

# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

**Scientific Journal**

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## Scientific Journal

# INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS

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# INSPECTING COMPLIANCE TO MANY RULES: AN AGENT-BASED MODEL

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## ABSTRACT

Ever increasing scope and complexity of regulations and other rules that govern human society emphasise importance of the inspection of compliance to those rules. Often-used approaches to the inspection of compliance suffer from drawbacks such as overly idealistic assumptions and narrowness of application. Specifically, inspection models are frequently limited to situations where inspected entity has to comply with only one rule. Furthermore, inspection strategies regularly overlook some useful and available information such as varying costs of compliance to different rules.

This article presents an agent-based model for inspection of compliance to many rules, which addresses abovementioned drawbacks. In the article, crime economic, game-theoretic and agent-based modelling approaches to inspection are briefly described, as well as their impact on the model. The model is described and simulation of a simplified version of the model is presented. The obtained results demonstrate that inspection strategies which take into account rules' compliance costs perform significantly better than random strategies and better than cycle-based strategies. Additionally, the results encourage further, wider testing and validation of the model.

## KEY WORDS

ICARUS, compliance inspection, agent-based model, multi-agent system, compliance costs

## CLASSIFICATION

JEL: C63, C72, C73, D81, D83, K42

## INTRODUCTION<sup>1</sup>

In 1788 in *The Federalist* James Madison<sup>2</sup> famously stated: *If men were angels, no government would be necessary. If angels were to govern men, neither external nor internal controls on government would be necessary.* However, men, as we know, are not angels.

Human society of today is reliant on an ever-increasing mass of regulations, rules and other social norms. Changes in regulations reflect changes in society and in our understanding of the processes inside society as well as between society and the environment. Increase in society's interconnectedness and complexity is reflected by the rise in scope and complexity of regulations, particularly in areas such as financial regulation [1] and environmental regulation [2], and in particular in the US and the EU.

In democratic societies, regulations typically introduce limitations and rules of conduct that should, in the end, be beneficial to the society as a whole. However, to individuals and organizations that have to obey those regulations, they might seem overly cumbersome, useless or even counterproductive. If opportunity arises, individuals as well as companies might try to violate the rules or shirk from their responsibilities since, as Madison pointed out, men are not angels. Therefore, there usually has to be a detriment or penalty for noncompliant entities, for regulations to be effective. Since regulations are typically not self-enforcing, they require some sort of external coercion mechanism such as law enforcement agencies, inspection agencies, etc. Inspection agencies and similar organizations usually want to achieve maximum compliance with rules and regulations under their authority. However, those agencies are not omniscient and usually cannot know whether an entity is compliant or not, without performing some sort of inspection procedure. On the other hand, inspection procedures and available resources are rarely such to allow total coverage of all constituents. Therefore, one of the key challenges for any inspection organization is optimal selection of entities for inspection. This selection process should identify – as correctly as possible – violators, and punish them. Additionally, the inspection procedure should serve as a deterrent to unwanted behaviour. To complicate matters further, the expanding regulatory landscape and rise in complexity and numerousness of constituents is often not met with correspondingly expanding inspection resources. Therefore, rise in efficiency of the inspection selection process becomes paramount.

Inspection selection and inspection itself have been objects of extensive scientific inquiry. There are several approaches to the matter, and each has noteworthy weaknesses. Firstly, analysis of historical data delivers valuable insights, but it cannot establish causation and isolate variables. Real-world experimentation is often legally impossible or ethically unacceptable. Laboratory setups and surveys can be performed, but often encounter difficulties when trying to recreate real-world setups and incentives. Finally, various modelling approaches are often utilised, but they are also plagued with shortcomings such as overly simplistic assumptions, narrow outlook, limited application, analytical insolvability, lack of empirical validation, etc.

This article outlines several modelling approaches to analysis of the inspection problem and underlines their advantages and limitations. Moreover, the article presents ICARUS (acronym: Inspecting Compliance to mAny RULeS), an agent-based model for inspection of compliance to many rules. The model describes a generic environment in which one inspection agency inspects compliance of a set of entities to a group of rules. Finally, the article demonstrates a simplified simulation environment of the model and tests the working hypothesis that conduct of inspections based on knowledge of resource needs for compliance reduces total non-compliance in the system.

## **INSPECTION MODELS: ASSUMPTIONS AND APPROACHES**

Inspection models are based on numerous assumptions about human behaviour and motives. Some assumptions are intuitively understandable, while others less so. Moreover, inspection models' outcomes can be vastly different and even contradictory, depending on the underlying assumptions.

In this section, the basic assumptions arising from the human rationality and its boundedness are briefly explained and two main modelling approaches – inspection games and agent based modelling – are outlined.

### **CRIME ECONOMICS AND HUMAN RATIONALITY**

Traditionally, non-compliance with the established sets of social norms, including laws, was considered a sign of lack of character, mental illness or social inadaptability. 18<sup>th</sup> century economists and criminologists started to change that outlook by describing humans as rational beings who make decisions based on scrupulous analysis of potential benefits and costs of their actions. Cesare Beccaria in his seminal work *On Crimes and Punishments* in 1765 extended the utility theory to crime and argued that, in regards to criminal justice, people act with free will, in rational manner and try to achieve their own personal gratification. Accordingly, people will be deterred from crime when the punishment outweighs benefits of the crime.

The notion that human rationality is the foundation of decision-making dominates classical economics. In the context of economics of crime, Becker formalized that idea through his economic model of crime [3]. According to the model, potential criminals make their decisions based on comparison of benefits of crime and expected costs, which reflect sanction cost and probability of sanction's occurrence. This model does not apply only to individuals, but to companies and other organizations as well. It could be argued that companies, and especially publically traded companies, present an even more "natural" background for the economic model of crime since their utility function is unambiguous – maximization of shareholder profit. Hence, companies (or their management) are even more likely to try to objectively assess benefits and expected costs of crime (or non-compliance) and act accordingly.

The economic model of crime allows deduction of further conclusions. In complex environments such as banking, companies have to obey a myriad of rules. Violation of rules will, if detected, result in sanctions, and costs of those sanctions might vary in respect to the rule that was violated. However, in many regulated environments fines (sanctions) are pre-determined and are often the same for groups of rules or even for all the rules contained in a legal act. Although compliance with the rules might not have any direct benefits for the regulated entity, it will – almost certainly – incur certain costs. Those costs will vary, depending on the requirements of specific rules. Some rules might be inexpensive to comply with (e.g. rules with details on how to perform various administrative procedures), some could require considerable resources (e.g. establishment of certain processes or organizational units) and some could incur massive costs (e.g. additional capital requirements). In such setup, companies might objectively assess compliance cost, sanction cost and probability of its incurrance and decide to comply with some rules while violating others.

However, although the rationality hypothesis is very useful in analysis of human behaviour, it is also highly demanding. Perfect rationality requires complete knowledge of the environment – in our simplified case; it would imply perfect knowledge of potential benefits and sanctions of non-compliance, as well as perfect knowledge of probability of sanction's occurrence. Furthermore, it requires clear preferences, unbiasedness and ability

to calculate perfectly, in real time [4]. These strong requirements are often unattainable in real life, and the human rationality requirement is often softened via the concept of bounded rationality. Bounded rationality, which was first presented by Simon [5], relaxes rationality requirements by acknowledging that although humans try to make fully rational decisions, they are fallible in their decision-making – they have biases, they are unable to calculate probabilities perfectly, they make mistakes in logic and act in situations with incomplete information. Experimental evidence supports the idea that human rationality is bounded in the area of crime economics [6].

Inspection models mostly heavily rely on ideas of rationality and/or bounded rationality.

### GAME THEORY AND INSPECTION GAMES

Game theory can concisely be described as a formal study of conflict and cooperation [7] and was first applied to inspection problems by Dresher in 1962 [8]. Although it was initially applied to inspection of compliance to nuclear disarmament treaties, in subsequent years it was applied to a plethora of inspection problems. The applications include accountancy and auditing, tax inspection, enforcement of environmental regulations, crime control, smuggling (so-called smuggling game), relationship between politicians and bureaucrats (so-called oversight game), etc. Overview of literature on inspection game can be found in [9-11].

Inspection game, in game theoretic setting, presents a special class of non-cooperative games<sup>3</sup>. The basic interactions in the inspection game are shown in Figure 1. An entity that is obliged to comply with a certain rule decides (step I) to comply or violate that rule. In step II inspector decides whether to inspect the given entity or not. When deciding whether to violate or comply, the entity does not know with certainty whether the inspector will inspect. Correspondingly, inspector – when deciding whether to inspect or not – does not know whether the entity violates the rule or complies with it. Payoff matrix is shown in table in the Figure 1. The game has no stationary equilibrium since relationships between payoffs imply that players always have reasons to change their strategies. I.e. if the entity knew that inspector will inspect, it would comply with the rules. However, if inspector knew that the entity will comply, it would prefer not to inspect. And if the entity knew that inspector will not inspect, it would violate, which would then entice inspector to inspect, and so on. Arrows in the payoff matrix indicate the order of players' preferences.

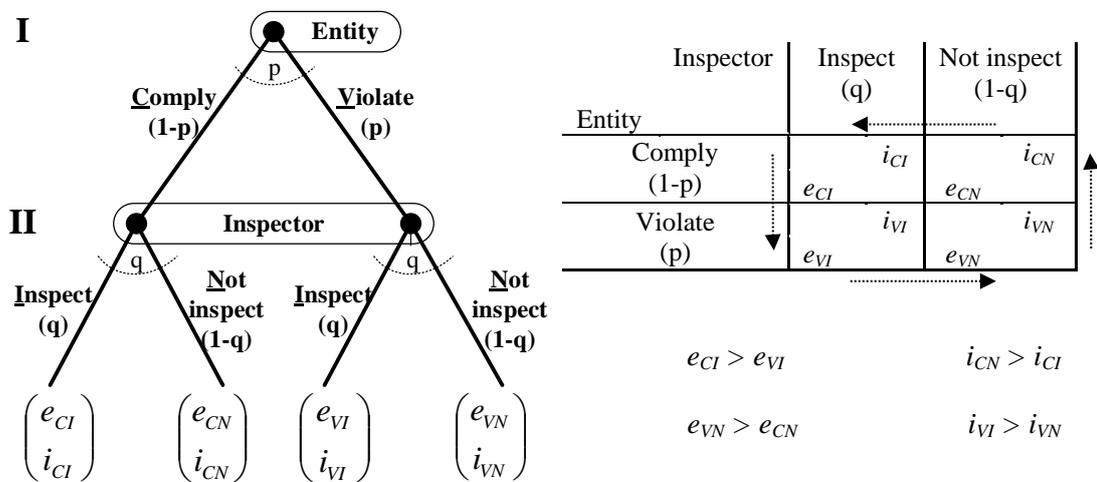


Figure 1. Inspection game in extended and in normal form.

In line with that, the only equilibrium strategies are mixed strategies. That is, if  $p$  is probability of violation and  $q$  is probability of inspection, the optimal mix of pure strategies was calculated by Tsebelis [12] and is given by (1)<sup>3</sup>:

$$p^* = \frac{i_{VN} - i_{VI}}{i_{CI} - i_{CN} + i_{VN} - i_{VI}} \quad q^* = \frac{e_{CN} - e_{VN}}{e_{CN} - e_{VN} + e_{VI} - e_{CI}}, \quad (1)$$

The values of  $p^*$  and  $q^*$  reflect the essence of the game-theoretic approach to inspection. Game theory is analysis of strategic interaction between players and, in line with that, decisions of each player are primarily influenced by what that player believes will be the actions of the other player. Accordingly, entity's probability of violation ( $p^*$ ) is influenced exclusively by inspector's payoffs, and inspector's probability of inspection ( $q^*$ ) is influenced exclusively by entity's payoffs. Significant complications arise with such result, namely, it is a very bold assumption that entity will know all inspector's payoffs and vice-versa. Because of described relationship between payoffs, both players in an inspection game have incentives to hide their motivations and, hence, payoffs from the other player. Furthermore, stated solution implies that entity's decision whether to violate ( $p^*$ ) is completely independent of the size of penalty ( $e_{VI}$ ) which is in contradiction with the economic model of crime and some experimental results [6]. Tsebelis' conclusions have been studied and analysed (interestingly, Tsebelis' model was rarely tested experimentally [13; p.156]) and it has been concluded that the analytical results are valid, but that they reflect a very simplified setup (one-off game with only 2 players and with complete information) [14]. Furthermore, divergences between results of the simplified model and reality can be explained by players' bounded rationality [6]. It is interesting to note that the situation where one centralised inspection agency inspects compliance of numerous agents fits Becker's model much better than Tsebelis' [15].

Many classic inspection games consider similarly simple setup. However, most real-life scenarios (e.g. financial inspection, inspection of environmental protection, compliance audits, etc.) require introduction of more complex parameters such as:

- 1) the game is played repeatedly,
- 2) there is more than one entity that can be inspected (one-inspector-n-inspectees or m-inspectors-n-inspectees scenarios),
- 3) players' rationality is not perfect but bounded,
- 4) players possess imperfect and incomplete information,
- 5) players are heterogeneous (e.g. entities are characterized by different payoffs, possess different information and their rationality is bounded in different ways), and
- 6) players learn and adapt.

Introduction of more complex parameters to the inspection game makes the model more realistic, but also significantly complicates or even thwarts its analytical solvability. Authors who study inspection games continually add complexity to their models, however, even recently developed inspection game models that introduce complications and better correspond to the situation that is analysed in this article still include very limiting assumptions. E.g. in a setup that is somewhat similar to the one analysed in this article, Deutsch and Golany describe a finitely repeated inspection game with single inspector and several agents, where inspector tries to optimally allocate limited inspection resources [16], but the game still has very limiting assumptions such as complete information.

Although inspection games might be limited in their ability to introduce complexity and remain solvable, elements of inspection game and general circumstances described in the inspection game can be successfully used in other modelling approaches such as agent-based modelling. Furthermore, game-theoretic setting has a great value in highlighting strategic relationship between actors.

## **AGENT-BASED MODELLING (ABM)**

Agent-based modelling (ABM)<sup>5</sup> arose from research on complex adaptive systems. Agents are independent, commonly software-implemented entities, which have a set of characteristics, take actions depending upon certain conditions and interact with each other [17]. There are numerous sources that describe when [17], how [18-20] and why [17] to use ABM.

ABM is being used in the field of social sciences, because of its unique capabilities for describing complex systems, knowledge discovery and hypothesis testing [21]. The ABM approach is particularly relevant in cases where it is not possible to conduct experiments to test certain social phenomena because of ethical or legal considerations. In addition, the cost of model development and running of simulations is typically significantly lower than the cost of conducting experiments that test certain social phenomena.

ABM and game theory are, in many ways, considerably different approaches to modelling. While the game theory is structured, analytical and demanding in constructing and solving of the models, the ABM allows great flexibility in devising models and setting their parameters. ABM can also easily incorporate ideas and concepts from game theory, crime economics, etc. Parameters such as those mentioned in the ordered list in the previous chapter are easy or even trivial to apply and implement. ABMs that model inspections often use assumptions from the inspection game and particularly the assumption about bounded rationality of agents. Interestingly and more generally, vast majority of agent-based models implement bounded rational agents [17]. It is no wonder then that ABM has been extensively used for analysis of inspection problems.

The area where ABM is particularly extensively used is analysis of tax compliance and tax inspection. Tax inspection models are often very different, particularly in relation to their complexity. For example, in [22-27] a number of models have been described and analysed, each with different assumptions, different levels of complexity (models that were developed earlier are simpler, and later models are more complex), different validation methods and are developed in different simulation environments. ABM has also been used in analysis of (inspection of) crime [6, 28] and banking supervision [29]. All cited ABM inspection game models (with exception of [29]) model situations in which compliance to only one rule is analysed (one-entity-one-rule).

The most significant downside in use of the ABM is difficult rigorous validation of the model. Most ABM models are very specific and are validated (if at all) against a limited set of narrow, field-specific data.

## **THE ICARUS MODEL**

The main motivation for development of the ICARUS model is the fact that inspection models almost exclusively consider one-entity-one-rule environments, which is a very bold assumption, especially when considering inspection of regulatory compliance in highly regulated areas (financial services, environmental protection, etc.). Further motivation is to have a polygon for testing the hypothesis that use of knowledge about resource needs in conduct of inspections can reduce the total number of violations in the system, which might be particularly relevant for regulatory compliance in highly regulated areas.

The proposed ICARUS model is an agent-based model of compliance inspection in which one inspection agency (inspector) inspects whether a set of entities (agents) are compliant with a set of rules. The main motivations for use of the ABM approach are flexibility of such approach, legal restrictions concerning experimental approach to regulatory compliance and limitations of purely game-theoretic approach.

In the following sub-chapters, modelling environment, assumptions and formal representation of the ICARUS model are presented.

## **MODEL ASSUMPTIONS AND THE GENERAL OUTSET**

The basic idea and driver for development of the resource requirements focused model is the assumption that costs of compliance can be used as signals that are available to inspector (with some degree of accuracy). Based on those signals, inspections could be directed in a way that would detect non-compliance with higher precision and, in turn, reduce total non-compliance in the system.

