1,037***	0,872***	1,950***
A1 Top management takes care of strategic planning in your organization.	-	0,173**	0,248***	-
A2. Strategic planning in your organization is a priority activity.	0,129***	-	-	-
A3. The organization follows a defined set of procedures in strategic planning.	-	0,146*	-	-
A4. The organization ensures that all managers are involved in the planning process.	0,159**	-	-	-
B1. The organization has a clearly written mission statement.	-	-	-	-0,278***
B2. All employees and all levels of management in the organization understand the mission.	0,309***	-	-	0,255***
B3. The organization has written short-term (one year) and long term (3-5 years) goals.	-	0,124**	-	0,184**
B4. The organization, as needed, defines the goals by location or geographical area (municipalities, regions ...).	0,151***	-	0,171***	-
F1. The organization takes care to use the talent and time of each individual board member.	1,471***	0,124**	0,412***	0,324***
F2. The organization ensures adequate involvement of each strategic plan development team member.	-	0,282***	-	-
F3. The organization takes care of the development of new members of the strategic plan development team.	-	-	-	-
F4. The strategic plan development team strives to get all employees involved in the strategic planning process.	-	-	-	-
R2 adjusted for the number of independent variables	0,571	0,612	0,517	0,282

***statistically significant at 1 % probability,
**statistically significant at 5 % probability,
*statistically significant at 10 % probability

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UTJECAJ STRATEŠKOG PLANIRANJA NA UPRAVLJANJE JAVNIM ORGANIZACIJAMA U BOSNI I HERCEGOVINI

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SAŽETAK

Suvremene javne organizacije u svom poslovanju trebaju poznavati unutarnje i vanjske faktore koji utječu na njihovo poslovanje. Uspostava ravnoteže između ovih faktora osnovna je pretpostavka izgradnje uspješnog koncepta poslovanja u današnjim uvjetima brzih promjena i povećane konkurencije. Cilj ovog rada je utvrditi kakav je utjecaj strateškog planiranja na upravljanje javnim organizacijama. Provedeno je empirijsko istraživanje u javnim organizacijama u Bosni i Hercegovini. Rezultati istraživanja pokazuju kako primjena strateškog planiranja u javnim organizacijama omogućava racionalnije, učinkovitije i efektivnije upravljanje resursima organizacije. Strateškim planiranjem definiraju se određeni vidovi mjerenja performansi što smanjuje mogućnost alociranja resurse na temelju subjektivnog stava ili osjećaja, ambicije ili svojevrsnog odgovora na određeni politički pritisak. Time se doprinosi transparentnom, racionalnom, učinkovitom i efektivnijem upravljanju organizacijom u pružanju kvalitetnih javnih usluga.

KLJUČNE RIJEČI

strateški menadžment, strateško planiranje, javna organizacija, menadžment

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QUALITY SERVICE EVALUATION THROUGH THE SYSTEM OF COMPLAINTS AND PRAISE

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ABSTRACT

Complaint, as the expression of customer dissatisfaction with the quality of products or services, is very valuable information. Well-built system for collecting, processing and analysis of complaints allows organizations to create the information base for making decisions based on facts. This data base provides an effective adoption and implementation of measures for continuous improvement of products/services quality. To make the system work effectively, it is necessary to continuously use the same methodology for collecting and processing complaints to be able constant comparisons from period to period. Greater investment in quality of products/services does not mean reducing the number of complaints in the same time due to the effects of the phenomenon of "unrealistic expectations". In addition to complaints, a valuable source of information on customer satisfaction is the system of praise.

KEY WORDS

complaint, quality, customer satisfaction, phenomenon of unrealistic expectation, scrap, praise

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INTRODUCTION

Organizations that have adopted the concept of quality as a strategic guideline in the business take into account the degree of customer satisfaction with the service/product and do not neglect the opportunities offered by a good system of collecting, processing and analysis of complaints and praises. The development of such a system is one of the effective methods of evaluating the service quality by measuring the level of customer satisfaction. Customer satisfaction can be measured in various ways: measurement based on questionnaires and interviews, based on the results of audits by the external and internal interested groups, based on the standards typical of certain services specific for a certain organizations, as is the case of health care services.

This article deals with the complaints and praises system in the health care service as one of the elements that can be used in the monitoring of clinical and managerial practices to early identify areas of concern. Complaints are, in fact, warning signs, often present before the patient/customer suffered significant damage, other sources of information include clinical indicators, incidents and results of clinical audit. In addition, the patient/customer does not have to complain when it is not satisfied with the service. On the other hand, the patient/customer himself may be satisfied, but it is noticed that the quality of service that he has experienced was not at the ideal level.

Complaint is not enough just to record, but it should be a trigger to start the whole process that involves: analyzing, communicating with a person who has submitted a complaint, making decisions and planning for improvements, implementation of planned activities aimed at eliminating the causes of permanent dissatisfaction, capturing the specific complaints through periodic reports based on measurement of customer satisfaction.

For example, carefully examined complaints must result in real changes, such as improvement of policies, procedures and processes to improve care, about this customers should be certainly informed, on the basis of complaints preventive actions should prevent repetition of these complaints in the future. The complaint, therefore, can be a catalyst for improving the quality system and point out the special needs of certain groups of patients, for example, children or elderly persons.

Thus, quality improvement for a specific organization is based on several quality management principles: customer orientation, decision-making based on facts and principles of continuous improvement.

WHAT IS A COMPLAINT?

The practical definition says that the complaint is “expression of dissatisfaction that requires response” [1]. However, patients/customers do not always use the word “complaint”. They will wrap their complaints, sometimes, in terms of “comments” or “suggestions” to sound less blatant. For health facility it is important to recognize these „comments“' as a complaint. Specifically, it is important to recognize the situation of patient/customer dissatisfaction with the services in relation to the standards, no matter how they call them. Therefore, in the category of complaints should include any defect or failure to provide medical services at acceptable standards from the perspective of the customer [2].

Complaints are an important indicator of what customer dissatisfaction is constituted, what problems they face during the provision of service. The value of complaints is that they are an excellent additional source of data for determination a value for customers, what are they complaining about and for elimination of complaints sources in order to achieve higher customer satisfaction level.

In the case of this research, complaints were divided into two categories: informal and formal. The informal complaint, regardless of whether it is addressed by telephone or verbally, in personal contact or through the questionnaires it can be solved by careful treatment quickly and efficiently. A formal complaint has a form of a letter, fax or e-mail, usually refers to serious problems, for example, the unconscionable treatment and must go through a formal process of solving the problem. To submit a complaint means to express dissatisfaction. Thus, the formal complaint is a written protest. The mutual interest is to minimize the effect of the complaint and settle it quickly and in a sensitive manner.

Healthcare facility in this research has its own policies and procedures for complaint cases and follows these policies and procedures [3]. While doing so, a system for tracking complaints particularly took following situations into account:

- sources of data about customer dissatisfaction. Sources analysis for given period were the same, so that the number of complaints could be comparable. Any change in the source of data should be taken into account in the study,
- methodology for collecting and processing complaints, must be the same, in order to ensure comparability. It is necessary to determine what is considered the complaint, i.e. which form of expression of dissatisfaction (whether to consider only expressions of dissatisfaction expressed in writing or orally, and in this case how should they be appropriately recorded),
- evaluation of quality service through the quantification of the degree of customer satisfaction in quality of products or services. The creation of information base as a starting point to explore the root causes of decline in the quality of products or services, and customer dissatisfaction of quality in general,
- interpretation of the number of complaints and the performance of the conclusions,
- purpose that it is not only determination of the number of complaints and simplification of the problem, but the analysis that will lead to the development plan of corrective actions that will act on the causes of customer dissatisfaction.

As it was stated before, the organization that is in focus of this research is health care organization. Dental Clinic Zagreb has accepted the monitoring and analysis of complaints, as the way of quality service evaluation. Top management has made a decision according to which all properly received written complaints from customers, should be delivered to the quality management for processing. Quality team assignments are [3]:

- each complaint must be carefully examined,
- a record of each complaint must be kept in the book of complaints, complaints must be classified into specific categories,
- complaints must be periodically analyzed,
- periodic report on evaluation of quality service based on the analysis of complaints as one of the indicators must be made,
- actions for improvement must be proposed in the report,
- report must be submitted to the quality management representative.

Based on the report of the quality management representative the information for the top management is prepared and proposed with necessary decisions for further actions, primarily to eliminate the cause of the complaint.

COMPLAINTS WITH THEIR SOURCES AND COLLECTING METHODOLOGY

To develop the system, the sources of collecting complaints were determined. Consequently the top management adopted a Procedure of quality named *Complaints of customers – patients* which was established way of collecting, monitoring and analyzing customers complaints [3]. The purpose of this procedure is to establish a methodology that will not change for a long period, to ensure the comparability of results of analysis of complaints from period to period. This action established the obligation of contacting the applicant of the complaints, if it is reasonably possible to establish contact.

There were a different ways of collecting complaints such as: via e-mail, through the official website, in writing to the registry book, through records in the book of praise and complaints, through questionnaires.

The beginning of collecting complaints, based on established methodologies, and their processing and analysis, dates back to July 1st 2006. The first report on the results of the analysis of customers complaints was made in 2006 year, and it referred to the period from July 1st 2006 to December 12th 2006. Subsequently, complaints were regularly collected, and reports were drafted and discussed after each received complaint.

Thus, there are three sources of collecting complaints. These are patients' complaints in written (*formal complaints – FC*), complaints written in the polls *(poll complaints – PC*) and records of complaints to the finished product (*finished product complaints – FPC*), such as the complaints on orthodontic or prosthetic product.

Reports on complaints include, besides determination of the number of complaints, causes analysis, identification of poor processes and determining the level of *the scrap* [4].

Table 1. Number of service complaints in Dental Clinic Zagreb in the period from year 2006 to year 2013 (formal complaints – FC, poll complaints – PC, finished product complaints – FPC). Source: E. Krstić Vukelja research (Management review's records from 2006-2013).