There are 2 types of actors in the model (inspector and entities) with conflicting interests. Relationship between the inspector and entities can be described, in game-theoretic terms, as a non-cooperative, non-zero sum, finitely repeated inspection game between one inspector and several inspectees (entities), with incomplete information. The model operates in discrete time intervals. In each interval entities make violate/comply decisions for each rule, and inspectors make inspect/not inspect decisions for each entity-rule pair.

Entities and inspector behave as follows:

- 1) each entity is characterized by a decision-making process that includes an internal component, which reflects entity's compliance resource requirements, and an external component by which entity assesses probability of inspection, based on some known inspector-related parameters, and
- 2) inspector's decision-making process is determined by the selected inspection strategy. In essence, it is an optimal assignment problem since the inspector is trying to optimally allocate his limited inspection resources to achieve the lowest total number of violations in the system. However, an important drawback for inspector is that he does not know the total number of violations in the system at any time (unless he can inspect all entity-rule pairs simultaneously, which is not a realistic prospect and trivializes the inspection problem).

The model fully utilizes advantages the ABM approach. First and foremost, agents (entities) are heterogeneous:

- 1) entities are characterized by different resource requirements needed for compliance (costs of compliance) with each of the rules. These differences in costs of compliance reflect differences in entities' internal organization, complexity, size, business model, etc. However, although there are some differences in resource requirements between entities, resource requirements also have an underlying orderedness across entities, which is also known to inspector (e.g. in any bank, costs of capital requirements are higher than costs associated with some administrative procedure), and
- 2) entities differ in their risk appetite (or risk preference), which influences their assessment of the inspection probability. Risk appetite variable reflects two findings: Firstly, risk preferences differ among people and might present a stable personality trait [30]; secondly, decision-makers are not perfectly rational when assessing risk [31]. This bias also models agents' bounded rationality in decision-making.

Furthermore, the model assumes imperfect and incomplete information. A limited set of variables are known to all players (inspector and all entities), while the majority of variables are known only locally. Every entity knows only its own payoffs. Entity's utility function is based on the economic model of crime and is influenced by costs of compliance, costs of mandatory punishments and entity's own assessment of probability of inspection of each rule.

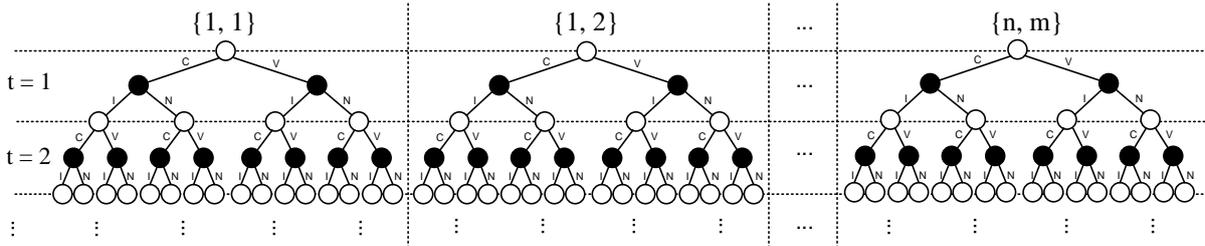
When assessing the probability of inspection, entities consider their own inspection history record and based on that and some general information about the system, try to estimate probability of inspection in that time interval. Furthermore, each entity is locally connected to several other entities and shares information with them. Of particular interest is information whether violations were punished or not, which either reinforces compliance (if violations are

punished) or noncompliance (if violations are unpunished). This mechanism mimics experimentally observed “broken windows” dynamics [13].

The model presumes that entities and inspector use their historical data (memories) about inspections to make assumptions about the future. Such approach is in contrast with the idea of forward-looking decision-making by a rational individual, but fits well with the idea of boundedly rational behaviour and empirical findings related to learning from experience [32].

## FORMAL DESCRIPTION OF THE MODEL

Let  $\mathcal{E} \equiv \{1, \dots, n\}$  be a set of  $n$  entities (agents, organizations),  $n \in \mathbb{N}$ , where each entity in  $\mathcal{E}$  is obliged to comply with all the social norms (rules) contained in  $\mathcal{O} \equiv \{1, \dots, m\}$ ,  $m \in \mathbb{N}$ . Each entity in  $\mathcal{E}$  at every discrete time interval  $t$  in set  $\mathcal{T} \equiv \{1, \dots, \tau\}$  decides whether to comply or violate each of the rules contained in  $\mathcal{O}$ . Compliance with the rules is monitored by the inspection agency (inspector)  $\mathcal{J}$ . In every  $t \in \mathcal{T}$ ,  $\mathcal{J}$  decides whether it will inspect each of possible combinations (pairs) of entities and rules. When entity  $i$  decides whether it will comply or violate rule  $j$  in  $t$ , it does not know whether  $\mathcal{J}$  will inspect  $\{i, j\}$  at  $t$ . Analogously, when  $\mathcal{J}$  decides which pairs of entities and rules it will inspect at  $t$ , it does not know the state of compliance. However, after inspecting  $\{i, j\}$  at  $t$ ,  $\mathcal{J}$  knows, with certainty, whether  $i$  complied with or violated  $j$  at that time. The described interaction is displayed in the Figure 2.



**Figure 2.** Inspector-entity interaction displayed in the extended-form.

If the inspector detects violation of a rule, it will punish respective entity with a fine. Fines can be rule-specific and are contained in the vector:

$$\mathbf{k} \equiv (k_1, \dots, k_m), \{k_j \mid k_j \in \mathbb{Z} \wedge k_j < 0\}, \forall j \in \mathcal{O}. \quad (2)$$

### Features and behaviour of entities (agents)

Entity  $i$  is characterized by a vector of resource needs for fulfilling each rule in  $\mathcal{O}$ :

$$\mathbf{c}_i \equiv (c_{i1}, \dots, c_{im}), \{c_{ij} \mid c_{ij} \in \mathbb{R} \wedge c_{ij} < 0\}, \forall i \in \mathcal{E}, \forall j \in \mathcal{O}. \quad (3)$$

Furthermore, entities differ in their risk appetite, which influences their assessment of inspection probability. Risk appetite of the entity  $i$  is given with (4):

$$r_i \in \mathbb{R}, \forall i \in \mathcal{E}. \quad (4)$$

Decision whether to comply or violate the rules is further influenced by the information acquired from other connected entities. Association of entity  $i$  with other entities is defined by the vector:

$$\mathbf{g}_i \equiv (g_{i1}, \dots, g_{in}), g_{ip} \in \{0,1\}, \forall i, p \in \mathcal{E}. \quad (5)$$

Associations between entities are symmetric (if entity  $i$  is connected to entity  $p$ , then  $p$  is also connected to  $i$ ):

$$\{g_{ip} \mid g_{ip} \in \{0,1\} \wedge g_{ii} = 1 \wedge g_{ip} = g_{pi}\}, \forall i, p \in \mathcal{E}. \quad (6)$$

Possible values of the association vector are:

$$g_{ip} = \begin{cases} 0, & \text{if } i \text{ and } p \text{ are not connected,} \\ 1, & \text{if } i \text{ and } p \text{ are connected,} \end{cases} \quad \forall i, p \in \mathcal{E}. \quad (7)$$

Entities also keep track of inspections' history. At  $t$ , entity  $i$  knows the results of inspections to which it was subjected in the last  $l$  time intervals. The results of inspections known to  $i$  at  $t$  are given with (8).

$$\begin{aligned} \chi_i(t) &\equiv (h_i(t-1), \dots, h_i(t-l)), \\ h_i &\in \{0, a, b\}, \{t, l \mid t \in \mathcal{T} \wedge l \in \mathcal{T} \wedge 1 \leq l \leq t\}, a, b \in \mathbb{N}, \forall i \in \mathcal{E}. \end{aligned} \quad (8)$$

Possible values of the inspection history vector of entity  $i$  are:

$$h_i(t-u) = \begin{cases} a, & \text{if inspector detected violation at } t-u, \\ 0, & \text{if } i \text{ was not inspected at } t-u, \\ b, & \text{if inspector detected compliance at } t-u, \end{cases}, \quad \{u \mid u \in \mathcal{T} \wedge 1 \leq u \leq l\}. \quad (9)$$

Entities make rational decisions whether to comply with or violate each rule in  $\mathcal{O}$ , by comparing cost of compliance and expected punishment disutility. The expected punishment disutility of entity  $i$  for violating rule  $j$  at  $t$  is given with the product of proscribed penalty  $k_j$  and entity's subjective assessment of the inspection probability at  $t$ :  $p_{ij}(t)$ .  $p_{ij}(t)$  is a valuation function:

$$p_{ij}(t) = f(c_{ij}, r_i, g_i, \chi_i(t), I_C), \quad \forall i \in \mathcal{E}, \forall j \in \mathcal{O}, \forall t \in \mathcal{T}. \quad (10)$$

The subjective assessments of inspection probability at  $t$  of all the rules in  $\mathcal{O}$  for the entity  $i$  are contained in the vector:

$$\mathbf{p}_i(t) \equiv (p_{i1}(t), \dots, p_{im}(t)), \{p_{ij} \mid p_{ij} \in \mathbb{R} \wedge 0 \leq p_{ij} \leq 1\}, \forall i \in \mathcal{E}, \forall j \in \mathcal{O}, \forall t \in \mathcal{T}. \quad (11)$$

In line with that, the subjective expected utility (SEU) function for the entity  $i$  at  $t$  is:

$$\pi_i(t) \equiv \sum_{j=1}^m \max [c_{ij}, p_{ij}(t) \cdot k_j], \quad \forall i \in \mathcal{E}, \forall t \in \mathcal{T}. \quad (12)$$

In line with the subjective assessments of the inspection probability and the resulting decisions, the state of compliance of entity  $i$  with all the rules in  $\mathcal{O}$  at  $t$  is given with the vector:

$$\mathbf{o}_i(t) \equiv (o_{i1}(t), \dots, o_{im}(t)), o_{ij} \in \{-1, 1\}, \forall i \in \mathcal{E}, \forall j \in \mathcal{O}, \forall t \in \mathcal{T}. \quad (13)$$

Possible values of the compliance vector are:

$$o_{ij}(t) = \begin{cases} -1, & \text{if } c_{ij} < p_{ij}(t) \cdot k_j \text{ (violation),} \\ 1, & \text{if } c_{ij} > p_{ij}(t) \cdot k_j \text{ (compliance),} \\ \sim U\{-1, 1\}, & \text{if } c_{ij} = p_{ij}(t) \cdot k_j \text{ (random selection),} \end{cases} \quad \forall i \in \mathcal{E}, \forall j \in \mathcal{O}, \forall t \in \mathcal{T}. \quad (14)$$

## Features and behaviour of the inspector

Inspector's main objective is to reduce the total number of violations in  $\mathcal{T}$ . The total number of violations is given with (15):

$$\Pi \equiv \sum_{t=1}^T \sum_{i=1}^n \sum_{j=1}^m o_{ij}(t). \quad (15)$$

It is important to note that the inspector does not know the total number of violations in  $\mathcal{T}$  and that he can, in general, observe only a very limited set of data. Namely,  $\mathcal{J}$  is not aware of entities' true preferences and compliance resource needs; it can only make estimates based on the known inspection history and his own knowledge about compliance resource needs. On the other hand, inspector is aware that entities are rational in their decision-making. Inspector's knowledge of the compliance resource needs is contained in the vector:

$$\mathbf{d} \equiv (d_1, \dots, d_m), \{d_j \mid d_j \in \mathbb{R} \wedge d_j < 0\}, \forall j \in \mathcal{O}. \quad (16)$$

$\mathcal{J}$  keeps track of inspections' history for the last  $l$  intervals,  $l \in \mathcal{T}$ . The entire history of inspections that is known to  $\mathcal{J}$  at  $t$  is contained in the three-dimensional matrix  $S(t)$ . Correspondingly, the history of inspections of the entity  $i$  at  $t$  is:

$$S_i(t) \equiv \begin{bmatrix} s_{i1}(t-1) & \dots & s_{i1}(t-l) \\ \vdots & \ddots & \vdots \\ s_{im}(t-1) & \dots & s_{im}(t-l) \end{bmatrix},$$

$$s_{ij} \in \{-1, 0, 1\}, \{t, l \mid t \in \mathcal{T} \wedge l \in \mathcal{T} \wedge 1 \leq l \leq t\}, \forall i \in \mathcal{E}, \forall j \in \mathcal{O}. \quad (17)$$

In other words, at  $t$ , inspector is aware of the result of inspection of the rule  $j$  in entity  $i$  that was performed before  $u$  intervals and is given with (18):

$$s_{ij}(t-u) = \begin{cases} -1, & \text{if at } t-u \text{ } i \text{ was inspected and } o_{ij}(t-u) = -1 \\ 0, & \text{if at } t-u \text{ } i \text{ was not inspected} \\ 1, & \text{if at } t-u \text{ } i \text{ was inspected and } o_{ij}(t-u) = 1 \end{cases},$$

$$\{u \mid u \in \mathcal{T} \wedge 1 \leq u \leq l\}. \quad (18)$$

Furthermore, inspector's inspection capacity is limited and at  $t$  it can perform only  $I_C$  inspections of entity-rule pairs  $\{i, j\}$ ,  $i \in \mathcal{E}, j \in \mathcal{O}$ ,  $\{I_C \mid I_C \in \mathbb{N} \wedge 0 \leq I_C \leq mn\}$ . Therefore, at  $t$ ,  $\mathcal{J}$  selects  $I(t)$  for inspection where  $I(t) \subseteq \mathcal{E} \times \mathcal{O}, t \in \mathcal{T}$ . Or, in other words, inspection selection is a function  $f(I_C, \mathbf{d}, S(t))$ , where  $f: \mathcal{E} \times \mathcal{O} \rightarrow I(t)$ .

Taking into account inspector's knowledge and limitations, its goal is to use the available inspection resources as efficiently as possible and to make, at every time interval, an optimal selection of  $I_C$  pairs of entities and rules that it will inspect. To achieve that, inspector can decide to use a number of strategies – from very basic strategies such as random or cyclic selection<sup>4</sup> of inspection pairs, to more advanced strategies where future inspections are based on inspections' history, perceived resource needs of compliance or some combination thereof. It is important to note that the ICARUS model is strategy-agnostic and allows use of a plethora of inspection strategies.

The relationship between inspector and entities, as it was already stated, is characterized by a significant information asymmetry and incomplete information. Table 1 summarizes the data known to each of the parties at  $t$ .

**Table 1.** Knowledge of entities (agents) and inspector at  $t$ .

<b>Entity <math>i</math></b>	$r_i, c_i, g_i, \chi_i(t), p_i(t), o_i(t)$
<b>Inspector</b>	$S(t), \mathbf{d}, I(t)$
<b>Common knowledge (all entities and the inspector)</b>	$n, m, \mathbf{k}, I_C$

## SIMULATION

A proof-of-concept simulation of a simplified variation of the ICARUS model was devised and constructed to:

- verify the basic ideas and assumptions behind the proposed model,

- to analyse whether different approaches to inspection will lead to significantly different outcomes (primarily number of observed violations), and
- to examine whether constructing a simulation of the general model and its validation would be justifiable.

Process flowchart of the simulated model with specification of variables that change values in particular steps as well as their context is shown in Figure 3.

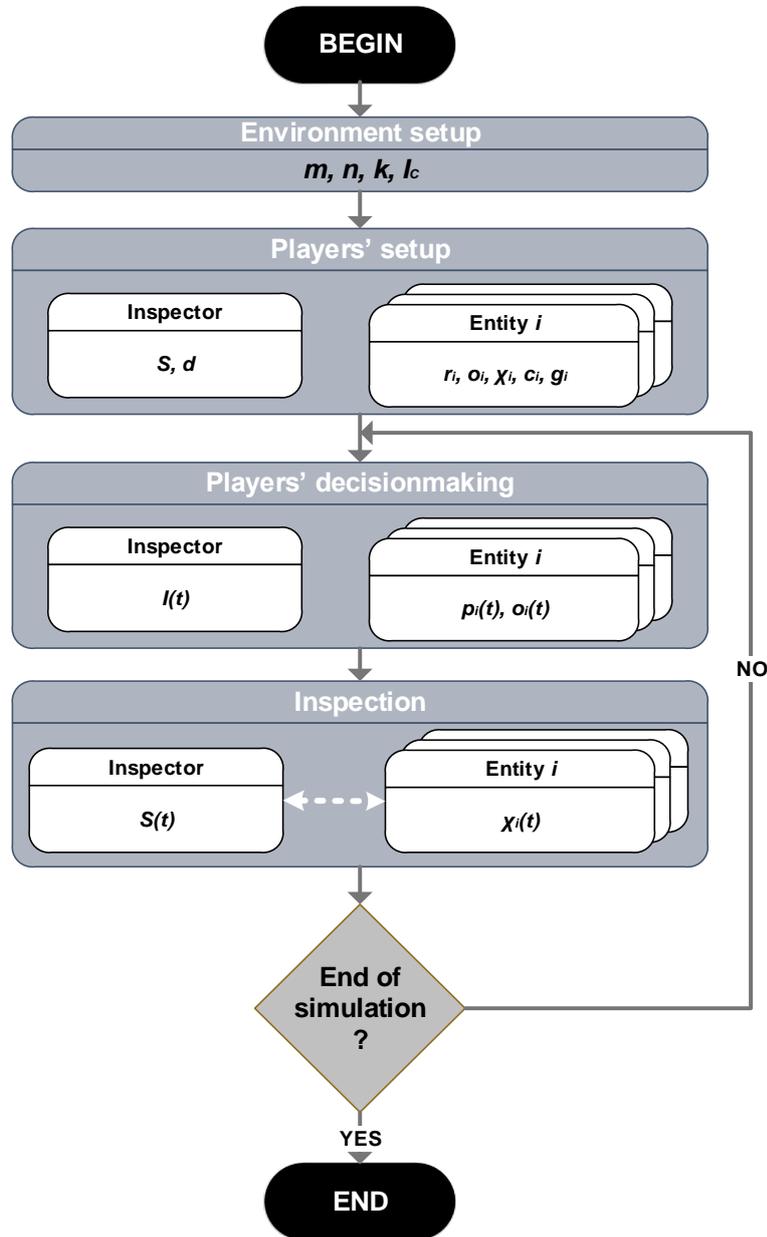


Figure 3. Process flowchart.

The simulation was performed in Microsoft Excel and Visual Basic for Applications while statistical analysis was performed in R [37]. The full data set, complete simulation results and statistical analysis can be found at <http://dx.doi.org/10.6084/m9.figshare.2009472>.

### SIMULATION SCENARIOS

Performed simulation incorporates 4 scenarios. Each scenario models different approach to inspection (different inspection strategy). The scenarios can be described as follows:

- 1) **random** scenario. Inspector completely randomly (uniform distribution) selects  $I_C$  discrete combinations of entities and rules  $\{i, j\}$  for inspection at every discrete time interval  $t$ . The only limitation is that the same combination cannot be inspected more than once in one interval. Random scenario is the baseline inspection strategy,
- 2) **resource** scenario. Inspector randomly selects entities and rules, but while all the entities have the same probability of selection, probability of selection of different rules is defined by the inspector's opinion on the cost of compliance (vector  $d$ ). The selection process can be thought of as a variation of the Fitness proportionate selection algorithm [33], where  $d_j$  has the role of fitness of the rule  $j$ . Hence, the probability of inspecting rule  $j$  of a given entity  $i$  in  $t$  is  $p_{ij} = \frac{d_j I_C}{n \sum_{k=1}^m d_k}$ ,
- 3) **cycle** scenario. Inspector iteratively selects all combinations of entities and rules, in a cyclical fashion. That is, every entity-rule combination will be inspected before any inspection combination is repeated. Cycle scenario reflects some traditional audit practices e.g. that all audit areas have to be reviewed cyclically, at least once every three to five years [34; p.250], and
- 4) **cycle\_resource** scenario. As its name implies, this scenario is a combination of the Resource and the Cycle scenarios. The entities are selected in a cyclical manner, while the rules are selected on a resource-weighted principle. Hence, if entity  $i$  is selected for inspection at  $t$ , the probability of inspection of rule  $j$  of that entity in that interval is  $p_j = \frac{d_j}{\sum_{k=1}^m d_k}$ .

## SIMULATION PARAMETERS

Scenarios have 10 discrete time intervals (“turns”) and each scenario was run 20 times. The scenarios were run for only 10 intervals since in practice, successful inspection strategies would have to produce good results rather quickly to be politically viable. Scenarios were run 20 times to account for randomness (e.g. depending on the order in which rules are selected for inspection, total number of violations might vary considerably) and provide averaged results. Furthermore, repeated runs enable statistical analysis of end results (violations’ totals after 10 turns).

Scenarios are characterized by a set of static initial parameters which are the same for all scenarios and all the runs. In the described simulation scenarios, inspection history has no influence on the inspector's actions.

Table 2 presents simulation parameters that are not entity-specific, while entity-specific parameters are displayed in Table 3.

**Table 2.** Simulation parameters that are not entity-specific.

Variable	Value
Number of entities ( $n$ )	5
Number of rules ( $m$ )	3
Inspector's inspection capacity ( $I_C$ )	4
Impact of detected non-compliance ( $a$ )	2
Impact of detected compliance ( $b$ )	1
Fine for entity caught in violation ( $k_1 = k_2 = k_3 = k$ )	-10
Inspector's assessment of cost of compliance with 1. rule	-1
Inspector's assessment of cost of compliance with 2. rule	-2
Inspector's assessment of cost of compliance with 3. rule	-3

**Table 3.** Entity-specific simulation parameters.

Entity ( $i$ )	Resource requirements ( $c_{ij}$ )			Risk appetite ( $r_i$ )
	$j = 1$	$j = 2$	$j = 3$	
1.	-1,50	-2,40	-3,00	1,2
2.	-0,90	-3,20	-2,70	0,8
3.	-0,90	-2,40	-3,90	1,0
4.	-0,60	-1,60	-2,10	1,1
5.	-1,40	-2,00	-4,50	1,3

The parameters were set according to empirical observations and some practical considerations. The  $n$  and  $m$  parameters were given low values to simulate simple environment but to still allow heterogeneity across entities' through their risk appetite ( $r$ ) and across rules through related compliance costs ( $d, c$ ). The  $I_C$  was set to be in line with the three to five years inspection cycle (see section Simulation Scenarios). Risk appetites of entities vary  $\pm 30\%$  around the risk neutral position, where 3 entities are risk-takers, 1 entity is risk-neutral and 1 entity is risk-averse. Fines or regulatory punishments for violation of all 3 rules were set to the same value, to mimic a simple but realistic setting (see chapter Crime economics and human rationality). Resource requirements (compliance costs) vary significantly across different rules, and even across the same rule, but for different entities, to reflect differences in internal organization, complexity, size, business model, etc. of different organizations. However, relative order of rules' compliance costs is the same for each entity. The values of compliance costs and fines are meaningful in relation to each other.

In this setting, connections between entities are not set, i.e.  $g_{ip} = 0, \{i, p \mid i \in \mathcal{E} \wedge p \in \mathcal{E} \wedge i \neq p\}$ . Entities are not aware of the inspection strategies. Entity  $i$  can estimate inspection probability at  $t$  based on its knowledge of the following variables:  $n, m, k, I_C, r_i, c_i, \chi_i(t)$ . The influence of inspection history and other parameters on entities and their subjective assessment of inspection probability is defined by (19):

$$p_{ij}(t) = \frac{I_C}{nm r_i} \left[ 1 + \sum_{u=1}^l \frac{h_i(t-u)}{k+1} \right], l = 5, i \in [1,5], j \in [1,3]. \quad (19)$$

The subjective assessment of inspection probability (19) also takes into account temporal discounting [35] of results of previous inspections.

Short simulation timeframe (10 turns) prevents introduction of more advanced learning strategies that could be honed during a longer simulation run.

## RESULTS AND DISCUSSION

On micro-level, simulation demonstrates anticipated behaviour of agents, which is expected since micro-level behaviour is determined by the inbuilt rules of behaviour. Several illustrations of predicted micro-level behaviour are:

- 1) Risk-taking entities violate rules more often than risk-neutral or risk-averse entities (*ceteris paribus*). To provide an example: although entity 1 and entity 3 have the same cost of compliance for rule 2, since entity 1 is a risk-taker and entity 3 is risk-neutral, entity 1 sometimes violates rule 2, while entity 3 never does.
- 2) Rules with higher cost of compliance are violated more often than rules with lower cost of compliance (*ceteris paribus*). For example, entity 3 is risk-neutral and entity 4 has slight preference for risk-taking. Cost of compliance with rule 3 is, however, significantly higher for entity 3. Hence, entity 3 violates rule 3 around 26% of times (across all runs and all simulation scenarios), and entity 4 never violates the respective rule.

3) Punishment (caught violation) deters entities from violation for some time, but if there are no subsequent inspections, entities relaps to violation.

On macro-level, simulation again demonstrates expected behaviour. Inspections, in general, reduced the number of violations in the system. Figures 4 and 5 show the results of all simulation runs, grouped by analysed scenarios (inspection strategies). Each diagram presents results of 20 runs of a 10-step simulation for each of 4 simulation scenarios. Abscissae represent the number of steps, while ordinates represent the **cumulative** number of violations observed in each step. Size of the dot represents the number of observations in a certain point, while the line connects means of cumulative violations in each step. The shading around the line presents 95 % CI (confidence interval).

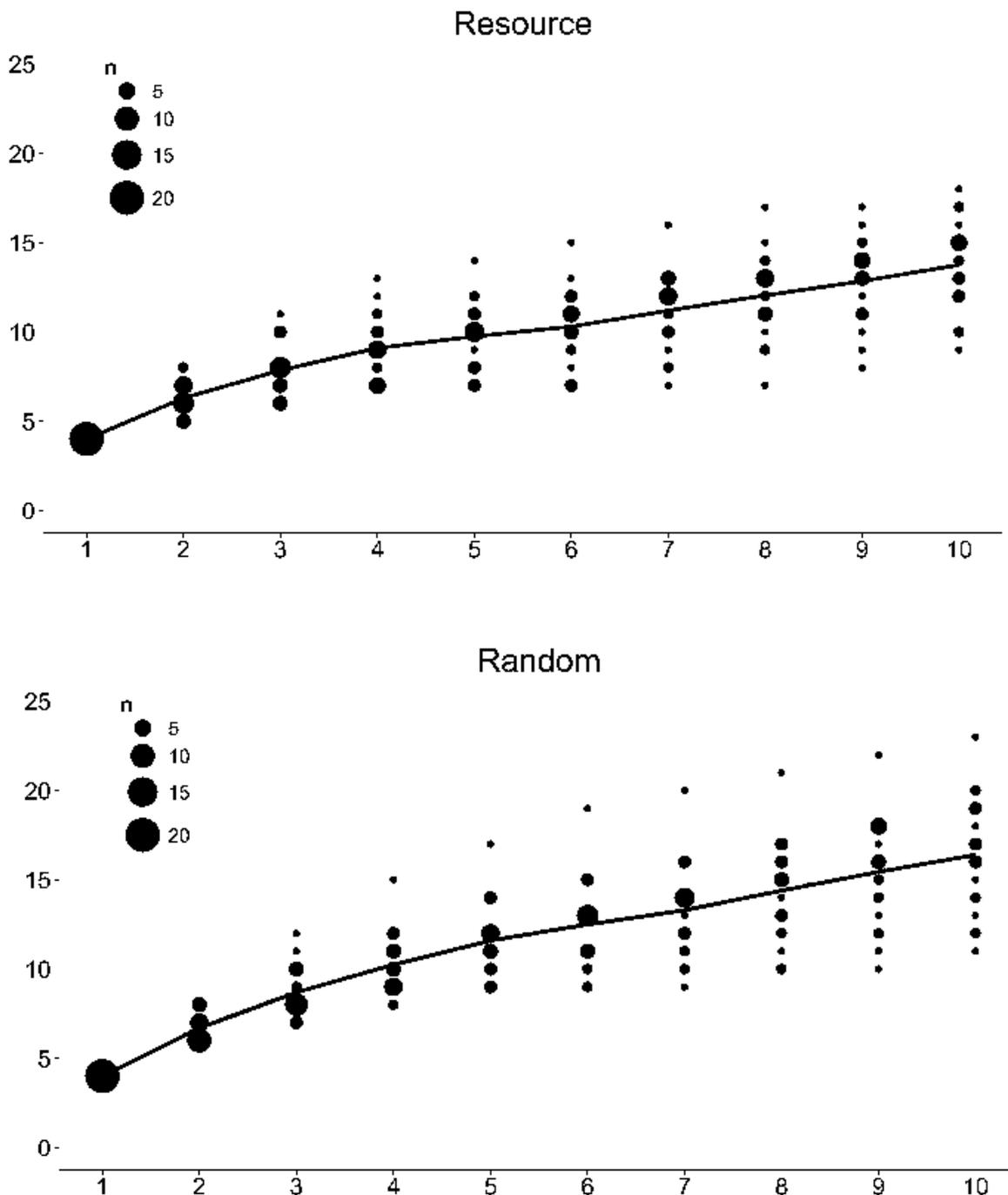
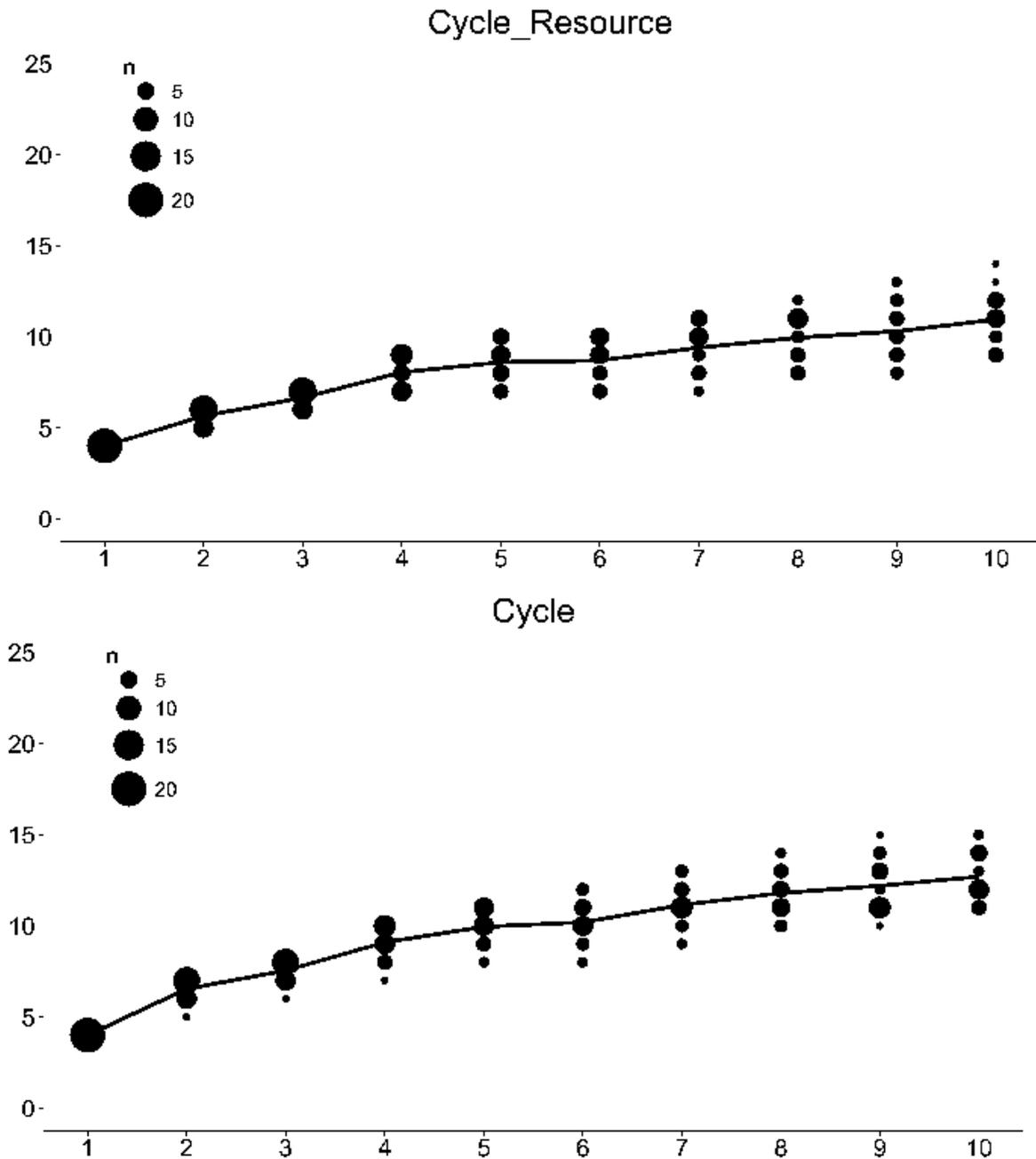


Figure 4. Results of all runs for each scenario.

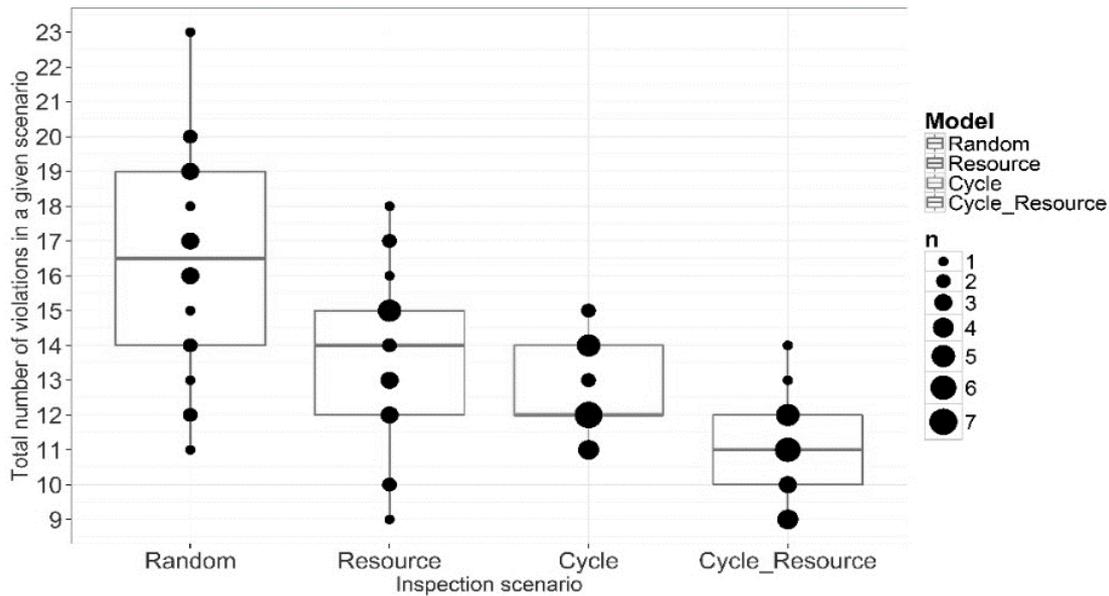


**Figure 5.** Results of all runs for each scenario.

Figure 6 shows violation totals for each run (after 10 steps), grouped by inspection strategies. Boxplots present averaged values, while the dots present results of every single scenario. The size of the dot represents the number of observations in a certain cross-section of the diagram. Respective numerical results are presented in Table 4.