Year	Number of complaints				Number of patient's visits	Amount of investments (kn)	Number of complaints per 10.000 patinets			
	FC	*PC*	*FPC*	*Total*			*FC*	*PC*	*FPC*	*Total*
2006	5		0	5	208287	3.660.000	0.24	0	0	0.05
2007	2	177	5	184	196402	3.394.000	0.10	9.01	0.25	9.37
2008	5	71	3	79	205546	2.984.816	0.24	3.45	0.15	3.84
2009	7	31	2	40	231352	1.913.726	0.30	1.34	0.08	1.73
2010	9	65	2	76	196590	2.555.000	0.46	3.31	0.1	3.87
2011	11	16	3	30	179676	2.737.890	0.61	0.89	0.17	1.67
2012	6	71	5	82	176226	1.950.000	0.34	4,02	0.28	4,65
2013	10	39	1	50	100444	2.250.000	0.99	3.88	0.1	4.97
Total	**55**	**470**	**21**	**546**	**1,494,523**	**21.445.432**	**0.37**	**3.14**	**0.14**	**3.65**

According to the data in Table 1 in the period from the 2006 up to the 2013 year, a total number of 55 *formal complaints (FC)* were received, a total number of 470 *poll complaints (PC)* were received and 21 *finished product complaints (FPC)* were received. Finally, a total number of 546 complaints were received. Observed by years most complaints were received during 2007 year (184), then during 2012 year (82), and so on. Interesting information that can be observed, for a given period, refers to the significant decrease in patient's visits in 2013 year and an increase in investment in the same year. These investments here include not only the implementation of new technologies, investments in continuing education, work environment restoration but also a large amount of investment for construction interventions realization that was supposed to facilitate access to departments for people with disabilities. This fact is particularly taken into account in further analysis by referring to the data in Tables 4, 5, 6, 7 and 8.

However, the absolute number of complaints in one year does not say much and comparison of absolute numbers per year can be misleading about the customers satisfaction level. This means that the interpretation of the number of complaints in a given period should be approached cautiously.

A small number of complaints does not necessarily mean a small number of dissatisfied customers. These customers have decided to have their complaints heard and to be sure that something will be done about it. Most people do not complain, and therefore those who complain should be approached very seriously.

In the analyzed period, the number of patients/customers visits changed. That is why, it is necessary to calculate the relative indicator so that the correct conclusion, about the customer satisfaction level, based on the number of complaints could be made. The relative indicator is calculated as the complaints number per 10 000 patients/customers visits. It can be concluded that the customers satisfaction level on this indicator was the worst in the 2007 year, when it was received the largest number of complaints, because that year relative indicator of customer satisfaction was 9,37 complaints per 10 000 patients/customers visits. The customers satisfaction level was lowest in 2006 year when it was recorded the smallest number of complaints during that year, which corresponds to the value of relative indicator of 0,05. This result was expected due to the fact that in 2006 Dental Clinic Zagreb began with the introduction of a quality management system and its real swing was experienced is 2007 year. This could be noticed through the seriousness in collecting data.

COMPLAINTS AND THEIR CAUSES

Analysis of the nature of complaints is important to identify the causes of the problem and eliminate them. Reviewing the literature there is a "countless" kind of complaints, in fact as much as one can imagine different situations and events in patient contact with the health service. For the purpose of analysis different types of complaints were brought together and thus they were categorized. From results and analysis of customer satisfaction in secondary health care service some categories and types of complaints causes were divided in several groups [2]: (i) process control (organization, planning, etc.), (ii) the human factor, (iii) equipment, (iv) building (facilities) and (v) other, Table 2.

Table 2. The causes of complaints in the period from year 2006 to year 2013 (total number of all complaints). Source: E. Krstić Vukelja research (Management review's records from 2006-2013).

Year	Process Guidance (%)	Human Factor (%)	Equipment (%)	Building (%)	Other (%)	Total (%)
2006.	40	20		0	40	100
2007.	3,26	11,41		28,8	56,52	100
2008.	5,06	13,92		37,97	43,04	100
2009.	7,5	10		35	47,5	100
2010.	6,57	19,73		30,26	43,42	100
2011.	16,67	40		10	33,33	100
2012.	9,75	28,05		29,26	32,92	100
2013.	4	6		28	62	100
Total:	**6,41**	**16,48**	**0**	**29,48**	**47,61**	**100**

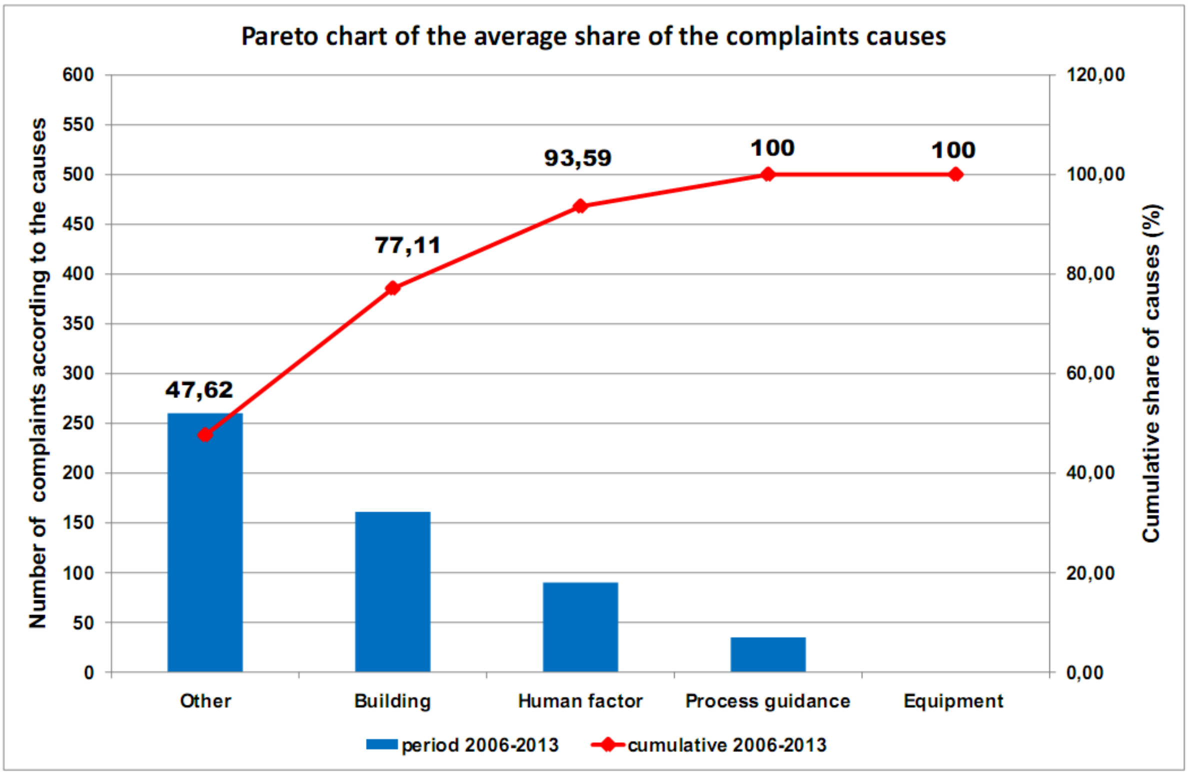


Figure 1. Pareto chart of the average share of the complaints causes. Source: E. Krstić Vukelja research (Management review's records from 2006-2013).

From the cause complaints analysis it can be seen in which direction to take improvement actions. However, it could be noted when analyzing total sum of all of complaints it might be difficult to distinguish the most influential cause of the problem. Viewed separately for each source of complaints it might be easier to reach certain conclusions about implementation of corrective actions.

It can be concluded that in the analyzed period, most of complaints were caused by the concept of "other" 47,62 % (such as: long waiting lists, information about the canceled receipt was not provided, failure to comply with the agreed time of receipt, fear of the doctors, insufficient number of employees, survey questionnaires too long, receive out of turn, no e-mail communication with the doctor, unclear division of numbers etc). The quality of building (such as: sufficient number of seats, neatness, air conditioning, good directions) causes on average 29,5 % of the total number of complaints. Human factor (lack of courtesy, professionalism, communication, for questions often received vague or even rough and arrogant answers, attitudes and behavior of staff etc.) has made an average of 16,5 % complaints during the provision of services. The quality of process guidance (poor maintenance of medical documentation, lack of substitute physician, poor planning and implementation of process, poor diagnostic tests etc.) has caused an average of 6,4 % complaints, and finally the quality of equipment caused none complaint.

Pareto diagram shows which causes are generating the most problems. For this particular example it can be conclude that the concept of "other", building quality and human factor constitute approximately 93,59 % of all causes of complaints. This means that in designing the program of corrective actions and improvements it is necessary to emphasize those measures and activities that will improve the quality control of the main and additional processes and competence of employees. Any change for the better in these two segments will significantly contribute to improving the quality of services, and reducing the number of

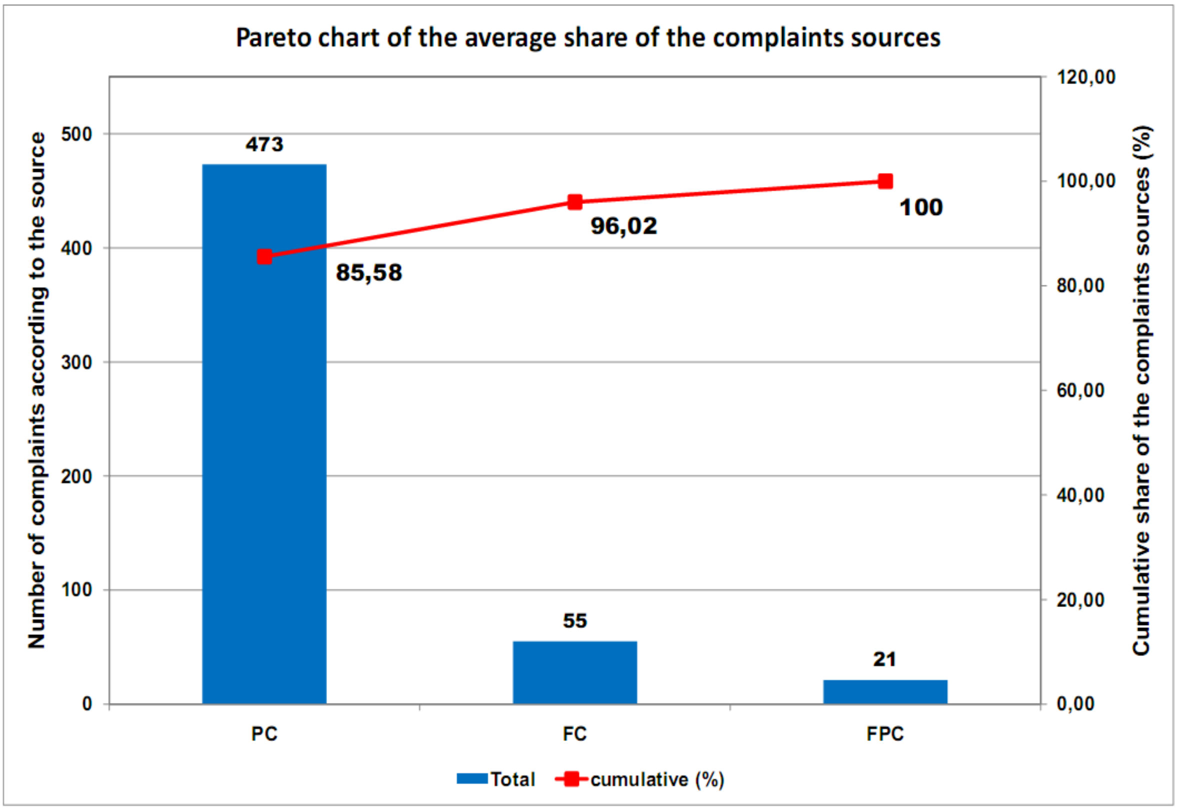


Figure 2. Pareto chart of the average share of the complaints sources. Source: E. Krstić Vukelja research (Management review's records from 2006-2013).