Results shown suggest that there is a significant difference in the cumulative number of violations, dependent on the inspection strategy. One-way ANOVA was performed to test the significance of observed differences in violations. The results confirm that significance,  $F(3,76) = 21,27$ ;  $MSE = 104,03$ ;  $p \ll 0,001$ . Further examination of observed differences was performed via the Tukey's HSD test. Test results are displayed in the Table 5.

Presented results confirm that inspection strategies which considered resource needs performed significantly better than the random strategy. Cycle-based strategies also achieved good results.



**Figure 6.** The violations totals for each of 4 observed inspection strategies.

**Table 4.** Simulation Data Summary.

Simulation Scenario	<i>n</i>	Mean	SD	SE
Random	20	16,40	3,136	0,701
Resource	20	13,75	2,447	0,547
Cycle	20	12,70	1,342	0,300
Cycle_Resource	20	10,95	1,395	0,312
<b>Total</b>	<b>80</b>	<b>13,45</b>	<b>2,942</b>	<b>0,329</b>

**Table 5.** Results of the Tukey's HSD test<sup>6</sup>.

Simulation Scenario Comparison	Difference of means	95 % CI		Adjusted <i>p</i> -value
		Lower	Upper	
Resource - Random	-2,65	-4,487	-0,813	0,0017*
Random - Cycle	3,70	1,863	5,537	0,0000**
Random - Cycle_Resource	5,45	3,613	7,287	0,0000**
Resource - Cycle	1,05	-0,787	2,887	0,4417
Resource - Cycle_Resource	2,80	0,963	4,637	0,0008**
Cycle_Resource - Cycle	-1,75	-3,587	0,087	0,0676

\*adjusted *p*-value < 0,01

\*\*adjusted *p*-value < 0,001

However, the result of cycle-based strategies should be taken with caution, since the relationship between  $I_C$ ,  $m$  and  $n$  in this setup is such that full cycle (inspection of all rules at all entities) can be performed in less than 4 periods, which might be overly optimistic in some inspection environments [26]. Furthermore, it is interesting to observe that Cycle\_Resource scenario achieved better results than the Cycle scenario, albeit with borderline statistical significance.

## CONCLUSION

This article presented ICARUS – an agent-based model for inspection of compliance to many rules. The model was created to address the shortcomings of often-used approaches to the inspection problem such as over-idealization of assumptions, narrowness of application and, in particular, limitation to inspection of the one-inspectee-one-rule situations. The presented model was implemented in a somewhat simplified setting (this includes simplification of values of some parameters as well as rather small number of entities and runs) and the

working hypothesis that conduct of inspections based on knowledge of resource needs for compliance reduces total non-compliance in the system was tested.

The results are promising as conducted simulation demonstrated expected micro-level and macro-level behaviour and showed that resource-focused inspection strategies perform significantly better than random strategies and better than cycle-based strategies. Furthermore, attained results encourage development of a full-scale model and related simulation that should be subjected to extensive testing and validation. Further research should also empirically validate significance and test characteristics of association between compliance resource requirements and observed violation rates. The developed full-scale model and its simulation should enable comparison of effectiveness of various inspection strategies and, through parameter estimation and sensitivity analysis, identification of parts of the parameter space in which those strategies achieve the best results.

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## **REMARKS**

<sup>1</sup>The views expressed in this article are those of the author and do not necessarily reflect the views of the Croatian National Bank.

<sup>2</sup>James Madison was the fourth President of the US and had the key role in development of the US Constitution and the Bill of Rights. He recognized a need for strong central government while, on the other hand, promoting rights of individuals. His work left a lasting effect on legal theory and on our understanding of proper principles and procedures of democratic government [36].

<sup>3</sup>Description and explanation of game-theoretic concepts such as cooperative and non-cooperative game, imperfect and incomplete information, payoffs, pure and mixed strategies etc. are beyond the scope of this article and can be found in [37].

<sup>4</sup>Calculations can be found in [12].

<sup>5</sup>Several terms and acronyms are used in literature: *Agent-Based Modelling* (ABM), *Multi Agent-Based Modelling* (MABM), *Agent-Based Simulation* (ABS), *Multi Agent Simulation* (MAS). The terms are sometimes used interchangeably, although their meanings are not identical.

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# TRUST MODEL FOR SOCIAL NETWORK USING SINGULAR VALUE DECOMPOSITION

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## ABSTRACT

For effective interactions to take place in a social network, trust is important. We model trust of agents using the peer to peer reputation ratings in the network that forms a real valued matrix.

Singular value decomposition discounts the reputation ratings to estimate the trust levels as trust is the subjective probability of future expectations based on current reputation ratings.

Reputation and trust are closely related and singular value decomposition can estimate trust using the real valued matrix of the reputation ratings of the agents in the network.

Singular value decomposition is an ideal technique in error elimination when estimating trust from reputation ratings. Reputation estimation of trust is optimal at the discounting of 20 %.

## KEY WORDS

singular value decomposition, reputation, trust, social network, discounting

## CLASSIFICATION

JEL: C65

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## **INTRODUCTION**

Social networks permeate our lives as no agent life in a vacuum; it must interact with other agents to achieve its goals. With so much user interactions, the question of whom to trust has become an increasingly important question. We rely on trust in our day to day interactions and activities with each other [1-3]. Social networks are central to transmission of information as these networks embed dynamic, stochastic and interdependency behaviour of the agents [3-5]. An agent is likely to encounter agent generated content, some of which the agent uses to make decisions and develop context within a community with respect to whom to trust and why. For agents to interact, they require information on whom to trust [1].

The history of past interactions forms a basis for the agents' abilities and dispositions that can be aggregated together through a good reputation system [6]. Reputation is a collective measure of trustworthiness and an important factor in performing trust decisions. Reputation is what is generally said or believed about a person's or thing's character or standing [3]. In social networks, reputation is a quantity derived from the underlying network and the agent's reputation is visible to all agents [7]. Each agent rates all the other agents in the network to form a real valued matrix of the agents' reputation ratings. These peer to peer reputation ratings are used to estimate trust levels of the agents in the network using SVD technique.

## **TRUST**

A number of methods and models for computing trust have been developed [1, 5, 8-11]. The works agree that trust levels play a central role in interactions of agents. Social interaction structures formed over time tend to separate agents into small interaction groups [10]. These social interactions on networks affect agent activities that should be incorporated in social network models to develop optimal models [12].

In the work of [8], agents optionally expressed some level of trust for the other agents. The expressions become entries for a real valued matrix that is used to predict a known trust value between any two users. [1] computes trust using a path probability in a random graph. For each pair of users  $(x, y)$ , they placed an edge between them with some probability that depends on the direct trust between them denoted by  $t_{x,y}$ . Trust is represented in the interval  $(0, 1)$  by [9]. A value of 1 indicates that the agent is highly trusted and hence blind trust. This model reflects members of social network and differentiates them according to their disposition to trusting somebody.

The basic criteria for judging the quality and soundness of reputation computation engines are highlighted by [13]. Reputation is a perception that an agent has of another's intentions and norms [14]. It is a social quantity calculated based on actions by a given agent and observations made by others in an "embedded social network". Trust is a particular level of subjective probability with which an agent assesses that another agent or group of agents will perform a particular action both before he can monitor such action [15]. We can conclude that trust is a subjective probability or expectation an agent has about another's future behaviour.

We use singular value decomposition (SVD) to extract trust levels from the real valued matrix of the agents' reputation ratings. We discount the reputation ratings by estimating the approximation errors of the matrix. SVD is a matrix approximation technique that is widely researched and a common tool used heavily in recommendation systems, bioinformatics, computer vision and text processing among other applications [3, 16-18]. SVD has an efficient algorithm for its computations; a stable and effective method to split the system into a set of linearly independent components, each of them bearing its own energy contribution [19].

We model the trust levels of agents in the social network using the SVD method which has wide appeal, highly versatile and used extensively in many applications in different areas of research. No known research work that has used SVD to estimate trust from reputation ratings of a real valued matrix. Section three introduces social network and rank application based on SVD. The results are analyzed in section 4 and conclusions are in section 5.

## SOCIAL NETWORKS

We assume that reputation ratings about current interactions are captured and distributed and agents are willing to provide the ratings. Consider a set  $\mathbf{N} = \{1, 2, \dots, N\}$  of agents whose state and interactions in a social network evolve in discrete time  $t$ . We assume that the agents are connected to each other at any given time  $t \in [0, T]$  and thus we have a peer to peer review system for the agent's reputation ratings in the network. Let  $R_i = \{r_{1i}, r_{2i}, \dots, r_{(n-1)i}\}$  be the reputation ratings agent  $i$  receives from the other  $N - 1$  agents in the social network. This peer to peer reputation rating is based on the five star scale: 1 - lowest, 2 - low, 3 - medium, 4 - good and 5 - high, that is,  $R \in [1, 2, 3, 4, 5]$ :

$$R = \begin{cases} r_{ij} = 5, & \text{if } i = j, \\ 1 \leq r_{ij} \leq 5, & \text{if } i \neq j. \end{cases} \quad (1)$$

Each agent is expected to rate the other  $N - 1$  agents. As would be ideal in real life situations, if we were to rate ourselves, we would likely give ourselves a maximum score of 5. The ratings form the entries of a real valued matrix,  $R$  and are bidirectional. These entries of matrix  $R$  are assumed to be the 'raw' trust values of the agents as [1] notes that trust and reputation are closely linked.

Let  $T$  be the matrix of "raw" trust values of the agents. As trust is the future expectation of the current reputation ratings, then  $E(T) = \alpha R = R_k$ . We estimate the "raw" trust values of the matrix  $T$  by discounting the singular values obtained from the SVD with a factor  $\alpha$ . This eliminates the noise which represents the future expectations based on current observations in trust estimation. The noise is eliminated by adopting different accuracy threshold from 10 % to 90 %. The error is defined as [20]:

$$\alpha = \sqrt{\frac{\sum_{i=1}^k \sigma_i^2}{\sum_{i=1}^N \sigma_i^2}}. \quad (2)$$

This is the relative error for a sum of the first  $k$  terms of the SVD outer product expansion [7]. SVD is used for optimal low rank approximation and a partial SVD can be used to construct a rank  $k$  approximation. Given a matrix  $R$ , we can represent it as the product of two orthonormal matrices  $U$  and  $V$  and a diagonal matrix  $S$  as  $R = USV$ . The low rank approximation is to make  $\|\tilde{T} - VV^T \tilde{T}^T\|$  as small as possible for the estimation of the trust levels from the reputation ratings [9]. The matrix  $\tilde{T} = R_k$  can be decomposed with SVD and thus the discounted matrices. We use simulation with  $U \sim (0,1)$ , for  $1 \leq R \leq 5$  based on Matlab version 7.0.1.

We compare trust and reputation levels based on the simulated peer to peer reputation ratings which are a real-valued matrix. SVD extracts the trust matrix by discounting the reputation matrix using SVD.

**Table 1:** Glossary of all matrix names used in the study.

Name	Meaning
$T$	“Raw” trust levels matrix
$R$	Reputation ratings matrix
$\tilde{T} = R_k$	Estimated trust levels matrix (discounted with $\alpha$ )

## RESULTS AND DISCUSSIONS

In Table 2, the Friedman test is for the first seven values of the error term  $\alpha \in [0,1;0,7]$ . We have truncated the last two values in the Table 2 as they basically do not add more information in our analysis. At  $\alpha = 0,1$  the test is statistically significant indicating that in estimating trust from reputation, the noise component is evident. At  $\alpha = 0,2$  and above, the test is not statistically significant. This shows that we can only discount reputation ratings with an error term of 20 % to achieve our desired results in estimating trust from the reputation ratings. Increasing the error term above 30 % does not improve on the model performance. Thus, 20 % is the optimal level of discounting of reputation ratings to estimate trust values of agents in a social network.

**Table 2.** Friedman tests comparison between Trust, Reputation and Noise.

Error term, in %	10	20	30	40	50	60	70
<b>Reputation &amp; Trust</b>	0,0184	0,1573	0,2513	0,1573	0,1083	0,1573	0,1573
<b>Trust &amp; Noise</b>	0,0073	0,1083	0,2513	0,1573	0,1083	0,1573	0,2513
<b>Reputation &amp; Noise</b>	0,0073	0,1083	0,1573	0,1573	0,1573	0,1573	0,2513

In Figure 1, when the three columns are compared, the first is different from the second and the third. These results are similar to those in Table 2 which shows that after the discounting of 20 %, there is no added value with continued noise elimination. This shows that rank approximation in this scenario is ideal when the error term is set at 20 %, any discounting above this level does not improve the model.

In Figure 2, as evident in Table 2, the second and third columns are similar while the first column is different. We observed that the discounting is optimal when  $\alpha = 0,2$  which is depicted in the second column of Figure 2. Generally, there is a high similarity between trust and reputation as noted in [1]. Reputation rating is thus an important factor in estimating trust and this rating is an accurate expected value of trust.

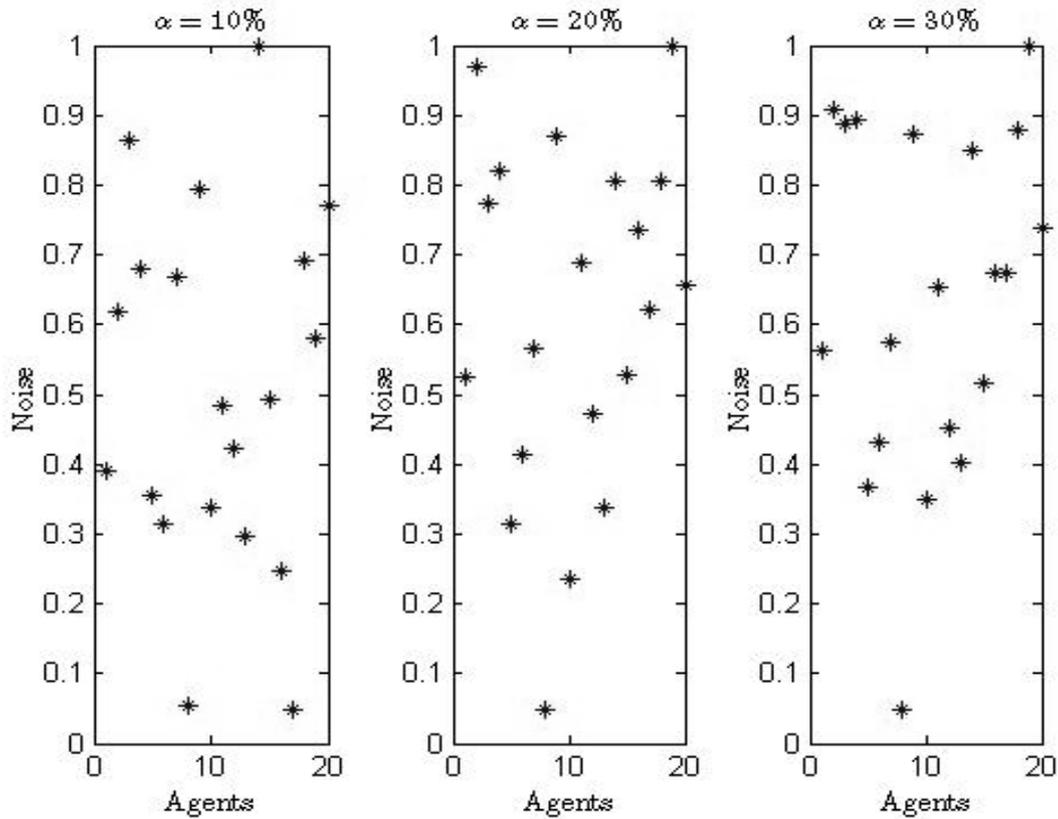


Figure 1. Noise levels for the first three error terms.