complaints. Other causes, in this case: process guidance and equipment, constitute an important minority of causes of complaints and will certainly not be a priority in solving problems related to the quality services evaluation measured by the number of complaints as an indicator.

It can be concluded that in the analyzed period, the largest source of complaints was from survey questionnaire (*poll complaints* – PP) 85,58 %. The second source according to the share of prevalence was written complaints (*formal complaints* – FC) 10,44 %, and finally the last was from the *finished product complaint records* – FPC 3,98 %. This result was expected because the survey questionnaires were anonymous, and customers have full freedom to express whatever is troubling them without any fear of being recognized. The common characteristics for both analysis was that the biggest negative comments were placed on account of poor communication – the human factor.

This can be confirmed by another analysis that was conducted in the period 2007-2009, which dealt with customer satisfaction with the quality of health services, provided useful conclusions about what is really essential to customer. The analysis of that survey resulted with the conclusion of two groups of patients/customers. The first group, which comes to the medical treatment during a long period of time, and another group that comes to the health care institution for the first time. This is most easily shown by comparing observations of patients who has come for the first time and those who are regular users of health care services, Figure 3.

In other words, the group of patients who has experienced for the first time specific health care service was pleasantly surprised and thrilled with the first impression of the Clinic (neatness, equipment, courtesy of medical staff, well checked specialist departments), while patients who has experienced health care service for a long period of time has given greater importance to human factors (empathy, patient well-being as top priority, ensuring various forms of care to the patient etc.).

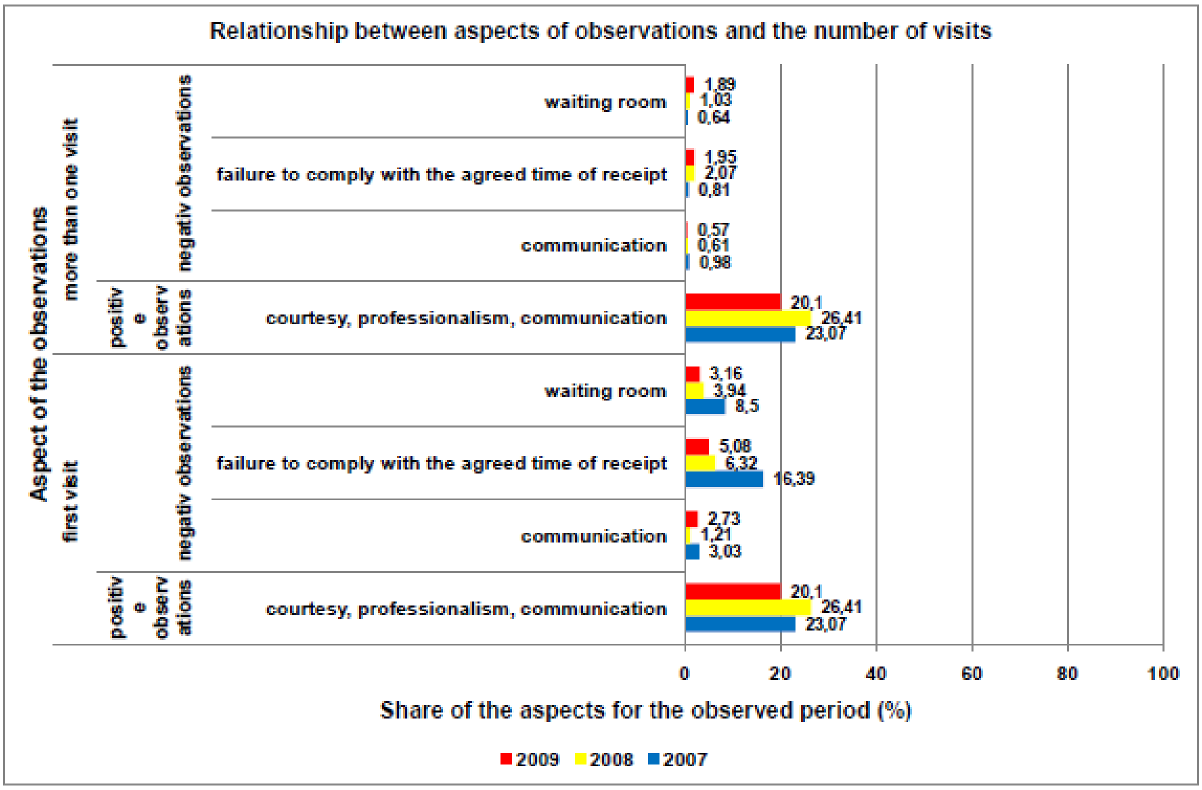


Figure 3. Relationship between aspects of observations and the number of visits. Source: E. Krstić Vukelja, B. Klaić, M. Vukelja, I. Duplančić: *Indicator of patient satisfaction – a model for managing, monitoring, evaluating and improving the quality of health care services*, UDK 616.31:658.562 : 10th Croatian Conference on Quality and 1st Scientific Symposium , May, 10 to 12, 2010th, Sibenik.

So, the more the customers came for health treatment the more they emphasized importance of communication, clarity of instructions, information about treatment, information after with the overall experience of health care quality. From this standpoint, the most important factors for patients were kindness, communication of health and non-health professionals.

Experience has shown that communication problems lie in the background of most complaints. When more experienced professionals were asked to provide a list of the factors that most contribute to raising the complaint, at first place they pu the poor communication between staff, poor communication with customers and poor maintenance of professional documentation. Good staff training, in terms of skills to answer the questions and concerns of customers, is the key for successful prevention. Emphasis is placed on understanding the process of complaints and the need for effective communication, observing the problem from the perspective of the customer and dealing with "difficult" customer. So, could the complaints be prevented? Not all. However, the complaint can be reduced to a minimum if clear and complete information is provided to customers, if they are involved in decisions about which service to choose, if they are fully informed about the service and if they are treated with due respect.

THE SCRAP

From the total number of complaints one part were related to services that are categorized as “scrap”, meaning that they were non-compliant requirements and expectations of the customer. For service it is not easy to determine what is scrap, and what it is not [4]. When the mistake is made during the service, sometimes it is possible to correct or mitigate the

Table 3. The share of the scrap in the total number of complaints in the period from year 2006 to year 2013. Source: E. Krstić Vukelja research (Management review's records from 2006-2013).

Year	The Scrap (%)	Fixable (%)	Total (%)
2006.	20	80	100
2007.	0	100	100
2008.	4	96	100
2009.	5	95	100
2010.	4	96	100
2011.	0	100	100
2012.	0	100	100
2013.	0	100	100
Average	**2**	**98**	**100**

consequences of its non-compliance in the first attempt (repeated action, efforts to mitigate the consequences, to apologize, to provide a service which partially fulfills the requirements, etc.). If all this fails, scrap service occurs.

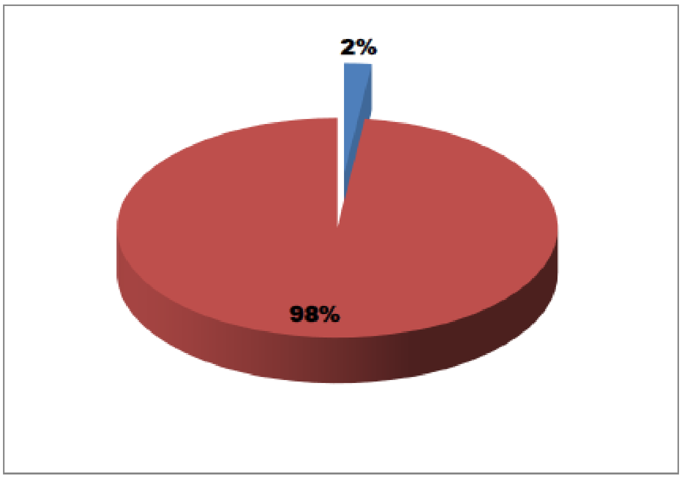
From the total number of complaints in the study period, 2 % or total number of 9 complaints have a character of the scrap, which means that provided services were completely incompatible and have implications at the customer satisfaction level. With such a service, the customer cannot be satisfied because it cannot meet his demands. This service requires certain actions in terms of correcting errors or mitigate its effects, and requires the involvement of specific resources (people, time, information, equipment, etc.).

Such actions may include:

- repetition of actions at no additional charge (generating costs due to the (non) quality),
- communication with the customer in order to clarify, apologies and inform,
- making concessions in price, to mitigate the consequences (costs due to (non) quality),
- payment of compensation (costs due to (non) quality) [3, 4].

Engaging the above resources through the implementation of corrective action causes costs due to the (non) quality. The amount of these costs can be relatively accurately determined.

Figure 4. The share of the scrap in the total number of complaints in the period 2006-2013 (scrap: 2 %, fixable: 98 %). Source: E. Krstić Vukelja research (Management review's records from 2006-2013).



CORRELATION OF NUMBER OF COMPLAINTS AND VISITS OF PATIENTS/CUSTOMERS

When talking about the number of complaints, for a thorough analysis it is not sufficient to determine only a relative indicator, it is essential to determine the degree of positive correlation between, on one side, the number of complaints in one year and some other selected size, for example, the number of patients, the amount of investments (infrastructure, education, new technologies, etc.). In order to establish the existence and intensity of positive correlation the Spearman's rank correlation coefficient is used. By determining the strength of the connection between the number of complaints and the number of patients/customers visits on one hand, and the number of complaints and the amount of investments, on the other hand, it will be determined which of these connections has a stronger intensity, and impact on the number of complaints.

Table 4. Calculation's coefficient of rank correlation for the data in Table 1 (Total number of complaints, formal complaints, poll complaints, finished product complaints and the number of patient's visits). Source: authors' research.

Year	Number of complaints				Number of patient visits	Rank					Deviation (X_{ri} . Y_{ri})							
	X_i				Y_i	X_{ri}				Y_{ri}	d_i				di^2			
1	*2*				*3*	*4*				*5*	*6*				*7*			
	FC	*PC*	*FPC*	*Total*		*FC*	*PC*	*FPC*	*Total*		*FC*	*PC*	*FPC*	*Total*	*FC*	*PC*	*FPC*	*Total*
2006	5		0	**5**	208287	6	8	8	8	2	4	6	6	6	16	36	36	36
2007	2	177	5	**184**	196402	8	1	1	1	5	3	-4	-4	-4	9	16	16	16
2008	5	71	3	**79**	205546	6	2	3	3	3	3	-1	0	0	9	1	0	0
2009	7	31	2	**40**	231352	4	6	5	6	1	3	5	4	5	9	25	16	25
2010	9	65	2	**76**	196590	3	4	5	4	4	-1	0	1	0	1	0	1	0
2011	11	16	3	**30**	179676	1	7	3	7	6	-5	1	-3	1	25	1	9	1
2012	6	71	5	**82**	176226	5	2	1	2	7	-2	-5	-6	-5	4	25	36	25
2013	10	39	1	**50**	100444	2	5	7	5	8	-6	-3	-1	-3	36	9	1	9
Total:	**55**	**470**	**21**	**546**	1,494,523										**109**	**113**	**115**	**112**

The empirical value of the Spearman rank correlation coefficient between the total number of complaints and the number of visits is *r*' = –1,0000. It is a bond of total correlation, which suggests that with the increase of number of patient visits there are grate possibility for increase of complaints during the year. The standard deviation in the total number of complaints is σ = 12,76994, which means that the average deviation from the average is about 12 complaints.

The same conclusion could be made when regarding separately different sources of complaints. The empirical value of the Spearman rank correlation coefficient between the number of *formal complaints* and the number of visits is *r*' = –0,9464, it is a bond of strong correlation. The empirical value of the Spearman rank correlation coefficient between the number of *poll complaints* and the number of visits is *r*' = –1,0179, it is a bond of strong correlation. The empirical value of the Spearman rank correlation coefficient between the number of *finished product complaints* and the number of visits is *r*' = –1,0536, it is a bond of strong correlation.

PHENOMENON OF “UNREALISTIC EXPECTATIONS”

The empirical value of the Spearman rank correlation coefficient between the total number of complaints and the amount of investment in quality is *r*’ = –0,6071. This is a moderate to good correlation of these values and means that the number of complaints depends on the amount of investment.

Table 5. Calculation’s coefficient of rank correlation for the data in Table 1 (Total number of complaints, formal complaints, poll complaints, finished product complaints and the amount of investments). Source: authors’ research.

Year	Number of complaints				Number of patient visits	Rank					Deviation (X_{ri} - Y_{ri})							
	X_i				Y_i	X_{ri}				Y_{ri}	d_i				di^2			
1	2				3	4				5	6				7			
	FC	PC	FPC	Total		FC	PC	FPC	Total		FC	PC	FPC	Total	FC	PC	FPC	Total
2006	5		0	**5**	208287	6	8	8	8	2	4	6	6	6	16	36	36	36
2007	2	177	5	**184**	196402	8	1	1	1	5	3	-4	-4	-4	9	16	16	16
2008	5	71	3	**79**	205546	6	2	3	3	3	3	-1	0	0	9	1	0	0
2009	7	31	2	**40**	231352	4	6	5	6	1	3	5	4	5	9	25	16	25
2010	9	65	2	**76**	196590	3	4	5	4	4	-1	0	1	0	1	0	1	0
2011	11	16	3	**30**	179676	1	7	3	7	6	-5	1	-3	1	25	1	9	1
2012	6	71	5	**82**	176226	5	2	1	2	7	-2	-5	-6	-5	4	25	36	25
2013	10	39	1	**50**	100444	2	5	7	5	8	-6	-3	-1	-3	36	9	1	9
Total:	**55**	**470**	**21**	**546**	1,494,523										**109**	**113**	**115**	**112**

The empirical value of the Spearman rank correlation coefficient between the total number of complaints and the amount of investments is *r*’ = –0.6071. It is a bond of moderate to good correlation, which suggests that the influence of other factors that have significant impact on complaints should be investigated.. The standard deviation in the total number of complaints is σ = 12.76994, which means that the average deviation from the average is about 12 complaints.

The same conclusion could be made when regarding separately different sources of complaints.

The empirical value of the Spearman rank correlation coefficient between the number of *formal complaints* and the amount of investments is *r*’ = –1,1250, it is a bond of total correlation. The empirical value of the Spearman rank correlation coefficient between the number of *poll complaints* and the amount of investments is *r*’ = –0,6071, it is a bond of medium strong correlation. The empirical value of the Spearman rank correlation coefficient between the number of *finished product complaints* and the amount of investments is *r*’ =-0,7321, it is a bond of strong correlation.

At first glance it seems illogical that with the increase of the amount of investments the number of complaints increases too. It is expected that the increase in the quality of infrastructure, people, etc. should result in higher levels of service quality and result in fewer complaints.

However, this positive correlation makes sense. The fact is that an increase in investment in infrastructure, in the people (education) or in the development of new services, increases the range of facilities and services provided to customers (renovation of the waiting room environment, renovation of the doctor's office, implementation of new technologies, easier access for the invalid people, and many others), and increase the possibility of a large number dissatisfied patients with some of the additional facilities or services, on the one hand.

It is the phenomenon of "unrealistic expectations" [5]. Lately expectations of customers are strongly increased faster than the capabilities of different service providers to please them. Although, today's customers, due to the investments in new materials and new processes, have more than ever highly efficient and highly effective treatment. While most customers are satisfied with the service/product they receive, still there are many who have bad experiences in contact with the specific service provider, and a minority of them complaint. Thus, the increase in the number of complaints does not necessarily mean that there has been the deterioration in the quality of services provided. Simply, this number may be a result of rising customer expectations. Reporting and analysis of complaints in combination with other information and indicators can be used in assessing the performance and quality of some services.

The fact that the organization has a system for the collection and analysis of complaints represents some progress in quality service evaluation through the system of complaints. But this is not enough. This evaluation should be combined with other forms of quality service evaluation in the order to create information base for decisions to improve the level of quality.

QUALITY SERVICE EVALUATION THROUGH THE PRAISE SYSTEM

Praise is also an indicator of quality service level. For their collection and analysis the same system and instrumentation can be used [2]. Sources of collecting praise are the same as for collecting complaints, with one exception there are no praise in case of finished product record. From period to period it is necessary to use the same methodology to ensure comparability, just to make review about the increase or decrease of praise and its influence on overall customer satisfaction level with present quality service.

Table 6. Number of praise on service in the period 2006 – 2013 (*Total number of praises* – T, *formal praises* – FP, *poll praises* – PP and the number of patient's visits and the amount of investments).

Year	Number of praises			Number of patient's visits	Amount of investments (kn)	Number of praises per 10 000 patient		
	FP	PP	T			FP	PP	T
2006	0	0	0	208287	3.660.000	0	0	0
2007	19	195	214	196402	3.394.000	0,97	9,93	10,89
2008	9	180	189	205546	2.984.816	0,44	8,76	9,19
2009	4	86	90	231352	1.913.726	0,17	3,72	3,89
2010	0	107	107	196590	2.555.000	0	5,44	5,44
2011	0	32	32	179676	2.737.890	0	1,78	1,78
2012	4	72	76	176226	1.950.000	0,23	4,08	4,31
2013	4	118	122	100444	2.250.000	0,39	11,75	12,15
Total:	**40**	**790**	**830**	**1.494.523**	**21.445.432**	**0,27**	**5,28**	**5,55**

As the complaints number, absolute number of praise does not show much, so the relative praise number was calculated. In this case it was the number of praises per 10 000 patients. Thus, for example, in 2007 there were a total number of 214 compliments, which is more than in 2013 year (122). But, the relative number of praise per 10 000 patient was greater in 2013 year (12.15), than in 2007 year (10.89). Based on data collected on the praise, it is possible to conduct analysis of their structure according to a source of recording (formal praise through the book of praise, praise recorded through the polls) and according to the interested group (patient, patient's relatives, visitors, external interested groups), and by cause (process, infrastructure, general impression, kindness, and human factors, etc.)

Table 7. Calculation's coefficient of rank correlation for the data in Table 6. Symbols have the same meaning as in Table 6. Source: authors' research.

Year	Number of praises			Number of patient's visits	Rank				Deviation $(X_{ri} - Y_{ri})$					
	X_i			Y_i	X_{ri}			Y_{ri}	d_i			di^2		
1	2			3	4			5	6			7		
	FP	PP	Total		FP	PP	Total		FP	PP	Total	FP	PP	Total
2006	0	0	0	208287	6	8	8	2	4	6	6	16	36	36
2007	19	195	214	196402	1	1	1	5	-4	-4	-4	16	16	16
2008	9	180	189	205546	2	2	2	3	-1	-1	-1	1	1	1
2009	4	86	90	231352	3	5	5	1	2	4	4	4	16	16
2010	0	107	107	196590	6	4	4	4	2	0	0	4	0	0
2011	0	32	32	179676	6	7	7	6	0	1	1	0	1	1
2012	4	72	76	176226	3	6	6	7	-4	-1	-1	16	1	1
2013	4	118	122	100444	3	3	3	8	-5	-5	-5	25	25	25
Total:	**40**	**790**	**830**	**1,494,523**								**82**	**96**	**96**

By determining the level of correlation calculating the Spearman rank correlation coefficient between the total number of praises per year and the number of patient's per year, the value of *r*' = –0,7143 is obtained. It is therefore related as strong correlation, but with a negative sign, which means that the observed influence values are of moving in different directions. The Spearman rank correlation coefficient between the number of formal praises per year and the number of patient's per year is *r*' = –0,4643, it is relatively poor correlation. And, finally the Spearman rank correlation coefficient between the number of poll praises per year and the number of patient's per year is *r*' = –0,7143, and it is related as strong correlation. The standard deviation in the number of praise is σ = 72,3459, which means that the average deviation from the average is about 72 praises.