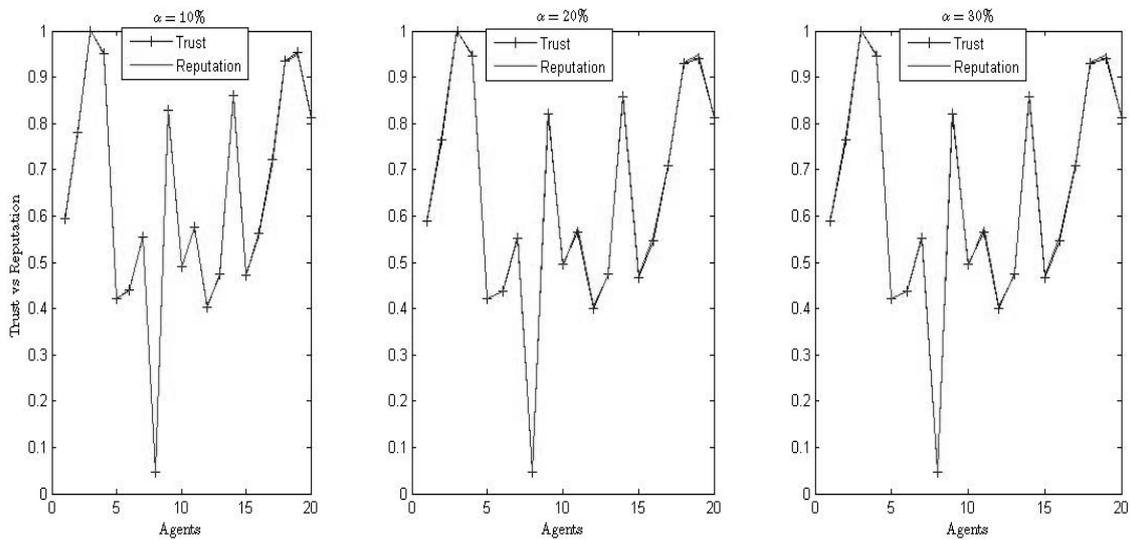


Figure 2. Relationship between trust and reputation levels.

## CONCLUSIONS

Trust values in a social network are estimated based on a real-valued matrix of the agents' reputation ratings. The ratings are discounted using SVD as trust is the future expectations based on the current reputation ratings. An optimal discounting of 20 % is optimal with the noise levels remaining constant after this value. Trust and reputation are observed to be closely related. In general, SVD is a versatile technique that is ideal for estimating trust using reputation ratings. Further research in using it to estimate trust and distrust levels in a social network can be applied.

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# EMPLOYEE LOYALTY: DIFFERENCES BETWEEN GENDERS AND THE PUBLIC AND THE PRIVATE SECTOR

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## ABSTRACT

This article presents the results of the research on important parameters of the loyalty of employees in the public and the private sector. The research was conducted in the Republic of Croatia as a part of the research focused on the essential parameters of the loyalty of employees that contribute to building and retaining loyal behaviour. Loyalty and employee satisfaction are the key parameters that influence the success of a company. In addition, loyal and satisfied employees are important in building good relationships with customers, suppliers and all stakeholders involved in business processes of an organization or a company. The purpose of this article is to define whether there is a difference in parameters of the loyalty of employees in the public and the private sector in Croatia, as well as whether there are certain differences based on gender.

## KEY WORDS

loyalty, employee loyalty management, quality management, employee motivation

## CLASSIFICATION

JEL: I21, J21

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## **INTRODUCTION**

Gaining loyalty of an employee is the highest level of positive interpersonal relationships that a manager in a company can achieve. Sometimes achieving loyalty requires a number of positive circumstances. It is commonly believed that loyalty is achieved only through money and providing employees with high salaries or benefits, but nowadays the situation has changed. The research presented in this paper focussed on determining parameters which incite an individual to become loyal to his or her company or boss and remain at the company where he or she works. The paper provides an explanation on each parameter that affects loyalty and the parameters are ranked according to their importance. Respondents are divided into two groups, those coming from the private and those coming from the public sector. One of the goals of the research was to determine whether there are differences in the perception of the parameters between people working in those two sectors. Data analysis grouped respondents by age, gender and education, and provided the ranking of parameters. Further research should establish to what extent the amount of salary is a motivating factor for employee loyalty and to what extent job security and a regular monthly salary are also motivating factors for employee loyalty.

The aim of this article is to draw attention to the importance of employee loyalty for a company. The article addresses the issue of how employees should be treated in order to become loyal and which conditions that a company provides result in employees' loyalty to a company.

## **LITERATURE REVIEW**

### **EMPLOYEE MOTIVATION**

One of the essential and the most important functions of managers is to manage human resources, which means to employ intelligent and quality people, motivate and educate them, provide them with useful information and knowledge and to help them develop their skills to eventually achieve better results benefiting the company. Human resources management, which implies managing people, their knowledge, creativity and skills, as well as their specific concerns and working climate, is the only feature that is unique for each specific company and can not be copied [1]. Human Resources Management in contemporary organizations has two main tasks: (1) it needs to support the achievement of competitive advantages with the help of employees, with the emphasis on employee motivation and investment in the development of knowledge, skills and behavior, and (2) it needs to contribute to the success of an organization [2]. Research shows that the success of organizations and companies is linked to the quality of performance of the functions and activities such as: compensation and benefits, creating a good organizational climate and culture, and the existence of training programs and learning [3]. Studies also show that the success of organizations is also linked with factors related to employees, such as: (1) security of the transaction, (2) job satisfaction, (3) organizational commitment, (4) the creation of knowledge and (5) the ability of managers and their relationship to employees [4]. Motivation is defined as a generic term for all internal factors that consolidate the intellectual and physical energy, initiate and organize individual activities, direct behavior and determine the direction, intensity and duration of the behaviour. Eventually, it provides an answer to the question why someone behaves in a certain way, which consequently results in operating performance at a certain level [1]. People are different, but the most important thing for managers is to discover how to get people interested. Some people wish to achieve a high status, some are motivated by money, some are satisfied with the pleasant working environment and interpersonal relationships, and some just look for a little respect from colleagues or superiors. There are five basic factors related to behavior used to examine the

theory of motivation: needs, encouragement, knowledge, type of business, sensations / emotions. There are theories according to which an individual is motivated by his or her unsatisfied needs, i.e. if the social life (and the position) of an individual is not satisfactory, the person will be motivated for more activities aimed at satisfying the needs for promotion. This theory is based on empirical findings that show that people tend to repeat behavior that has had positive effects, and to avoid behavior that has had negative effects. According to the theory of cognitive motivation behavior is not only influenced by external factors, but the motivation also involves cognitive processes – values, expectations, beliefs [5]. Work itself holds the key to motivation. It is believed that boring and monotonous jobs stifle motivation, while demanding and diverse jobs encourage it.

## **FINANCIAL PARAMETERS OF EMPLOYEE LOYALTY**

Motivational techniques can be divided into financial and non-financial. Financial techniques are divided into direct: (1) salaries, (2) fees, (3) travel costs, and indirect: (1) social benefits, (2) education, (3) security and (4) other benefits such as a paid annual leave [6]. Many studies show that money has become an obsolete means of motivation, and indirect financial rewards such as seminars, trainings, courses, involvement in specific projects, free use of professional literature, etc. [7] are being increasingly appreciated. All serious organizations therefore need to explore what motivates their employee. Otherwise, companies could invest in the benefits that will not have the desired effect on employee motivation. Buying loyalty by offering attractive salaries has proved to be a short-term measure, so the more conscientious managers are nowadays turning to non-financial rewards, improving the communication skills within the company and working in a positive working environment. The website 'MojPosao' has conducted a research on a sample of 900 respondents examining which parameters affect loyalty the most. Only 41 % of respondents pointed out money and benefits as the important motivational factors. If a company wants to motivate employees by salary and rewards, it should certainly have a fair reward system. Otherwise, it can cause a distortion of human relations in the company [8]. Not only research on employee loyalty but research on customer loyalty has also proved that satisfied and motivated employees are important for a company. A study has tested all important parameters of the loyalty of consumers, and the results have shown that consumers place high-quality after-sales services, such as dealing with complaints, the speed and efficiency of service and respect for customers above the prices of products and services [9]. Therefore, it can be argued that without loyal and satisfied employees there are no loyal and satisfied customers either. In order for the material rewards to increase the performance and efficiency, they should be linked to those indicators of work and performance which individuals or groups can influence by their behavior [10]. The plans that are set need to be achievable, and there must be a clear link between the performance and the reward. Incentives, such as salaries, are divided into individual and group incentives. Individual incentives encourage and give priority to individual results of employees and encourage targeted behavior, which contributes to better quality and a better customer feedback. A disadvantage of individual incentives is that they do not facilitate teamwork and they can cause conflicts that may have adverse effects on the working environment and business operations. Group incentives encourage teamwork, unity and synergy within the working group. They also incite individuals to identify themselves with the group and the company. Teamwork enables the transfer of knowledge and information, which results in higher productivity than individual work. In contrast, a negative effect of collective incentives is that an unsuccessful employee remains unrecognised within a group. The responsibility of an individual for a mistake can be attributed to the whole group or to another member of the group, and it is difficult to level out the commitment of all the members of the group. There are also different types of rewards. The first form of rewarding is a fixed salary. The

advantage of this form of rewarding is that organizations can easily change their work plan without any major resistance of employees, the administrative department calculates salaries faster and the system of net pay is more comprehensible. In case of fixed salaries, the annual budget is predicted easily and the employees are also satisfied with a 'guaranteed' salary. On the other hand, the fixed salary does not motivate workers to work harder. This can lead to conflicts between the employees that work hard and those working less diligently since they all receive the same fixed salary. Workers who are capable and motivated to work hard often leave such companies because their fixed salary does not provide enough satisfaction, and they are aware that somewhere else they could be paid a lot more for their efforts and work [11]. The so-called clean fee reward system is usually used in the sales industry. An advantage of this system is that it encourages employees to work to their maximum capacity and commitment. It encourages creativity, innovation and entrepreneurship. It develops a competitive spirit among employees, tightly linking the movement of marketing costs with the overall results of the work. It allows for a differentiated approach, various types and percentages of reimbursement for various tasks. It also supports diverse policies and strategies of the organization. Research shows that the participation of employees in company profits is also a huge motivation factor [12]. In addition to financial rewards, there are indirect ones as well. They can include a variety of benefits, social benefits, vacations, education and etc. It is especially important to mention that those kinds of material benefits are awarded when hiring employees in a particular company, and they not allocated based on the criteria of company performance or the performance of employees. This system generally aims to keep employees in the company, rather than to improve performance and success [13]. Indirect financial rewards are well suited to achieve the loyalty to the company.

## **NON-FINANCIAL PARAMETERS OF EMPLOYEE LOYALTY**

The first non-financial parameter that affects employee loyalty is job satisfaction. Job satisfaction is a complex attitude which involves certain assumptions and beliefs about the job (a cognitive component), feelings towards work (an affective component), and the work assessment (an evaluation component). A satisfied worker is a productive worker, and the organization's success can not be achieved having dissatisfied employees. The parameters that affect job satisfaction can be divided into two groups, the personal and the organizational parameters.

Organizational parameters that affect job satisfaction are the following:

- 1) Work itself – people are happy if their work involves challenging tasks and not only simple and monotonous ones. Challenging work allows a worker to do a variety of tasks in which there is a freedom of action, but with a feedback on how the job is done.
- 2) Compensation system – in general, people are satisfied with their job if they have a high salary, but the reward system must be fairly built as well.
- 3) Favourable working conditions – if employees have better working conditions their satisfaction increases. Better working conditions create a greater opportunity for better expected operational performance.
- 4) Colleagues at work – work satisfaction is greater if employees work with colleagues with whom they have a good relationship and if there is a pleasant social atmosphere. A correct relationship with a supervisor also contributes to the satisfaction. Employees like having an open relationship with their superiors.
- 5) The organizational structure – if the organizational structure is clear, stable and familiar, employees have a clearer picture of the hierarchy in the company, and they are happy. But if it is not clear or there is no hierarchy, mutual human relations in the company can be disrupted. Some studies have shown that employees prefer decentralized organizations, because in such organisations they have a greater possibility of participation in the decision-making process [14].

Personal parameters that affect job satisfaction are the following:

- 1) The harmony between personal interests and work – when personal interests, knowledge and skills are in correlation with the needs of the workplace, employees are more satisfied with their job. They feel that their workplace allows them to express their opinions, knowledge and personality.
- 2) Length of service and status – older people with more years of service are generally satisfied with their job. This happens not only because a person eventually gets better at his or her job, gives better results and receives more rewards, but also because of the effect of cognitive dissonance. Even those who were not initially satisfied with their work, if they remain at the job for a long time, they convince themselves that they are satisfied with the job. Research has shown that job satisfaction increases with years of service and age, but the increase is not linear. First, job satisfaction increases sharply among the employees in their thirties, because at that age people become more successful at their jobs and their career advances. At the age of 40, people lose many illusions and are less satisfied. In the second half of the fifties and before retiring, job satisfaction of employees grows again.
- 3) Position and status – a higher position, better income, social status and power result in greater job satisfaction.
- 4) Overall satisfaction with life – job satisfaction and overall life satisfaction influence each other. Overall life satisfaction has a positive impact on job satisfaction, and vice versa, job satisfaction has a positive effect on overall happiness [15].

Research conducted in Croatia has shown that even 40 % of employees in Croatia are not satisfied with their job. The majority of employees believe that they have chosen a wrong profession, many people can not get a job in their profession, and some are forced to do the things they are not supposed to do according to the rules their profession. 30 % of respondents say that they do not like their jobs, and 38 % of respondents claim that their satisfaction in the workplace is hindered by poor relations with superiors or colleagues. Only 26 % of respondents are currently working and love everything about their job. The National Happiness Index shows that the least satisfied Croats are the ones between the age of 45 and 54. Men are generally more satisfied with their jobs or their work environment than women, and even 60 % of respondents believe that there is no possibility to make decisions and influence the work they do, which is one of the important factors for job satisfaction [16]. Other factors influencing loyalty include social responsibility [17], as well as free usage of communication channels, such as social media [18].

## **METHODOLOGY**

Respondents were divided into two groups, the respondents from the private and those from the public sector. A questionnaire was distributed among 50 respondents in each sector. The respondents had to rate the importance of each parameter of the loyalty of employees on a scale from 1 to 10, 1 being completely irrelevant and 10 being extremely important. In the second part of the research, participants had to evaluate the significance of the salary for their loyalty to a company. Checking the reliability of the parameter obtained by research we have found that Cronbach's Alpha = 0,925. This leads to the conclusion that the selected parameter has a relatively high reliability that enables further analysis of parameters of employee loyalty.

Respondents in the private sector included 50 % of women and 50 % of men, while in the public sector 25 % of respondents were women and 75 % were men, which amounts to 37 % of female and 63 % of male respondents in total. As far as the age of the respondents is concerned, in the private sector, 45 % of respondents were between the age of 20 and 30, 10 % of respondents were between the age of 31 and 40, 30 % of respondents were between the age of 41 and 50, and 15 % of respondents were between the age of 51 and 60. In the

public sector, 25 % of respondents were between the age of 20 and 30, 30% of respondents were between the age of 31 and 40, 25 % of respondents were between the age of 41 and 50 and 20 % of respondents were between the age of 51 and 60. The research participants also differ by education. In the private sector, 15 % of participants have finished only primary school, 40 % have secondary education, 30 % have a bachelor's degree, 15 % have a master's degree and there are no respondents with a PhD degree. In the public sector, 45 % of participants have completed primary school, 40 % have secondary education, 5 % have a bachelor's degree, 10 % have a master's degree and there are no respondents with a PhD degree. In the public sector, 65 % of respondents are just workers in the public administration, 25 % of them perform clerical work, 5 % have positions in the middle management, and 5 % of them belong to the top management. In private companies, 55 % of employees are just workers, 30 % of employees have a civil service position, 5 % of them belong to the lower management and 10 % to the senior management.

## RESULTS

Table 1 compares the public and private sector according to the average grade and standard deviation of the responses related to the employee motivations. Results reveal that workers in the private and the public sector estimate the parameters of loyalty quite differently. The most important parameter in the public sector is the respect from superiors, and in the private sector good business communication and a pleasant working environment.

**Table 1.** Employee motivations for the public and the private sector (Likert 1-10)

Item	Public sector	Private sector
Good motivation for work	7,80 (1,576)	8,55 (1,356)
Good business communication and a pleasant working environment	8,30 (1,261)	9,20 (1,795)
A successful team leader / organization	8,25 (1,832)	9,10 (1,373)
Job satisfaction	8,40 (1,273)	8,80 (0,894)
Direct financial rewards	8,20 (1,361)	8,95 (2,064)
Indirect financial rewards	6,75 (2,074)	8,10 (2,382)
Opportunity for advancement	7,55 (1,605)	8,75 (1,943)
Status	6,55 (2,114)	7,65 (1,981)
The sense of achievement and success	8,00 (1,376)	8,00 (1,777)
Achievement of objectives	7,80 (1,361)	8,10 (1,447)
Respect from co-workers	8,85 (0,875)	8,25 (1,943)
Respect from superiors	8,90 (0,968)	7,80 (2,093)
Ability to work autonomously	8,10 (1,447)	8,15 (1,182)
Good relationship with superiors	7,95 (1,701)	8,50 (1,395)

Table 3 shows parameters of loyalty for women and men, regardless of whether they work in the private or the public sector. For women, the most important parameter refers to good working conditions, while for men the first place was shared by two parameters, i.e. successful team leader / organization and respect from co-workers. For women, the least important parameter refers to indirect financial rewards, and for men the status which they have in the company. Direct financial rewards took the third place among women and the sixth place among men. It can be assumed that direct financial rewards are not the crucial and the key factor in building employee loyalty.