Table 8. Calculation's coefficient of rank correlation for the data in Table 6. Symbols have the same meaning as in Table 6. Source: authors' research.

Year	Number of praises			Number of investments (kn)	Rank				Deviation $(X_{ri} - Y_{ri})$					
	X_i			Y_i	X_{ri}			Y_{ri}	d_i			di^2		
1	2			3	4			5	6			7		
	FP	PP	Total		FP	PP	Total		FP	PP	Total	FP	PP	Total
2006	0	0	0	3.660.000	6	8	8	1	5	7	7	25	49	49
2007	19	195	214	3.394.000	1	1	1	2	-1	-1	-1	1	1	1
2008	9	180	189	2.984.816	2	2	2	3	-1	-1	-1	1	1	1
2009	4	86	90	1.913.726	3	5	5	8	-5	-3	-3	25	9	9
2010	0	107	107	2.555.000	6	4	4	5	1	-1	-1	1	1	1
2011	0	32	32	2.737.890	6	7	7	4	2	3	3	4	9	9
2012	4	72	76	1.950.000	3	6	6	7	-4	-1	-1	16	1	1
2013	4	118	122	2.250.000	3	3	3	6	-3	-3	-3	9	9	9
Total:	**40**	**790**	**830**	**21.445.432**								**82**	**80**	**80**

By determining the level of correlation calculating the Spearman rank correlation coefficient between the total number of praises per year and the amount of investments per year, the value of *r*' = –0,4286 is obtained. It is therefore related as poor correlation. The Spearman rank correlation coefficient between the number of formal praises per year and the amount of investments per year is *r*' = –0,4643, it is relatively poor correlation; and, finally the Spearman rank correlation coefficient between the number of poll praises per year and the amount of investments per year is *r*' = –0,4286, and it is related as relatively poor correlation.

The reasons for this may be several. One can be explained with the previously introduced phenomenon of unrealistic expectations. In time, customers get used to the new services and to a higher level of quality of these services. Thus, meaning that increase of quality service level is considered normal and not extraordinary event. Therefore, an increase in number of patients does not necessarily mean an increase in the absolute number of praises. By calculating the rank correlation of relative number of praise and the number of patient's visits the value of $r' = -1{,}2143$ is obtained, which means that there is total correlation between these two values. The Spearman's rank correlation coefficient for the relative number of compliments and the amount of investment is $r' = -0{,}6429$. This is a medium strong correlation and it may be interesting for further analysis.

CONCLUSION

There are different ways of evaluating the quality service. One way is by the analysis of customer complaints and praises. This analysis provides valuable information on quality of products/services based on customer opinion through their dissatisfaction or satisfaction. The process of collecting, processing and analyzing of complaints clearly define the sources of complaints and their causes, thus generating the best decisions about the actions for identification and elimination of the causes of the complaint and thus improving the quality service.

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OCJENA KVALITETE USLUGE KROZ SUSTAV ŽALBI I POHVALA

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SAŽETAK

Reklamacija je, kao izraz nezadovoljstva korisnika kvalitetom proizvoda ili usluge, vrlo dragocjena informacija. Dobro izgrađen sustav prikupljanja, obrade i analize reklamacija omogućuje organizaciji stvaranje informacijske osnovice za donošenje poslovnih odluka na temelju činjenica. Ta informacijska osnovica omogućuje učinkovito donošenje i provedbu mjera za kontinuirano poboljšanje kvalitete proizvoda/usluge. Da bi sustav bio učinkovit, potrebno je u kontinuitetu koristiti istu metodologiju prikupljanja i obrade reklamacija radi mogućnosti stalne usporedbe iz razdoblja u razdoblje. Veće investicije u kvalitetu proizvoda/usluge ne znače istovremeno smanjenje broja reklamacija zbog djelovanja fenomena "nerealnog očekivanja". Osim reklamacija, vrijedan izvor informacija o zadovoljstvu korisnika usluge/proizvoda jest i sustav pohvala.

KLJUČNE RIJEČI

žalba, kvaliteta, zadovoljstvo korisnika, fenomen nerealnog očekivanja, škart, pohvala

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EXISTENCE OF SUPER CHAOTIC ATTRACTORS IN A GENERAL PIECEWISE SMOOTH MAP OF THE PLANE

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ABSTRACT

In this paper we give some rigorous conditions for the occurrence of super chaotic attractors in a general piecewise smooth map of the plane.

KEY WORDS

piecewise smooth map, super chaotic attractor, rigorous proof of chaos, Lyapunov exponents

CLASSIFICATION

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INTRODUCTION

There are many works that focused on the topic of chaotic behaviours of a discrete mapping. For example it has been studied from a control and anticontrol (chaotification) schemes or from the use of Lyapunov exponents [1-4], or by the use of several modified versions of the Marotto theorem [5], to prove the existence of chaos in n-dimensional dynamical discrete system, where the results in some way are to making an originally non-chaotic dynamical system chaotic, or enhancing the existing chaos of a chaotic system.

Robust chaos is defined by the absence of periodic windows and coexisting attractors in some neighbourhood of the parameter space, since the existence of these windows in some chaotic regions imply that with small changes of the parameters would destroy the chaotic behaviour. This effect implies the fragility of this type of chaos. Contrary to this situation, there are many practical applications as in communication and spreading the spectrum of switch-mode power supplies to avoid electromagnetic interference [2-9], where it is necessary to obtain reliable operation in the chaotic mode where the robustness of chaos is required. A practical example can be found from electrical engineering to demonstrate robust chaos as shown in [6]. If all Lyapunov exponents are positive throughout the range, then the resulting attractors are called super-chaotic attractors. The importance of these attractors is that are more non-regular, and the iteration points are seemingly “almost” full of the considered space, which explains one of applications of chaos in fluid mixing, for example, refer to [7, 8]. A super-chaotification (or hyper-chaotification) scheme by making all Lyapunov exponents of a controlled dynamical system positive via the controller of some simple triangular function is given in [2].

In this paper, we shall determine rigorously a range in the parameters space for the existence of super chaotic attractors in a general 2-dimensional piecewise smooth discrete mapping, using an equivalence relation between the two matrices of the corresponding normal form of the considered map, and hence we compute analytically and assure the positivity of all the Lyapunov exponents. Since many practical applications require piecewise smooth map under discrete modeling [1, 6, 10], we consider a general two-dimensional piecewise smooth map $f(x, y; \rho)$, which depends on a single parameter ρ. Let Γ_ρ, given by $x = h(y, \rho)$ denotes a smooth curve that divides the phase plane into two regions R_1 and R_2. The map is given by:

$$f(x, y; \rho) = \begin{cases} f_1(x, y; \rho), & \text{if } x, y \in R_1, \\ f_2(x, y; \rho), & \text{if } x, y \in R_2. \end{cases} \quad (1)$$

For obtaining the general conditions of occurrence of super-chaotic attractors, it is assumed that the functions f_1 and f_2 are both continuous and have continuous derivatives. The map f is continuous but its derivative is discontinuous at the line Γ_ρ, called the “border”. It is further assumed that the one-sided partial derivatives at the border are finite, and in each subregions R_1 and R_2 the map (1) has one fixed point P_1 and P_2, for certain value ρ_* of the parameter ρ. A normal form theory for border-collision bifurcations of two-dimensional piecewise smooth maps has been developed in [10]. Obviously, it has been shown that it is possible to choose an appropriate coordinate transformation so that the choice of axis is independent of the parameter. In so doing, the normal form of the map (1) is given by:

$$N(x, y) = \begin{cases} \begin{pmatrix} \tau_1 & 1 \\ -\delta_1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\mu, & \text{if } x < 0, \\ \begin{pmatrix} \tau_2 & 1 \\ -\delta_2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\mu, & \text{if } x > 0. \end{cases} \quad (2)$$

where μ is a parameter and τ_i, δ_i, i = 1, 2 are the traces and determinants of the corresponding matrices of the linearised map in the two subregions R_1 and R_2 evaluated at P_1 and P_2, respectively.

Assume that:

$$\tau_1 > 1 + \delta_1, \text{ and } \tau_2 < -(1 + \delta_2). \tag{3}$$

Then there are no fixed points for the map (1) when $\mu < 0$, and there are altogether two fixed points, one in R_1 and the other in R_2, for $\mu > 0$, given by:

$$P_1 = \left(\frac{\mu}{1-\tau_1+\delta_1}, \frac{-\mu\delta_1}{1-\tau_1+\delta_1}\right), \quad \left(\frac{\mu}{1-\tau_2+\delta_2}, \frac{-\mu\delta_2}{1-\tau_2+\delta_2}\right). \tag{4}$$

Since as the parameter μ is varied through zero, the local bifurcation of the map (1) depends only on the values of τ_i and δ_i, (i = 1, 2), then it suffices to study the bifurcations in the normal form (2), and it is shown in [6] that the map (1) has a robust chaotic attractor for some parameter space region, when in first the condition (3) is verified. On the other hand, it is also shown in [6] that there is periodic attractor in the piecewise smooth map of the form (2) when $\tau_1 > 1 + \delta_1$, and $-(1 + \delta_2) < \tau_2 < (1 + \delta_2)$, thus, we exclude this region from our analysis when looking for super chaotic behaviour.