**Table 3.** Differences in ranking parameters by women and men.

<b>Women (37 %)</b>	<b>Men (63 %)</b>
Good business communication and a pleasant working environment	Successful Team Leader / Organization
Job satisfaction	Respect from co-workers
Direct financial rewards	Respect from superiors
Successful Team Leader / Organization	Good business communication and pleasant working environment
Good motivation for work	Job satisfaction
Opportunity for advancement	Direct financial rewards
Ability to work autonomously	Good relationship with superiors
Good relationship with superiors	The sense of achievement and success
Respect from co-workers	Ability to work autonomously
Achievement of objectives	Good motivation for work
The sense of achievement and success	Opportunity for advancement
Status	Achievement of objectives
Respect from superiors	Indirect financial rewards
Indirect financial rewards	Status

However, the direct question in the second part of the research was “To what extent is the amount of salary a motivating factor for employee loyalty, and loyalty to the employer?” Obtained replies clearly show the attitudes of respondents. The respondents had to rate two statements related to the amount of wages and job security on a scale of 1-5. The research results show that no one considers that salary is not a motivating factor. 7 % of people think that salary is a motivating factor of medium significance, 58 % of respondents believe that salary affects the loyalty to a significant extent, and 35 % of respondents said that salary affects employee loyalty to a company profoundly. These results show that the salary is still the most powerful factor that keeps an employee in a company.

Other important direct question in the second part of the research was “To what extent is job security and a regularly monthly salary are motivating factors for employee loyalty. Not any of the respondents claimed that job security and regular monthly salaries are not a motivational factor for employee loyalty. Only 2 % of people feel that they are an insignificant motivating factor, 20 % of people think that they are a factor of medium significance, 38 % of people think that they have a lot of influence on employee loyalty, and 40 % of people think that job security and a regular salary largely affect employee loyalty.

## **DISCUSSION**

Since the sample includes working-age citizens employed in the private and the public sector, and respondents are divided by gender, education and age, their ranking of important parameters of loyalty is compared according to these characteristics. With respect to that, in this study, we aim to test the following hypothesis:

**H1:** There is a difference in assessment of the key parameters of the loyalty of employees based on gender, age and the level of education and the position of respondents in an organization.

By examining correlations between parameters it has been found that among the respondents in the public sector, there is a very high positive correlation between the following parameters:

- Good motivation for work and ability to influence important decisions and behavior of others,  $r = 0,703$  with a significance level  $Sig. = 0,032$ ,

- Job satisfaction and ability to influence important decisions and behavior of others,  $r = 0,760$  with a significance level  $Sig. = 0,0001$ ,
- The possibility of promotion and the chance to influence the important decisions and behavior of others,  $r = 0,704$  with a significance level  $Sig. = 0,001$ ,
- Job satisfaction and ability to influence important decisions and behavior of others,  $r = 0,760$  with a significance level  $Sig. = 0,0001$ ,
- The possibility of progress and the status in the organization,  $r = 0,790$  with a significance level  $Sig. = 0,0001$ ,
- The possibility of progress and achievement of objectives,  $r = 0,703$  with a significance level  $Sig. = 0,001$ ,
- The possibility of promotion and the chance to influence the important decisions and behavior of others,  $r = 0,704$  with a significance level  $Sig. = 0,001$ ,
- The status of the organization and the possibility of promotion,  $r = 0,790$  with a significance level  $Sig. = 0,0001$ ,
- The sense of achievement and success and achievement of objectives,  $r = 0,702$  with a significance level  $Sig. = 0,001$ ,
- Respect from co-workers and respect from superiors,  $r = 0,789$  with a significance level  $Sig. = 0,0001$ ,
- Respect from co-workers and a good relationship with superiors,  $r = 0,737$  with a significance level  $Sig. = 0,0001$ ,
- Respect from co-workers and recognition for a job well done,  $r = 0,729$  with a significance level  $Sig. = 0,0001$ ,
- Recognition for a job well done and respect from co-workers,  $r = 0,729$  with a significance level  $Sig. = 0,0001$ ,
- Recognition for a job well done and a good relationship with superiors,  $r = 0,766$  with a significance level  $Sig. = 0,0001$ ,
- Good working conditions and a sense of achievement and success,  $r = 0,725$  with a significance level  $Sig. = 0,0001$ .

In the private sector, a high correlation was found to exist between the following parameters:

- Good motivation for work and direct financial rewards,  $r = 0,744$  with a significance level  $Sig. = 0,0001$ ,
- Good business communication and direct financial rewards,  $r = 0,855$  with a significance level  $Sig. = 0,0001$ ,
- Good business communication and career prospects,  $r = 0,785$  with a significance level  $Sig. = 0,0001$ ,
- Good business communication and status in the organization,  $r = 0,746$  with a significance level  $Sig. = 0,0001$ ,
- Good business communication and respect from co-workers,  $r = 0,770$  with a significance level  $Sig. = 0,0001$ ,
- Good business communication and a good relationship with superiors,  $r = 0,778$  with a significance level  $Sig. = 0,0001$ ,
- Direct financial rewards and a good motivation for work,  $r = 0,744$  with a significance level  $Sig. = 0,0001$ ,
- Direct and indirect financial rewards,  $r = 0,804$  with a significance level  $Sig. = 0,0001$ ,
- Direct financial rewards and career prospects,  $r = 0,758$  with a significance level  $Sig. = 0,0001$ ,
- Direct financial rewards and the status in the organization,  $r = 0,897$  with a significance level  $Sig. = 0,0001$ ,
- Good relationship with superiors and good working conditions,  $r = 0,754$  with a significance level  $Sig. = 0,0001$ .

This article utilizes the Kruskal-Wallis test. The null hypothesis of the test is based on the assumption that the middle population is equivalent for all groups of population. To test the null hypothesis, respondents are divided by gender, education and age. Using the Kolmogorov-Smirnov test for normality, it was established that no variable has a normal distribution of data. Since the significance level is less than  $\alpha = 0,05$  (*Sig.* < 0,05), and it is  $\sim 0,000$  for all the variables, the assumption of normal distribution of data can be rejected. The test results show that the hypothesis has a statistically significant coefficient, so we can conclude that among the respondents in the public sector, there is a statistically significant difference in key parameters of loyalty of employees related to the age. We have found that these are the following parameters:

- Job satisfaction at the significance level *Sig.* = 0,039,
- The amount of salary based on actual results at the significance level *Sig.* = 0,032.

When respondents employed in the public sector are observed taking into consideration their education, we can conclude that there is a statistically significant difference in key parameters of the loyalty of employees regarding the following parameter:

- Incentives to better, more intensive and more quality work at the significance level *Sig.* = 0,045.

When reviewing the respondents employed in the private sector, we can conclude that there is a statistically significant difference in key parameters of the loyalty of employees related to the gender. We have found that these are the following parameters:

- Job satisfaction at the significance level *Sig.* = 0,046,
- The possibility of promotion at the significance level *Sig.* = 0,044,
- The sense of achievement and success at the significance level *Sig.* = 0,047.

When observing and comparing the respondents employed in the public and the private sector together, we can conclude that there is a statistically significant difference in key parameters of the loyalty of employees related to their qualifications. We have found that these are the following parameters:

- The possibility of promotion at the significance level *Sig.* = 0,01,
- The amount of salary based on actual results at the significance level *Sig.* = 0,013.

## **CONCLUSION**

This research confirmed that there are a few parameters of the loyalty of employees dependent on education, gender and age, and the sector of employment, i.e. the private or the public sector. The sample included employees of the private and the public sector in Croatia. The results of the research presented in this paper can be used as guidelines for developing management of the loyalty of employees in the Republic of Croatia. The results of the research can also provide guidance for further research of key parameters of the loyalty of employees. When respondents are analysed by gender, the first three parameters are parameters that are related to working conditions, job security, recognition and respect by superiors. When respondents are analysed by gender and by sector in which they work, it is evident that the salary and direct financial rewards are important parameters for men and women employed in the private sector, while women in the public sector prefer job satisfaction, good working conditions, good business communication and a pleasant work environment, and men prefer respect from colleagues and superiors and recognition for a job well done. As far as the question of the importance of salary as a motivational factor for the employee loyalty is concerned, 35 % of respondents rated salary as the most important motivational factor for loyalty to the employer, and 40 % of respondents rated job security and a regular salary as the most important parameters of employee loyalty.

The correlation between the tested parameters of employee loyalty exists both in the private and the public sector, but only in the private sector there is a very high correlation between salaries or direct financial rewards, and (i) good motivation for work, (ii) good business communication, (iii) indirect financial rewards, (iv) opportunities for promotion and (v) the status of the organization. The scientific contribution of this paper is the definition of the parameters that employees in the private and the public sector assessed differently based on their education, gender and age, which is confirmed by a statistical test. These differently assessed parameters include the parameters of job satisfaction and salary among the respondents in the public sector analysed taking into consideration the age of the respondents. The parameter of incentives for better, more intensive and quality work in respondents in the public sector taking into consideration the qualifications of the respondents. The parameter of job satisfaction, advancement opportunities and a sense of achievement and success in respondents in the private sector taking into consideration their gender. When it comes to the private and the public sector together and taking into consideration the qualifications of the respondents, the significant difference is observable in the parameters of the opportunity for advancement and salary. Based on the above, we can confirm the hypothesis of this research saying that there is a difference in assessment of the key parameters of the loyalty of employees based on gender, age, the level of education and the position in the organization of respondents in the public and the private sector in Croatia.

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# A ROADMAP FOR A COMPUTATIONAL THEORY OF THE VALUE OF INFORMATION IN ORIGIN OF LIFE QUESTIONS

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## ABSTRACT

Information plays a critical role in complex biological systems. Complex systems like immune systems and ant colonies co-ordinate heterogeneous components in a decentralized fashion. How do these distributed decentralized systems function? One key component is how these complex systems efficiently process information. These complex systems have an architecture for integrating and processing information coming in from various sources and points to the value of information in the functioning of different complex biological systems. This article proposes a role for information processing in questions around the origin of life and suggests how computational simulations may yield insights into questions related to the origin of life.

Such a computational model of the origin of life would unify thermodynamics with information processing and we would gain an appreciation of why proteins and nucleotides evolved as the substrate of computation and information processing in living systems that we see on Earth. Answers to questions like these may give us insights into non-carbon based forms of life that we could search for outside Earth.

We hypothesize that carbon-based life forms are only one amongst a continuum of life-like systems in the universe. Investigations into the role of computational substrates that allow information processing is important and could yield insights into: 1) novel non-carbon based computational substrates that may have “life-like” properties, and 2) how life may have actually originated from non-life on Earth. Life may exist as a continuum between non-life and life and we may have to revise our notion of life and how common it is in the universe. Looking at life or life-like phenomenon through the lens of information theory may yield a broader view of life.

## KEY WORDS

origin of life, artificial life, life-like systems, information theory, reaction-diffusion systems

## CLASSIFICATION

JEL: O10, O35

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## INTRODUCTION

Information plays a critical role in complex biological systems. Complex systems like immune systems and ant colonies co-ordinate heterogeneous components in a decentralized fashion. How do these distributed decentralized systems function? How is the immune system able to find rare amounts of pathogens in infections and mount a response to eliminate them without any centralized control? One key component is how these complex systems efficiently process information. Previous research highlights the need for:

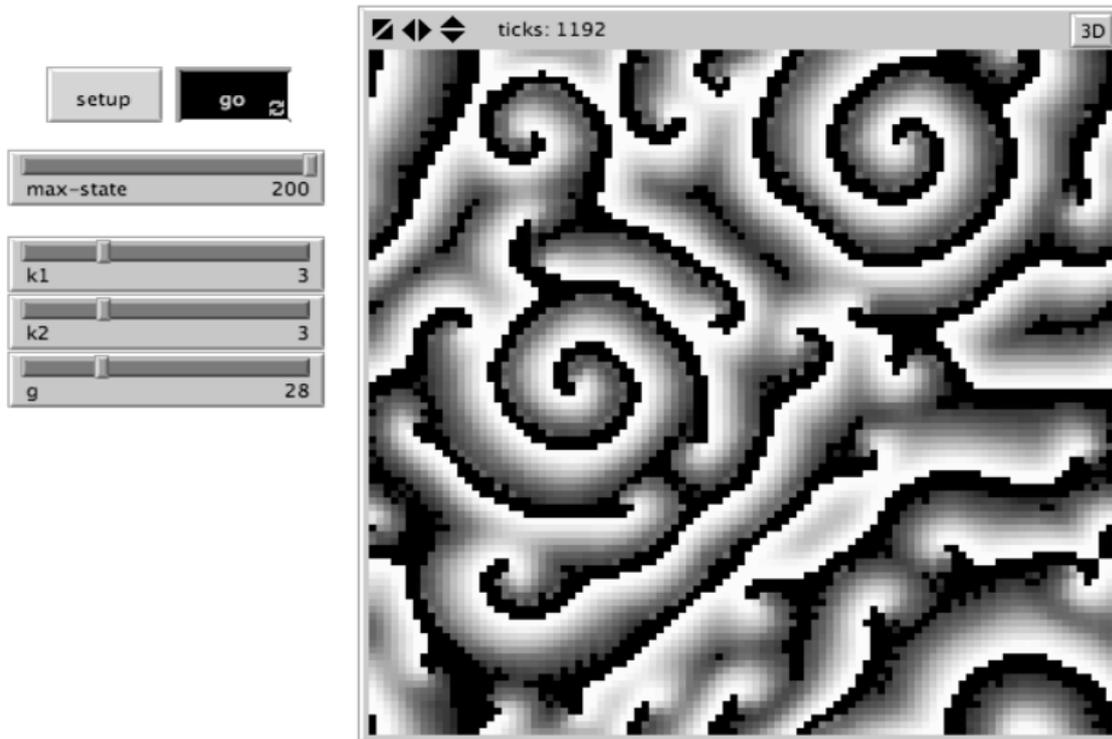
- 1) Specialized physical structures (called lymph nodes) that facilitate the otherwise serendipitous encounter of rare pathogen-specific immune system cells with their fated pathogens
- 2) Signals (chemicals called chemokines) that help in detecting pathogens and guiding immune system cells towards them, and
- 3) Associated infrastructure like receptors on cell membranes and signalling networks (“hardware”) for interpreting these diverse cues and orchestrating a response that is both decentralized and robust [1-4]. These complex systems have an architecture for integrating and processing information coming in from various sources and points to the value of information in the functioning of different complex biological systems. This paper proposes a role for information processing in questions around the origin of life.

It has been suggested previously that information processing capabilities distinguish life from other so-called non-living matter [5, 6]. Information processing is one amongst many key ingredients for life. We propose computationally testing the value of information in origin of life questions using spatially explicit models.

Living systems are complex adaptive systems. Physical space plays an important role in life. Specialized structures, for example cell membranes (“hardware”), serve both as an effective compartment as well as to integrate information (“software”) from extra-cellular and intracellular sources. The software and hardware of life have co-evolved to achieve efficient information processing.

Investigations novel computational substrates that allow information processing is important and could yield insights into novel non-carbon based computational substrates that may have “life-like” properties [7, 8]:

- 1) systems of oil droplets exhibit life-like properties: they are able to replicate and consume energy [9, 10]. Oil droplets also have been engineered to have compartments and derive propulsive power from an external energy source [11],
- 2) stars have an energy source and compartments. Disturbances from stars that undergo supernova at the end of their lifetimes lead to star formation in neighbouring galactic clouds and nebulae (which is conceptually similar to replication),
- 3) spherical droplets, self-propelled colloids [12] and motile crystals [13] move about in remarkably life-like ways. Microscopic beads of artificially engineered materials exhibit swarming behaviour under the influence of an electric field [12],
- 4) reaction-diffusion systems like the Belousov-Zhabotinsky (B-Z) reaction are chemical oscillators and also display complex properties reminiscent of life (Figure 1). Reaction-diffusion systems have been exploited to construct computational systems called “reaction-diffusion computers” [14, 15],
- 5) quantum systems are capable of transferring information like spin and have “life-like” properties [16], and
- 6) finally weather systems like hurricanes persist for long times [7]; even weather systems on other planets like the Great red spot on Jupiter has persisted for a very long time and displays complex behavior.



**Figure 1.** Screenshot from the NetLogo simulation tool for the Belousov-Zhabotinsky (B-Z) reaction showing wave like patterns that persist [17, 18].

Our ultimate aim is to develop a framework for an information theoretic definition of life that is independent of the subjective definitions of only carbon-based life. What life-forms could conceivably arise in our known universe subject to the known laws of physics? We hypothesize that carbon-based life forms are only one amongst a continuum of life-like systems in the universe.

Investigations into the role of computational substrates that allow information processing is important and could yield insights into novel non-carbon based computational substrates that may have “life-like” properties [7, 8] and even how life may have actually originated from non-life on Earth.

## AN INFORMATION THEORY OF THE ORIGIN OF LIFE

We hypothesize that the key components of an information-theoretic view of life are:

- 1) information processing (software),
- 2) information storage (memory),
- 3) the physical substrate (hardware) and the role of physical space,
- 4) information transfer (across both physical space and time),
- 5) persistence of information, and
- 6) energy and thermodynamics (energetic limits on information processing and life).

We expand on the possible properties of the substrate that allows information processing, information storage and information propagation:

- 1) Information processing. Systems of oil droplets are capable of complex behavior like replication [9, 10, 19]. Computational simulations of these systems may shed light on whether these systems would be capable of information processing. The system can be evolved using a genetic algorithm. The artificial “genome” would be the size of the droplet, its speed, charge on its surface, ambient air pressure and temperature. One test for

information processing would be verifying if an oil droplet can divide based on whether there are two other droplets in contact with it (this would function like a primitive AND gate). Such a simple model of information processing may allow for the emergence of co-operative behavior that would enhance survival. Similar approaches have been used in reaction-diffusion systems without compartments [14, 15].

- 2) Information storage and memory. Information storage and memory are critical components of life and life-like systems. We suggest testing if systems of oil droplets would be capable of storing information persistently. These systems are capable of a limited form of heredity [8] and some limited form of memory [10]. We suggest incorporating other factors in order to ensure that these systems remember their shape and have persistence. These factors would be additional physical structures (resembling a cellular cytoskeleton that would provide a physical scaffold on which to store information) and compartments that would allow richer dynamics within droplets to be coupled to cues outside the surface (similar to biological cell compartments and cell membranes).
- 3) The physical substrate (hardware) and the role of physical space. Physical space and structures play a role in facilitating information processing. We suggest testing if there are special shapes of oil droplets that may allow efficient information processing and propagation.

In the immune system, dendritic cells with a large surface area and protrusions specialize in information transfer (by maximizing the surface area over which interactions occur and the number of interactions with other cells). Computational simulations may yield oil droplets with specialized shapes that are optimal for information transfer.

The immune system also has physical structures called lymph nodes that are information processing centers: they facilitate the interaction and information transfer between immune system cells and pathogens. Additional constraints in the oil droplet simulation that allow for physical structures (“artificial lymph nodes”) would also facilitate information propagation between different types of oil droplets that are also spatially separated (discussed below).

- 4) Information transfer and propagation in space and time. Simulate propagation of information and the evolution of co-operative behavior. We recommend computationally simulating other “species”: smaller droplets that can propagate and relay information faster (like small molecules called chemokines in the immune system that are specialized for carrying information). It would also be fruitful to investigate the effect of introducing these “species”:
  - a) specifically testing if this would allow for the emergence of co-operative behavior and co-operative information processing amongst different droplets,
  - b) testing if faster propagation of information is good for the whole system since beneficial mutations can spread fast but so can deleterious mutations, and
  - c) testing the role of both short-range and long-range interactions in spreading innovative mutations (where mutations would be changes to the virtual “genome” mentioned earlier).
- 5) Persistence of information. A strong requirement for life and life-like systems is that information should persist. This would result in selection of attributes that may also be heritable. We note that conceptually, selection and heredity, and memory and persistence of information, are similar and point to the survivability and robustness of these systems. Thinking about the B-Z system we note that certain model parameters that lead to persistent wave-like patterns (Figure 1) may have a selective advantage, i.e. these behavior or parameters may become dominant in a population. Computational exploration of the

parameter space of these systems may give insights into if these systems are capable of long-term persistence and if there are particular behaviours or model parameters that have a “selective” advantage.

- 6) Energy: energetic limits on information processing. We suggest computationally simulating the energetic demands of systems like oil droplets. The simulations would have an explicit energy term, which would be varied over a realistic range. The effects of changing energy on the resulting dynamics and information processing capabilities of the system can then be readily observed.

Such a computational model should allow us to test:

- a) if features of information processing should emerge faster, and
- b) the rate of information processing itself is faster based on the energy or temperature of the system.

There may be an optimal range of energy and temperatures for information processing to be viable: too cold and molecular motion would cease; too hot and critical physical structures capable of sustaining information processing would dissociate faster than they would arise. Thermodynamics is thought to play an important role in the evolution of life and life-like systems [8, 20]. Simulations like these may help eventually unify thermodynamics and information processing under a common theory.

## DISCUSSION

One key characteristic of life is the ability to process information. We suggest a framework and roadmap of computational experiments that will give insights into the role of information processing and storage in non-living systems and life-like systems. This may yield insights into how the transition from non-life to life occurred on Earth and give insights into questions like:

- 1) How common is life in the universe? If information is fundamental to life [5, 6] then energetic limits on information processing and a physical substrate capable of supporting and storing information may be vital constraints on life. Research into an information theory of life would yield an understanding of the energetic limits on life and life-like systems with implications for how common life is in our current energy rich universe. If life exists as a continuum between non-life and life, we may have to revise our notion of life and how common it is in the universe.
- 2) What fundamental principles explain the emergence of life? Information and thermodynamics maybe some of the key principles that explain the emergence of life. This paper suggests a framework that would enhance our understanding of how the interaction of information processing with thermodynamics can lead to life-like properties.

## CONCLUSION

Information processing is fundamental to life. A computational model of the origin of life would unify thermodynamics with information processing. Such a model would enhance our understanding of how the interaction of information processing with energy and thermodynamics may lead to life-like properties. This would also lead to an understanding of the energetic limits on information processing in life-like systems. We hypothesize that the key components of an information theory of life should include:

- 1) information processing (software),
- 2) information storage (memory),

- 3) the physical substrate (hardware) and the role of physical space,
- 4) information transfer (across both physical space and time),
- 5) persistence of information (selection and heredity), and
- 6) energy and thermodynamics (energetic limits on information processing and life).

Such a theory would give us an appreciation of why proteins and nucleotides evolved as the substrate of computation and information processing in living systems that we see on Earth. Are other computational substrates viable and energetically feasible? What forms of substrates capable of information processing could conceivably exist? Answers to questions like these may give us insights into non-carbon based forms of life that we could search for outside Earth.

This could also conceivably give us an understanding of the viability of information processing on exoplanets outside the habitable zone and the kinds of “biospheres” that might be able to sustain such life or life-like systems. We note that our approach is complementary to a non-von Neumann computing paradigm that has been investigated in the context of artificial life and biologically inspired computing [21].

Based on our co-existence and co-evolution on Earth for millions of years we have been conditioned to believe that life should be similar to what we observe on this planet. Mathematical principles coupled with a mathematical definition of life, rather than observational evidence based on what we see on Earth, would better prepare us for life or life-like forms that may exist elsewhere in the Universe.

Life as we know it is based on the computational substrate of chemical bonds, e.g. protein interaction networks are based on the principle of shape recognition and chemical bonds: proteins bind to the surface of other proteins based on shape complementarity and thereby setup complex information cascades. Arthur C. Clarke wrote imaginatively about complex intelligent life arising from electrical currents in superconductors on a cold seemingly lifeless planet [22]. He imagined the electrical currents as being only slowly attenuated (due to superconductivity) and ultimately leading to neuron-like networks capable of intelligence. He proposed a completely different computational substrate: electrical currents in superconductors. The story challenges our imagination and although unlikely to be feasible, challenges the very notions of life.

Our ultimate aim is to develop a framework for an information theoretic definition of life that is independent of the subjective definitions of only carbon-based life. What life forms could conceivably arise in our known universe subject to the known laws of physics? We hypothesize that carbon-based life forms are only one amongst a continuum of life-like systems in the universe.

Life may exist as a continuum between non-life and life and we may have to revise our notion of life and how common it is in the universe. Looking at life or near-life through the lens of information theory may yield a broader view of the origin of life that the general public may find of interest.

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# MODELING, CONTROL AND NAVIGATION OF AN AUTONOMOUS QUAD-ROTOR HELICOPTER

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## ABSTRACT

Autonomous outdoor quad-rotor helicopters increasingly attract the attention of potential researchers. Several structures and configurations have been developed to allow 3D movements. The quadrotor helicopter is made of a rigid cross frame equipped with four rotors. The autonomous quad-rotor architecture has been chosen for this research for its low dimension, good manoeuvrability, simple mechanics and payload capability. This article presents the modelling, control and navigation of an autonomous outdoor quad-rotor helicopter.

## KEY WORDS

autonomous, quad-rotor helicopter, modelling control and navigation, 3D movements, sensor system

## CLASSIFICATION

JEL: Z19

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## **INTRODUCTION**

Autonomous outdoor quad-rotor helicopters increasingly attract the attention of potential researchers [1]. In fact, several industries require robots to replace men in dangerous, boring or onerous situations. A wide area of this research is dedicated to aerial platforms. Several structures and configurations have been developed to allow 3D movements, there are blimps, fixed-wing planes, single rotor helicopters, bird-like prototypes, quad-rotors, etc. The autonomous quadrotor helicopter is a small agile vehicle controlled by four rotors. The quadrotor architecture has low dimensions, good manoeuvrability, simple mechanics and payload capability [2, 3].

The quadrotor UAV is controlled by angular speeds of four motors, each motor produces a thrust and a torque, whose combination generates the main trust, the yaw torque, the pitch torque, and the roll torque acting on the quadrotor. Motors produce a force proportional to the square of the angular speed and the angular acceleration. Each motor produces a thrust and a torque, whose combination generates the main trust, the yaw torque, the pitch torque, and he roll torque acting on the quadrotor. Motors produce a force proportional to the square of the angular speed and the angular acceleration [4].

The quad-rotor helicopter configuration is well known and has been studied since the beginning of 1900s.

The study of kinematics and dynamics helps to understand the physics of the quadrotor and its behaviour [5-10]. Together with modelling, the determination of the control algorithm structure is very important.

The electrically powered four-rotor quad-rotor helicopter architecture has been chosen for this research for its low dimension, good maneuverability, simple mechanics and payload capability (Figure 1).

This structure can be attractive in several applications, in particular for surveillance, for imaging dangerous environments and for outdoor navigation and mapping [11, 12].



**Figure 1.** Quad-rotor helicopter.

The article is organized as follows:

Section 1: Introduction.

Section 2: The modeling of the autonomous quad-rotor helicopter is presented.

Section 3: The control strategy are presented.

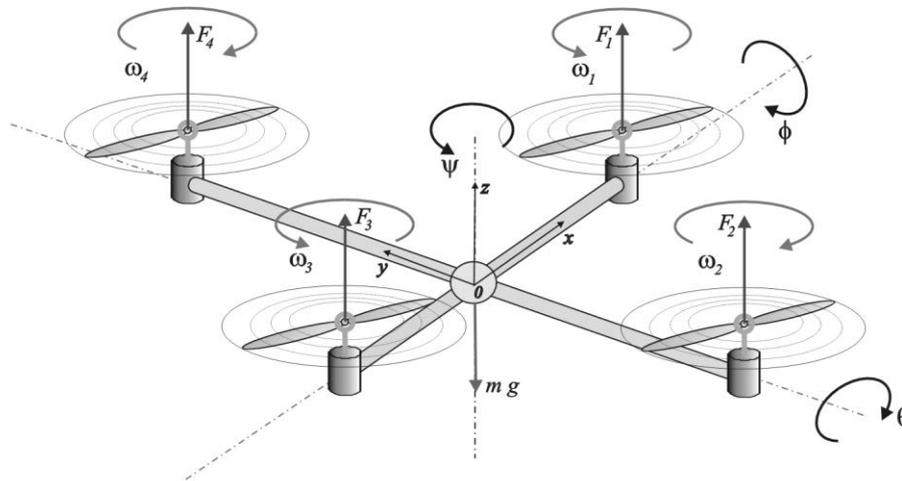
Section 4: The GPS navigation of the autonompus quad-rotor helicopter is illustrated.

Conclusions are given in Section 5.

## MODELING OF THE AUTONOMOUS QUAD-ROTOR HELICOPTER

The model of the quad-rotor helicopter and the rotational directions of the propellers can be seen in Figure 2. The rotor pair 2 and 4 rotates clockwise direction and the rotor pair 1 and 3, anticlockwise direction.

An autonomous quad-rotor helicopter has fixed pitch angle rotors and the rotor speeds are controlled in order to produce the desired lift forces.



**Figure 2.** The model of the autonomous quad-rotor helicopter.

The quadrotor helicopter has four actuators-brushless DC motors which exert lift forces  $F_1$ ,  $F_2$ ,  $F_3$ ,  $F_4$  proportional to the square of the angular velocities of the rotors. Actually, four motor driver boards are needed to amplify the power delivered to the motors. Their rotation is transmitted to the propellers which move the entire structure.

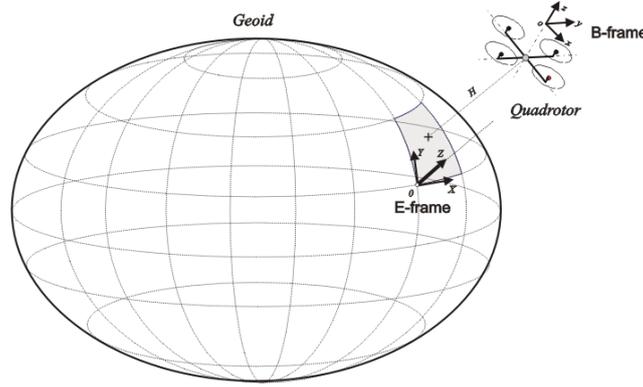
Two types of sensors are used for measuring the robot attitude and for measuring its height from the ground. For the first, an Inertial Measurement Unit (IMU) was adopted, while the distance was estimated with a Sound Navigation And Ranging (SONAR) and an InfraRed (IR) modules. There are: accelerometers and angular velocity sensors on the board of the quad-rotor helicopter.

The concept of the vision system is originated from motion-stereo approach. The camera is attached to the quadrotor helicopter. The data processing and the control algorithm are handled in the Micro Control Unit (MCU) which provides the signals to the motors [13-16].

## QUAD-ROTOR HELICOPTER COORDINATE SYSTEMS

To describe the motion of a 6 DOF rigid body it is usual to define two reference frames [17]:

- the earth inertial frame (E-frame), and
- the body-fixed frame (B-frame), Figure 3.



**Figure 3.** Earth- and Body-frame used for modeling of the quad-rotor system.

The equations of motion are more conveniently formulated in the B-frame because of the following reasons:

- the inertia matrix is time-invariant,
- advantage of body symmetry can be taken to simplify the equations,
- measurements taken on-board are easily converted to body-fixed frame, and
- control forces are almost always given in body-fixed frame.

The E-frame (OXYZ) is chosen as the inertial right-hand reference. Y points toward the North, X points toward the East, Z points upwards with respect to the Earth, and O is the axis origin. This frame is used to define the linear position (in meters) and the angular position (in radians) of the quad-rotor.

The B-frame (oxyz) is attached to the body, x points toward the center of gravity of the quad-rotor front, y points toward the quad-rotor left, z points upwards and o is the axis origin. The origin o is chosen to coincide with the center of the quad-rotor cross structure. This reference is righthand too. The linear velocity  $\mathbf{v}$  (m/s), the angular velocity  $\boldsymbol{\Omega}$  (rad/s), the forces  $\mathbf{F}$  (N) and the torques (Nm) are defined in this frame. The linear position of the helicopter (X, Y, Z) is determined by the coordinates of the vector between the origin of the B-frame and the origin of the E-frame according to the equation.

The angular position (or attitude) of the helicopter ( $\Phi$ ,  $\theta$ ,  $\psi$ ) is defined by the orientation of the B-frame with respect to the E-frame. This is given by three consecutive rotations about the main axes which take the E-frame into the B-frame. In this paper, the “roll-pitch-yaw” set of Euler angles were used. The vector that describes the quad-rotor position and orientation with respect to the E-frame can be written in the form:

$$\mathbf{s} = [X \ Y \ Z \ \Phi \ \theta \ \psi]^T. \quad (1)$$

The rotation matrix between the E- and B-frames has the following form:

$$\mathbf{R} = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\theta + c_\psi s_\theta s_\phi & s_\psi s_\theta + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\theta + s_\psi s_\theta s_\phi & -c_\psi s_\theta + s_\psi s_\theta c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}. \quad (2)$$

The corresponding transfer matrix has the form:

$$\mathbf{T} = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix}. \quad (3)$$

Where  $c_\theta$  and  $s_\theta$  represent  $\cos(\theta)$  and  $\sin(\theta)$  respectively.

## QUAD-ROTOR HELICOPTER KINEMATICS

The system Jacobian matrix, taking (2) and (3), can be written in the form:

$$\mathbf{J} = \begin{bmatrix} \mathbf{R} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{T} \end{bmatrix}, \quad (4)$$

where  $\mathbf{0}_{3 \times 3}$  is a zero-matrix. The generalized quad-rotor velocity in the B-frame has a form [18]:

$$\mathbf{v} = [\dot{x} \ \dot{y} \ \dot{z} \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T. \quad (5)$$

Finally, the kinematical model of the quad-rotor helicopter can be defined in the following way:

$$\dot{\mathbf{s}} = \mathbf{J} \cdot \mathbf{v}. \quad (6)$$

## QUAD-ROTOR HELICOPTER DYNAMICS

Dynamic modelling of the quadrotor helicopter is a well elaborated field of aeronautics. The dynamics of a generic 6 DOF rigid-body system takes into account the mass of the body  $m$  and its inertia matrix  $\mathbf{I}$ .

Two assumptions have been done in this approach:

- the first one states that the origin of the body-fixed frame is coincident with the center of mass (COM) of the body. Otherwise, another point (COM) should be taken into account, which could make the body equations considerably more complicated without significantly improving model accuracy,
- The second one specifies that the axes of the B-frame coincide with the body principal axes of inertia. In this case the inertia matrix  $\mathbf{I}$  is diagonal and, once again, the body equations become simpler.