MAIN RESULT

In this section, a rigorous proof for the occurrence of super chaotic attractors in the piecewise smooth map (1) is given, using an equivalence relation between the two matrices of the corresponding normal form (2), and hence we compute analytically all the Lyapunov exponents. The existence of fixed points P_1 and P_2 given in (4) imply that it is possible to write the map (1) under the normal form (2). Therefore, assume that

$$\delta_1 < \frac{\tau_1^2}{4} \text{ and } \delta_2 < \frac{\tau_2^2}{4}. \tag{5}$$

Then $\tau_1^2 - 4\delta_1 > 0$ and $\tau_2^2 - 4\delta_2 > 0$ and these inequalities imply that the eigenvalues in R_1 are $\lambda_{11} = (\tau_1 + \sqrt{\tau_1^2 - 4\delta_1})/2$, $\lambda_{12} = (\tau_1 - \sqrt{\tau_1^2 - 4\delta_1})/2$, while in R_2 the eigenvalues are $\lambda_{21} = (\tau_2 + \sqrt{\tau_2^2 - 4\delta_2})/2$, $\lambda_{12} = (\tau_2 - \sqrt{\tau_2^2 - 4\delta_2})/2$. On the other hand, there are many ways for realizing an equivalence between matrices A_1 and $A2$, with

$$A_1 = \begin{pmatrix} \tau_1 & 1 \\ -\delta_1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} \tau_2 & 1 \\ -\delta_2 & 0 \end{pmatrix}.$$

A simple way is to assume that their eigenvalues are equal. Suppose for example that $\lambda_{11} = \lambda_{21}$ and $\lambda_{12} = \lambda_{22}$, thus we obtain the following condition

$$2(\delta_1 + \delta_2) - \tau_1\tau_2 = -\sqrt{\tau_1^2 - 4\delta_1} \cdot \sqrt{\tau_2^2 - 4\delta_2}. \tag{6}$$

Hence equation (6) has a solution if $2(\delta_1+\delta_2) - \tau_1\tau_2 < 0$, or

$$\delta_2 < \frac{\tau_1\tau_2 - 2\delta_1}{2}, \tag{7}$$

so that equation (6) becomes

$$\delta_2^2 + (-\tau_1\tau_2 - 2\delta_1 + \tau_1^2)\delta_2 + \tau_2^2\delta_1 + \delta_1^2 - \tau_1\tau_2\delta_1 = 0, \tag{8}$$

with the discriminant $\Delta = (\tau_2 - \tau_1)^2(\tau_1^2 - 4\delta_1) > 0$. Since $(\tau_1^2 - 4\delta_1) > 0$, solutions of (8) with respect to δ_2 are

$$\begin{aligned} \delta_2^{(1)} &= \frac{\tau_1\tau_2 + 2\delta_1 - \tau_1^2 + \sqrt{(\tau_2-\tau_1)^2(\tau_1^2-4\delta_1)}}{2}, \\ \delta_2^{(2)} &= \frac{\tau_1\tau_2 + 2\delta_1 - \tau_1^2 - \sqrt{(\tau_2-\tau_1)^2(\tau_1^2-4\delta_1)}}{2}. \end{aligned} \tag{9}$$

For brevity, we consider only the case $\delta_2 = \delta_2^{(1)}$, the case $\delta_2 = \delta_2^{(2)}$ being quite similar.

Condition (7) with $\delta_2 = \delta_2^{(1)}$ gives the following inequality

$$\delta_1 < \frac{2\tau_1\tau_2 - 4\tau_2^2}{2}, \tag{10}$$

while condition (5) with $\delta_2 = \delta_2^{(1)}$ leads to

$$\sqrt{(\tau_2-\tau_1)^2(\tau_1^2-4\delta_1)} < -\left(2\delta_1 + \tau_1\tau_2 - \tau_1^2 - \frac{\tau_2^2}{2}\right), \tag{11}$$

with the additional condition

$$\delta_1 < \frac{-\tau_1\tau_2 + \tau_1^2 + \frac{\tau_2^2}{2}}{2}, \tag{12}$$

so that solution of the inequality (11) is possible. It is easy to verify that the inequality (11) still holds for all values of τ_1, τ_2, and δ_1, thus we consider only the condition (12) for this case.

Condition (3) with $\delta_2 = \delta_2^{(1)}$ gives

$$\delta_1 < \frac{\tau_1^2 - \tau_1\tau_2 - 2\tau_2 - 2}{2}. \tag{13}$$

Thus inequalities (3), (10), (12) and (13) imply that

$$\delta_1 < \min\left(\frac{2\tau_1\tau_2 - \tau_2^2}{4}, \frac{-\tau_1\tau_2 + \tau_1^2 + \frac{\tau_2^2}{2}}{2}, \frac{\tau_1^2 - \tau_1\tau_2 - 2\tau_2 - 2}{2}, \tau_1 - 1\right). \tag{14}$$

We remark that

$$\frac{2\tau_1\tau_2 - \tau_2^2}{4} - \frac{-\tau_1\tau_2 + \tau_1^2 + \frac{\tau_2^2}{2}}{2} = -\frac{(\tau_2-\tau_1)^2}{2} < 0.$$

because of what the condition (14) transforms into

$$\delta_1 < \min\left(\frac{2\tau_1\tau_2 - \tau_2^2}{4}, \frac{\tau_1^2 - \tau_1\tau_2 - 2\tau_2 - 2}{2}, \tau_1 - 1\right). \tag{15}$$

Note that $\lambda_{11} = \lambda_{21} > 1$, if (3) holds. i.e., $\delta_1 < \tau_1 - 1$, and $\lambda_{12} = \lambda_{22} < -1$, if and only if:

$$\delta_1 < -(\tau_1 + 1). \tag{16}$$

Lyapunov exponents of the map (1) in the region R_1 are $\varpi_1(X_0) = \ln|\lambda_{11}|$, and $\varpi_2(X_0) = \ln|\lambda_{12}|$, and in the region R_2 the Lyapunov exponents are $\eta_1(X_0) = \ln|\lambda_{21}|$ and $\eta_2(X_0) = \ln|\lambda_{22}|$, for all $X_0 \in R_2$, thus, according to (3) and (16) we obtain:

$$\varpi_1(X_0) = \eta_1(X_0) = \ln|\lambda_{11}| > 0 \text{ and } \varpi_2(X_0) = \eta_2(X_0) = \ln|\lambda_{12}| > 0 \tag{17}$$

Finally, the Lyapunov exponents are identical in both regions R_1 and R_2, then the map (1) has a super chaotic attractor when conditions (9), (15) and (16) are verified.

The main result of this article is now given.

Theorem 1. Consider the piecewise smooth map (1) written in the normal form (2), and assume the following:

(1) $\delta_1 < \min\left(\frac{2\tau_1\tau_2 - \tau_2^2}{4}, \frac{\tau_1^2 - \tau_1\tau_2 - 2\tau_2 - 2}{2}, \tau_1 - 1, -\tau_1 - 1 \right)$.

(2) $\delta_2 = \frac{\tau_1\tau_2 + 2\delta_1 - \tau_1^2 + \sqrt{(\tau_2 - \tau_1)^2(\tau_1^2 - 4\delta_1)}}{2}$.

Then the map (1) converges to a super chaotic attractor.

ELEMENTARY EXAMPLE

Let us consider the following piecewise linear map of the plane:

$$f(x,y) = \begin{cases} \begin{pmatrix} 1 & a \\ -b & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 0{,}2 \end{pmatrix}, & \text{if } x < 0, \\ \begin{pmatrix} 1 & -a \\ b & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 0{,}2 \end{pmatrix}, & \text{if } x \geq 0. \end{cases} \tag{18}$$

It is easy to verify that the map (18) is a special case of the map (1) with all its assumptions given in this paper. Indeed, using the main theorem of this article we find that if $a = 1{,}2$ then a portion of the range for the occurrence of super chaotic attractor is $b < -1{,}6667$, and in this case the Lyapunov exponents are

$$\varpi_1(X_0) = \ln\left|\frac{1}{2}\sqrt{1-4{,}8b} + \frac{1}{2}\right| > 0, \text{ and } \varpi_2(X_0) = \ln\left|\frac{1}{2} - \frac{1}{2}\sqrt{1-4{,}8b}\right| > 0.$$

On the one hand, the super chaotic attractor and the bifurcation diagram of the map (18) are shown in Fig. 1a) and Fig. 1b), respectively. On the other hand, we remark that the given attractor is more non-regular, and the iteration points are seemingly "almost" full of the considered space, which explains an application of chaos in fluid mixing, for example, see [7, 8].

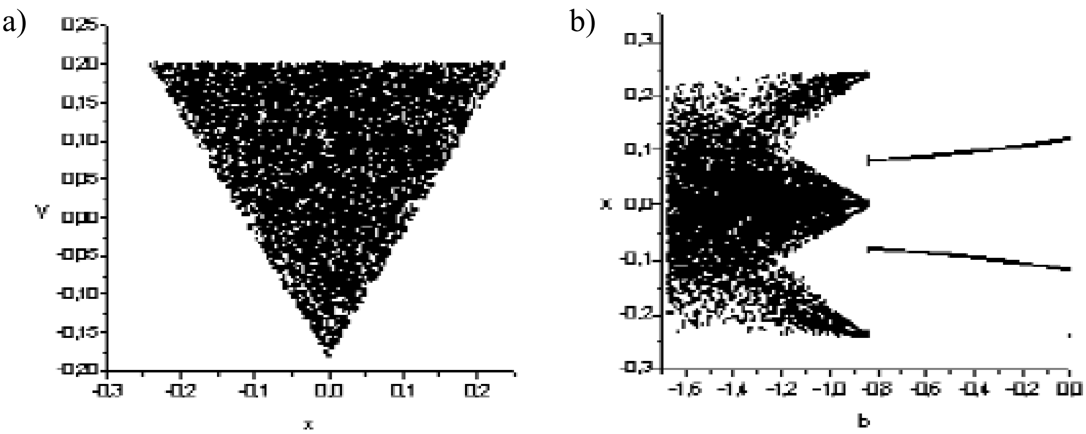

Figure 1. a) The super chaotic attractor obtained for $a = 1{,}2$, $b = -1{,}6668$, and the initial condition $x = y = 0{,}001$. b) The border collision bifurcation for the map (18) for $-1{,}66685 < b < 0$ and $a = 1{,}2$.

CONCLUSION

We have reported some analytical results on the existence of super chaotic attractors in a general piecewise smooth map of the plane. An elementary example is also given and discussed.

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EGZISTENCIJA SUPERKAOTIČNIH ATRAKTORA U PO DIJELOVIMA GLATKOJ RAVNINI

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Odsjek za matematiku – Sveučilište u Tebessi
Tebessa, Alžir

SAŽETAK

U radu su dani rigorozni uvjeti za pojavu superkaotičnih atraktora u po dijelovima glatkoj mapi ravnine.

KLJUČNE RIJEČI

po dijelovima glatka mapa, superkaotični atraktor, rigorozni dokaz kaosa, eksponent Lyapunova

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APPLYING SYSTEMS THINKING TO EXAMINE AND REDUCE DEPENDENCY ON FOOD BANKS

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ABSTRACT

Systems thinking is the art of understanding interconnections between various disciplines thereby unwinding the existing complexity. Most of the real world problems are complex, take for the example the increasing dependency rate on food banks. While various factors contribute towards it, not much has been done to bring the take off the number of dependents. By viewing this system from a holistic systems thinking lens, one explores the issue in depth. We realise the universally acceptable solution is not alleviating the problem in the long run. By applying systems thinking principles several hidden factors are brought to attention and subsequently can be dealt with more aptly. A movement that transcends disciplines results in delivering better solutions.

KEY WORDS

systems thinking, food bank, design thinking, social engineering

CLASSIFICATION

JEL: H31, I31

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INTRODUCTION

Systems thinking entails envisioning real life scenarios through a broad perspective while defying the laws of being systematic [1, 2]. It being a relatively new field of interdisciplinary studies, tries to mend the explanatory gap between the various fields of science, engineering and management. The concept might seem complex at first, however it owes its complexity due to admission of several basic concepts, all viewed under a single systems lens. To understand it better, let us take a real life scenario of traffic congestion, as that helps us best view the need and application for systems thinking [1]. While the transport department will view this situation as a need to increase or decrease automobiles, a city planning commission tackles the problem by providing more efficient customer friendly alternatives. The same traffic congestion would pose a different hazard to construction companies, environmental societies and public health institutions each viewing it differently. While each discipline can express its viewpoint and strategic solution to help solve the problem, for effective planning one needs to integrate these discipline specific viewpoints in order to co-design solutions. Needless to say, working towards a definitive goal across disciplines is not an easy task, especially when the viewpoints are expressed from experts in each domain. According to research published in Design Studies, the process of negotiation intrinsically leads to compromise in the design solution [1]. This is where a Systems Thinking perspective can provide a better insight towards to end goal while drawing appropriate methodologies and feedback mechanisms.

LEADING AREAS OF SYSTEMS CHANGE

Global health and Education are two key areas among the several areas of focus while considering the application of systems thinking methodologies. Several international development organisations and NGO’s deal with the problems associated with health and education in several parts of the world. While their efforts are commendable, it is intriguing to note that advancements in technology have not quite given the necessary tools to tackle these issues. We try to work around these problems part by part, analysing carefully how each sub part can be solved. While such an analysis is crucial to develop appropriate solutions, we fail to recognise the interdependent relations between several such subsystems. The understanding of these dependencies and relations is another such specialisation that can be better understood from a systems thinking lens. The relationship between disciplines is given a higher priority than the discipline itself.

The context of a system surpasses various fields of engineering, medicine, management, sciences, humanities and arts. Hence, it cannot be defined to belong to one particular niche. Universal issues when approached as a system will render a deeper as well holistic understanding. Hence, there is an immediate need to identify and analyse using systems thinking methodologies. While technicality is a prerequisite to coming up with innovative solutions, one cannot gauge the extent of the scenario without establishing an overall understanding.

RESEARCH QUESTIONS PERTAINING TO DEPENDENCY

The research provides a case study of how Systems Thinking was applied to a local community project. One of the food banks in the city was selected. The primary objective that was being addressed was *how to reduce dependency on the food bank.*

The research focus can be categorised as follows:
- examining the risks that result in dependency,
- exploring barriers; what prevents people from finding a better job,

- alleviation strategies; Moving people from dependency to independency,
- designing a system to prevent dependency.

PHASES OF THE PILOT PROJECT

DEVELOP AN INITIAL CONCEPTUAL MODEL

A causal loop diagram (CLD) was developed to represent the dynamics of the Food Bank under study [1, 2]. The process was simulated bringing in 4 main factors:

1) economic,
2) social,
3) environment,
4) policy.

This helped develop an overall understanding of how components within the system interact with each other. This system interconnectedness is reflected through the causal loop diagram. The model utility aims to reflect the behaviour patterns more closely rather than simply the units' interaction.

CONDUCT A WORKSHOP/MEETING WITH STAKEHOLDERS

In order to gain a deeper understanding of the system dynamics a meeting with the people associated with the food bank is crucial. These include the administration, the clients, the donors, the suppliers, volunteers, and all other individuals who are indirectly associated with its functioning.

This is the Research phase of the project, from which we aim to gather Data. Also, this will aid our next step of the project which is to identify leverage points.

A series of interviews were conducted with individuals of the Food Bank administration and volunteers. Their experience ranged from 1 to 3 years serving the food bank. The following areas were highlighted.

Emphasis on Unused Job Skills

1) Dependents are usually lacking language skills (for the job market),
2) equivalency issues,
3) refugees (education issues),
4) resume building is different (cultural shift),
5) communication/Internet to gain useful information (Unaware of this concept),
6) traumatized (revival clients need 7 years to come out of the cycle),
7) revenge mentality on the system.

The following questions arise on examining the system with a critical insight.

Q1. How to help them find a job to utilise their existing skills?

Q2. Some have lost hope in the system of obtaining jobs. There is a need for pushing them into the system to get them out of assistance

Q3. Incentive and platform to improve their skills set/language proficiency.

Q4. Discrepancy in the social needs versus the social assistance. The Food Bank has no system in place to assess clients. Some are in desperate need while some are taking advantage of the system (percentage of needs to be identified on a case by case basis).

Surveys from the Food Bank Volunteers revealed a more generalised view of issues surrounding food bank facilities.

Why do you think people depend on Food Banks?

1.) Low wages,
2.) big families,
3.) new to the country,
4.) hunger issues,
5.) poor wages,
6.) relieve financial stress,
7.) free food,
8.) social assistance.

What prevents people from finding a better job/pay scale?

1.) Language barriers,
2.) lack of education and qualifications,
3.) laziness,
4.) unfavoured due to un equivalent education,
5.) they don't want a job,
6.) lack of opportunities,
7.) lack of experience,
8.) lack of knowledge on where to find information/opportunities.

Other methods:

1.) better shelf space,
2.) bigger space,
3.) ticket machine,
4.) more training sessions for volunteers,
5.) better training sessions for the volunteers as everybody has different inputs,
6.) get more food into the bank.

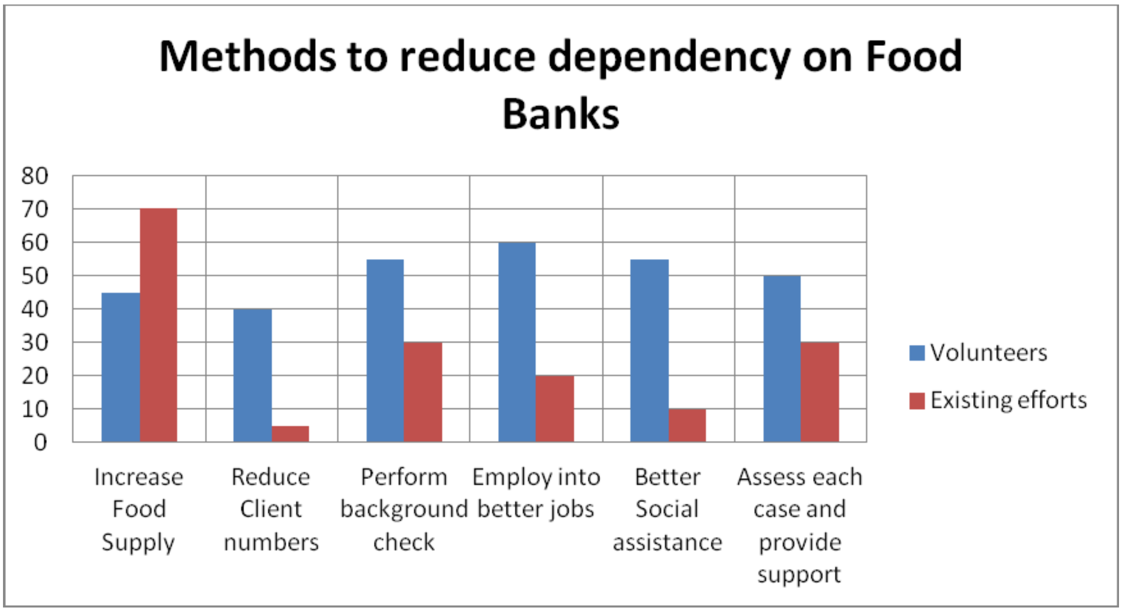

Figure 1. Volunteer surveyed on the best way to reduce dependency on food banks for the given clients versus the existing efforts to reduce dependency.

The clients interviewed were found to be undergoing training or looking for job opportunities. A few were unable to find jobs due to an increased number of responsibilities and dependencies at home. The following areas were found to be of prime focus. Measures directed towards providing services and awareness towards these areas should be increased to empower the women dependent on the food bank.

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By creating an empathy map, we employ design thinking to understand better the system from an empathetic point of view rather than purely non linear. This creates a cohesive environment to relate to the client in multidimensional ways. Empathy encompasses 4 aspects:

1.) what the clients "say",
2.) what the clients are "thinking",
3.) **the actions** that they have up taken to improve their issue; the "do",
4.) how do they "feel" about the current state of things.

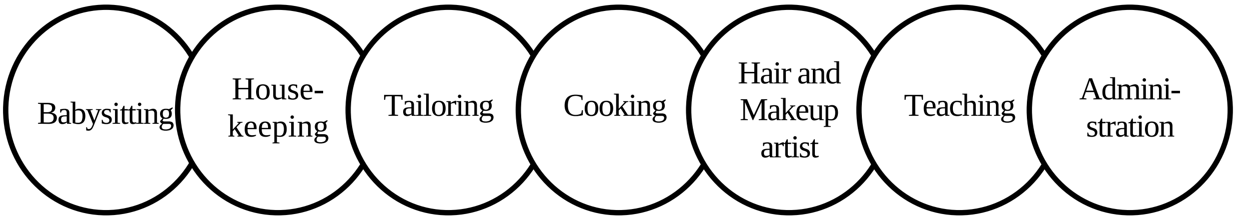

Figure 2. Areas to increase empowerment of women clients.

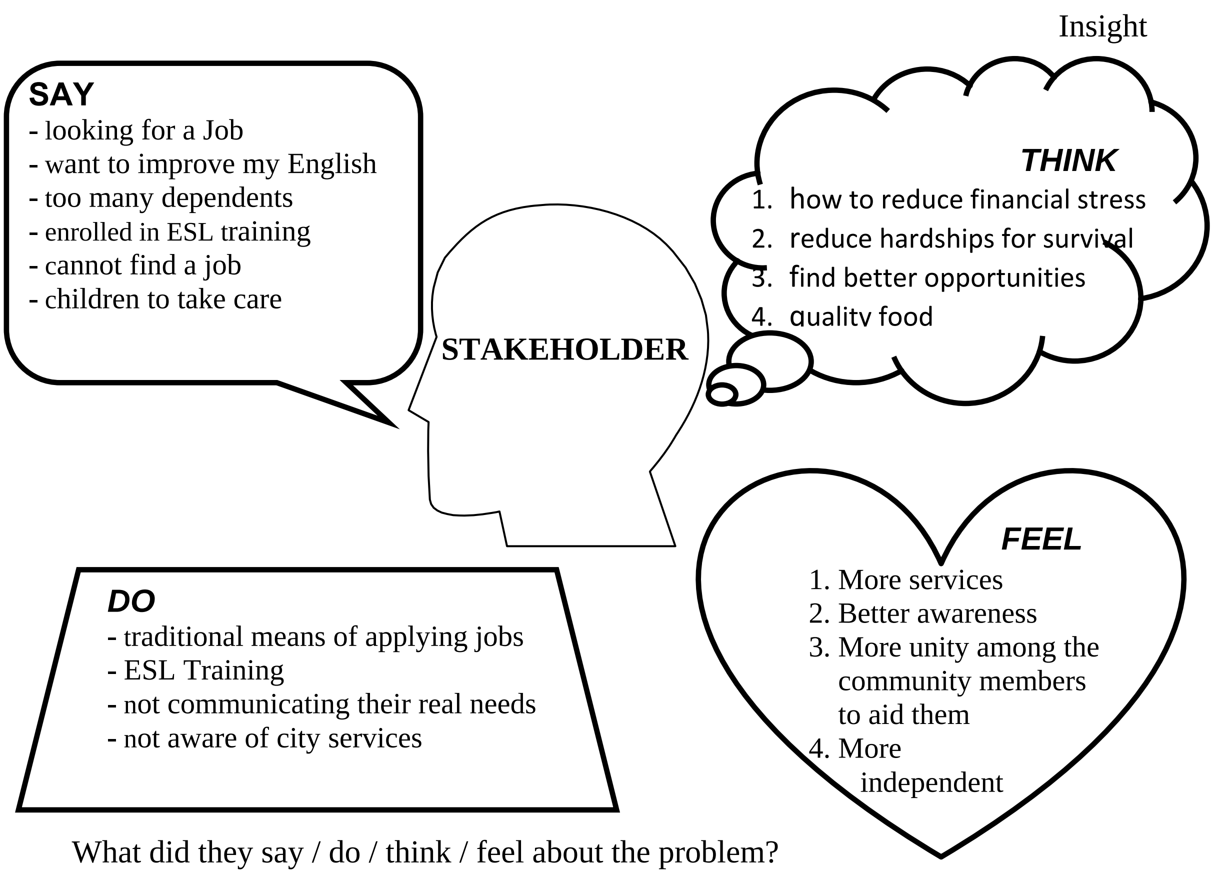

Figure 3. Empathy Map to help define problem statement for the given social problem [3].

The empathy map, Figure 3, highlights the factors that need to be considered in order to develop an effective problem statement. It is a statement that defines the problem from the clients' perspective and not as external elements of the system. Such a user centric problem definition is a crucial aspect of design thinking. The holistic understanding of the problem is necessary to analyse and deduce viable long term solutions.

Problem Statement Generated

A precise definition for the problem can be viewed in the figure below. Reducing dependency is not only beneficial as a health benefit but also as a means of better economical well being.

This statement clearly emphasis the need to develop immediate action plans to intervene into this system.

Figure 4. Problem statement generated from the Empathy map.

IDENTIFICATION OF LEVERAGE POINTS THROUGH SYNTHESIS AND ABSTRACTION OF DATA

We use Design Thinking principles to identify the leverage points in the system [3, 4]. While many areas in the systems can be intervened to bring about a significant difference in the customers well being, our design research is focussed towards our research question; how do we reduce the dependency on the existing food bank system. Design thinking entails 3 key factors.

Empathy

By interviewing the various stakeholders associated with the food bank system we are able to draw upon a suitable context which is user centric. This user defined problem defining is essential to design thinking principles.

Creativity

Idea generation requires the given problem statement to go through a creativity process to generate targeted solutions and tackle designs problems.

Rationality

Designs that are generated need to be fit to the context of the problem by prototyping and testing. This stage also includes creative elements to test against various factors and scenarios. Sketching, role playing, using low cost material [2] are few ways though which we can carry this process.

Based on the factors mentioned, we use design research to better understand the dependency needs of the food bank client.

Step 1. The problem statement was defined using the empathy problem map, as seen in the previous section. The following insights were generated:

1.) most of the cases are dependent owing to financial strain,

2.) an immediate plan to address employment is needed,

3.) lack of awareness of existing services is another hurdle to job search,

4.) incentives to direct the more critical cases in order to direct them to appropriate services,

5.) motivation is key.

Step 2. The next step involves:

- using the problem statement to design creative solutions,
- a means of prototyping, testing and applying the solutions to the given system.

The designing for our given problem statement is explained further in the text.

A layered chart is designed to get a better sense of our client needs and application areas. This prototype was developed to tackle the issue of unused jobs skills, as this was our key leverage point. By classifying the clients into a multilayer system, we are able to test different alleviation strategies.

It is important to note that each of the cases differ from one another, and while addressing them on a case by case basis would yield more results, the purpose of this study was to provide a useful template that is cost and time effective. Therefore, categorizing the client needs and backgrounds gave the study more structure to identify and work with our leverage point.

Job Centric

After careful examination, the study identified a distinct set of individuals who are motivated to find work; however the sheer difficulty of landing something stable and long term comes in the way of pursuing something worthwhile. The immediate action plan for reducing dependency should be directed towards this category of individuals. This is the most volatile layer in terms of transience.

Given the vast majority of existing career and employment services in the city, a plan to direct them towards finding quality employment is key.

Counsel Centric

While staying motivated gets difficult, given the inflexibility of the job economy, one cannot be unmindful of the complicated conditions some of the clients have lived or are living through. This issue is denser than simply directing or administering them to appropriate job openings. However, we need to carefully take these cases into consideration in order to understand what best way we can approach these individuals. While some might require trauma counselling and a mental health expert intervention would be more befitting, there are several others which might not need that level of counselling.

Deep centric

This category entails individuals whose dependencies are deeply rooted and justified. Their financial and social well being is directly related to their dependency on the food bank. Taking them off the food bank requires a long term action plan coupled with various intervention schemes as these cases are very critical. Hence, their dependency on the food bank system need not require immediate address.

GROUP MODEL BUILDING WITH PARTICIPANTS TO DETERMINE THE NATURE OF CAPACITY/SOLUTION TO ADDRESS THE ISSUE

The rationalisation aspect of design thinking is applied at this level of our pilot project. It includes elements of the creativity module as well as rationalisation module, together tested for feasibility with the various stakeholders. The method involved is group model building. It is an exercise developed specific to the system needs. In the case of our food bank scenario, in order to address the layered client data base, designing and structuring of solutions is required.

Group modeling building brings together stakeholders closely associated with the food bank system, policy makers, and donors to participate in a training environment or workshop which runs on systems dynamics methodologies. This creates an environment to holistically understand the problem statement at hand and develop effective solutions for it.

The nature of capacity building or leverage action also needs to be agreed upon. Hence, having a multidisciplinary participant database helps work through the barriers and several stagnant mental models. This makes the prototyping and testing of chosen solutions easier on the project coordinators as well as on the system.

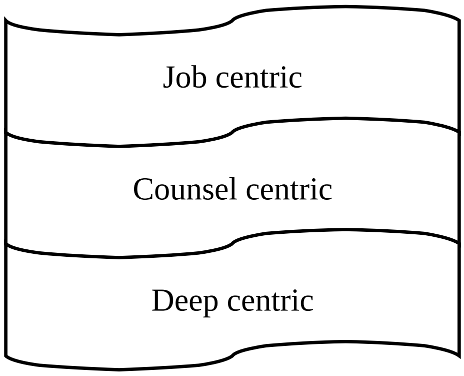

Figure 5. Schematic diagram representing different layers of the client database.

CONCLUSION

Systems Thinking is a phenomenal tool to which can be applied to various real life scenarios. Be it technical or social in nature, any system which entails elements, interrelations and a function can be viewed through the systems lens. Embracing the culture of systems thinking will rejuvenate our understanding of real life problems as we can observe through our case study with a food bank. Systems thinking methodologies aid the process of deriving solutions based on feasibility and long term improvement. Concept maps reflect segments of an organisation which are generally hidden or remain underscored due to overshadowing factors. Once the real problem statement is defined, it becomes easier to determine the intervention capacity. In conclusion, systems thinking specifically the concept of design thinking has tremendous advantage and by adhering to such an approach, organisations will soon be able to identify and replicate innovative models across several disciplines. Our disciplines have become far too disciplined and there is a need to break the barriers between them.

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PRIMJENA MIŠLJENJA O SUSTAVIMA ZA ISPITIVANJE I SMANJIVANJE OVISNOSTI O BANKAMA HRANE

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SAŽETAK

Systems thinking is the art of understanding interconnections between various disciplines thereby unwinding the existing complexity. Most of the real world problems are complex, take for the example the increasing dependency rate on food banks. While various factors contribute towards it, not much has been done to bring the take off the number of dependents. By viewing this system from a holistic systems thinking lens, one explores the issue in depth. We realise the universally acceptable solution is not alleviating the problem in the long run. By applying systems thinking principles several hidden factors are brought to attention and subsequently can be dealt with more aptly. A movement that transcends disciplines results in delivering better solutions.

KLJUČNE RIJEČI

mišljenje o sustavima, banke hrane, dizajniranje, društveno inženjerstvo

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