Each rotor produces moments as well as vertical forces. These moments were observed experimentally to be linearly dependent on the forces at low speeds. There are four input forces and six output states ( $x, y, z, \psi, \theta, \phi$ ) and, therefore the quad-rotor is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counter clockwise, in order to balance the moments and to produce yaw motions as needed.

The equations of motion can be written using the force and moment balance, yielding:

$$\ddot{x} = \frac{\left( \sum_{i=1}^4 F_i \right) (c_\phi s_\theta c_\psi + s_\phi s_\psi) - K_x \dot{x}}{m}, \quad (7)$$

$$\ddot{y} = \frac{\left( \sum_{i=1}^4 F_i \right) (s_\phi s_\theta c_\psi + c_\phi s_\psi) - K_y \dot{y}}{m}, \quad (8)$$

$$\ddot{z} = \frac{\left( \sum_{i=1}^4 F_i \right) (c_\phi c_\psi) - K_z \dot{z} - G}{m}, \quad (9)$$

$$\ddot{\psi} = l \cdot \frac{(-F_1 + F_2 + F_3 - F_4 - K_\psi \dot{\psi})}{J_x}, \quad (10)$$

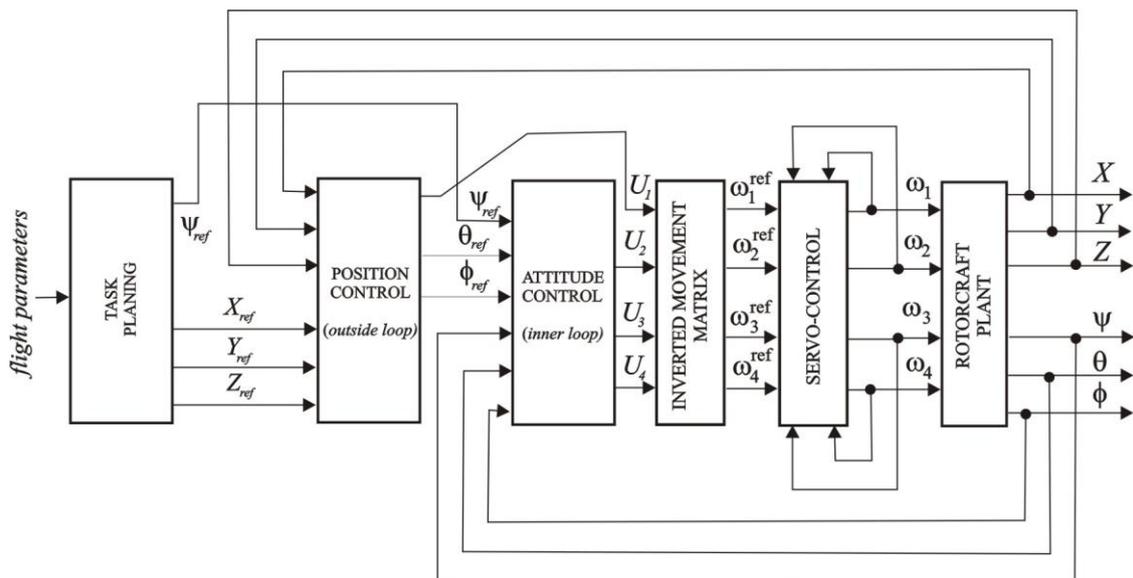
$$\ddot{\theta} = l \cdot \frac{(-F_1 - F_2 + F_3 + F_4 - K_\theta \dot{\theta})}{J_y}, \quad (11)$$

$$\ddot{\phi} = \frac{(-M_1 + M_2 + M_3 - M_4 - K_\phi \dot{\phi})}{J_z}. \quad (12)$$

The factors  $K_j$  in (7)-(12) given above are the air resistance coefficients to be determined experimentally.  $J_x$ ,  $J_y$ ,  $J_z$  are the moments of inertia with respect to the particular axes.

## MODELING OF THE CONTROL STRATEGY

Together with modeling, the determination of the control algorithm structure is very important for improving stabilization. Controlling an autonomous quad-rotor helicopter is basically dealing with highly unstable dynamics and strong axes coupling. In addition to this, any additional on-board sensor increases the autonomous quad-rotor helicopter total weight and therefore decreases its operation time. The control system of the autonomous quad-rotor helicopter requires accurate position and orientation information [14-18]. In this section we present a control strategy to stabilize of the quad-rotor. Figure 4 shows the block diagram of the quad-rotor control system.



**Figure 4.** The block diagram of the quad-rotor helicopter control system.

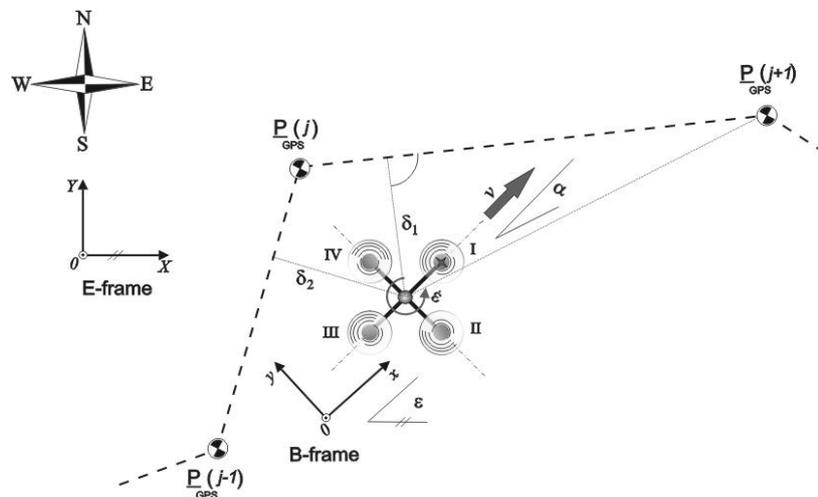
The task planning block is in debt to determine desired, i.e. referent 3D rotorcraft trajectory of flight as well as to propose the referent flight speed along the trajectory. The task planning block generates referent path based on flight parameters and quad-rotor task imposed. Position control block has to ensure accurate 3D trajectory tracking. It represents so called outside control loop. Based on sensory information (GPS, IR, SONAR) about the referent positions (speeds) and corresponding actual ones defined in the inertial coordinate system (E-frame), the position controller calculates referent attitude position of quad-rotor body (pitch and roll angle) that have to enable desired motion. Inner control block represents the core of the control scheme. It is responsible for the attitude control of quad-rotor system. Appropriate attitude control ensures in an indirect way required flight performances in the particular directions of motion such as longitudinal, lateral as well as vertical. Inner control block processes the task and sensor data and provides a signal for basic movements which

balances the position error. The essence of building control scheme presented in Figure 4 is that by controlling a body attitude (within an inner loop) it is enabled controlling of the rotorcraft movements in the coordinate directions co-linear with the axes of the inertial system. Inverted Movements Matrix block is used to compute the propeller's squared speed from the four basic movement signals.

Variety of control algorithms can be implemented within the flight controller presented in Figure 4. These are: (i) conventional PID regulator, (ii) backstepping method [19, 20] and (iii) knowledge-based Fuzzy Inference System (FIS) [4, 11]. The future of autonomous quad-rotor helicopter control lies in intelligent flight controllers that are programmed to specific flight missions [21-26].

## GPS NAVIGATION OF THE AUTONOMOUS QUAD-ROTOR HELICOPTER

The trajectory of the autonomous quad-rotor can be introduced by GPS coordinates (e. g.  $P_{GPS}(j)$ ) as shown in Figure 5. The autonomous quad-rotor helicopter is requested to track the imposed trajectory between the particular points ( $j=1, \dots, n$ ) with satisfactory precision, keeping the desired attitude and height of flight. The autonomous quad-rotor helicopter checks for the current position: X and Y by use of a GPS sensor and/or electronic compass. Also, the altitude is measured by a barometric sensor. An on-board microcontroller calculates the actual position deviation from the imposed trajectory given by successive GPS positions  $P_{GPS}(j)$ . It localizes itself with respect to the nearest trajectory segment, by calculation of the distances:  $\delta_1$  or  $\delta_2$ .



**Figure 5.** Quad-rotor localization and navigation with respect to the imposed GPS coordinates.

Gyroscopes provide angular velocity measurements with respect to inertial space. With recent developments in gyroscope technology, their usage in various fields is observably increasing. In combination with accelerometers, gyroscopes are used in position, velocity, and attitude computation in a variety of navigation and motion tracking applications for aircraft and robots. By providing angular velocity measurements, gyroscopes can also be used in angular orientation estimation. Using the gyroscope, the autonomous quad-rotor helicopter determines desired azimuth of flight  $\alpha$  (Figure 5) and keeps the desired direction of flight. The height of flight is also controlled to enable the performance of the imposed mission. The height of flight is also controlled to enable the performance of the imposed mission (task).

## CONCLUSIONS

In this article the modeling and navigation of an autonomous quad-rotor helicopter in a outdoor scenario is shown. The main aspects of modeling of rotorcraft kinematics and rigid body dynamics, spatial system localization and navigation of autonomous quad-rotor helicopter in outdoor scenario are considered in the paper. The control strategy is presented. The GPS navigation of the autonomous quad-rotor helicopter is illustrated.

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## **INSPEKCIJA USKLAĐENOSTI S VIŠE PRAVILA: VIŠEAGENTNI MODEL**

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### **SAŽETAK**

Stalan porast opsega i kompleksnosti regulative i drugih pravila koja upravljaju ljudskim društvom naglašavaju važnost nadzora usklađenosti s tim pravilima. Često korišteni pristupi inspekciji usklađenosti pate od nedostataka kao što su pretjerano idealističke pretpostavke te ograničeno područje primjene. Naime, modeli inspekcije često su ograničeni na situacije u kojima se nadzirani entitet mora uskladiti samo s jednim pravilom. Nadalje, modeli inspekcije redovito zanemaruju neke korisne i dostupne informacije, kao što su različiti troškovi usklađenosti s različitim pravilima.

Ovaj rad prezentira višegentni model inspekcije usklađenost s više pravila koji adresira prije navedene nedostatke. U radu je ukratko opisan pristup inspekciji kroz ekonomiku kriminala, teoriju igara te agentno modeliranje, kao i utjecaj tih pristupa na prezentirani model. Model je opisan te je prikazana simulacija pojednostavljene inačice modela. Dobiveni rezultati pokazuju da inspeksijske strategije koje u obzir uzimaju troškove usklađenosti postižu rezultate značajno bolje od nasumičnih i bolje od cikličkih strategija. Dodatno, rezultati ohrabruju daljnje, opsežnije testiranje i validaciju modela.

### **KLJUČNE RIJEČI**

ICARUS, inspekcija usklađenosti, višegentni model, višegentni sustav, trošak usklađenosti

## MODELIRANJE DOSTUPNOSTI INFORMACIJSKOG SUSTAVA BAYESOVOM MREŽOM POVJERENJA

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### SAŽETAK

Povjerenje je značajno za odvijanje učinkovitih međudjelovanja u društvenoj mreži. Modeliramo povjerenja agenata koristeć rangiranja ugleda u mreži koja tvori matricu realnih koeficijenata.

Dekompozicija singularnih vrijednosti uzima dio rangiranja ugleda kako bi procijenila razine povjerenja, čime je povjerenje subjektivna vjerojatnost budućih očekivanja temeljena na sadašnjim rangiranjima ugleda.

Ugled i povjerenje blisko su vezani i dekompozicija singularnih vrijednosti može procijeniti povjerenje pomoću matrice realnih koeficijenata pridružene rangiranju ugleda agenata mreže.

Dekompozicija singularnih vrijednosti idealna je tehnika za uklanjanje pogreške pri procjeni povjerenja na temelju rangiranja ugleda. Procjena ugleda i povjerenja optimalna je pri smanjivanju od 20 %.

### KLJUČNE RIJEČI

dekompozicija singularnih vrijednosti, ugled, povjerenje, društvena mreža, smanjivanje

## **LOJALNOST ZAPOSLENIKA: RAZLIKE IZMEĐU SPOLOVA I JAVNOG I PRIVATNOG SEKTORA**

I. Klopotan, K. Buntak i I. Drožđek

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Koprivnica i Varaždin, Hrvatska

### **SAŽETAK**

Ovaj članak prikazuje rezultate istraživanja bitnih parametara lojalnosti zaposlenika, u javnom i privatnom sektoru. Istraživanje je bilo provedeno u Republici Hrvatskoj kao dio istraživanja koje je usredotočeno na esencijalne parametre lojalnosti zaposlenika, a koji doprinose gradnji i zadržavanju lojalnog ponašanja. Lojalnost i zadovoljstvo zaposlenika su ključni parametri koji utječu na uspješnost tvrtke. Dodatno, lojalni i zadovoljni zaposlenici su važni za izgradnju dobrih veza sa kupcima, dobavljačima i dioničarima koji su uključeni u poslovne procese organizacije ili poduzeća. Namjera ovog članka je definirati da li postoji razlika u parametrima lojalnosti zaposlenika u javnom i u privatnom sektoru u Hrvatskoj, kao i postoje li razlike temeljene na spolu.

### **KLJUČNE RIJEČI**

lojalnost, upravljanje lojalnošću zaposlenika, upravljanje kvalitetom, motivacija zaposlenika

## PRIKAZ NUMERIČKE TEORIJE VRIJEDNOSTI INFORMACIJA U PITANJIMA PORIJEKLA ŽIVOTA

S. Banerjee

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Ronin Institut  
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Savez za kompleksne biološke sustave  
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### SAŽETAK

Uloga informacije u kompleksnim biološkim sustavima je značajna. Kompleksni sustavi poput imunološkog sustava i mravljih zajednica koordiniraju heterogene komponente decentraliziranim pristupom. Kako funkcioniraju takvi distribuirani i decentralizirani sustavi? Jedan značajan doprinos je to kako učinkovito kompleksni sustavi procesiraju informacije. Kompleksni sustavi imaju arhitekturu za integriranje i procesiranje informacije iz različitih izvora što upućuje na značajnost informacije u djelovanju različitih kompleksnih bioloških sustava. Ovaj rad predlaže određenu ulogu procesiranja informacija u pitanjima vezanim za porijeklo života te ikazuje kako numeričke simulacije mogu pružiti uvid u odgovore na takva pitanja.

Numerički model porijekla života mogao bi ujediniti termodinamiku i procesiranje informacija što bi nam omogućilo razumijevanje zašto su bjelančevine i nukleotidi evoluirali kao zamjene za komputaciju i procesiranje informacija u živim bićima na Zemlji. Odgovori na ta i slična pitanja mogu pružiti uvid u oblike života koji se ne temelje na ugljiku, a kakve se može očekivati van Zemlje.

Hipoteza rada je da su oblici života temeljeni na ugljiku samo jedan od kontinuuma sustava u svemiru, sličnih životu. Istraživanje uloge komputacijskih substrata koje omogućuje procesiranje informacija značajno je i može pružiti uvid u: 1) nove substrate, koji se ne temelje na ugljiku, a koji mogu imati svojstva slična živim bićima, 2) kako je život na Zemlji nastao iz neživoga. Život može postojati kao kontinuum između neživoga i živoga što može dovesti do potrebe za promjenom našeg poimanja života i činjenice koliko je čest u svemiru. Gledanje na život i životne pojave iz perspektive teorije informacija može dovesti do proširenog pogleda na život.

### KLJUČNE RIJEČI

porijeklo života, umjetni život, sustavi slični životu, teorija informacije, reakcijsko-difuzijski sustavi

## **MODELIRANJE, KONTROLA I NAVIGACIJA AUTONOMNOG KVADRIKOPTERA HELIKOPTERA**

D. Šoštarić

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Budimpešta, Mađarska

### **SAŽETAK**

Autonomni kvadrikopteri helikopteri za vanjski let sve više privlače pozornost potencijalnih istraživača. Nekoliko struktura i konfiguracija je razvijeno za 3D kretanje. Kvadrikopter helikopter se sastoji od krutog poprečnog okvira opremljenog sa četiri motora. Autonomna kvadrikopter arhitektura je odabrana za istraživanje zbog malih dimenzija, dobre upravljivosti, jednostavnog mehanizma i nosivih mogućnosti. Rad prikazuje modeliranje, kontrolu i navigaciju autonomnog kvadrikoptera helikoptera za vanjski let.

### **KLJUČNE RIJEČI**

autonomni kvadrikopter helikopter, modeliranje kontrola i navigacija, 3D kretanje, senzorski sustav



## MANUSCRIPT PREPARATION GUIDELINES

Manuscript sent should contain these elements in the following order: title, name(s) and surname(s) of author(s), affiliation(s), summary, key words, classification, manuscript text, references. Sections acknowledgments and remarks are optional. If present, position them right before the references.

**ABSTRACT** Concisely and clearly written, approx. 250 words.

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Include figures and tables in the preferred position in text. Alternatively, put them in different locations, but state where a particular figure or table should be included. Enumerate them separately using Arabic numerals, strictly following the order they are introduced in the text. Reference figures and tables completely, e.g., “as is shown in Figure 1,  $y$  depends on  $x$  ...”, or in shortened form using parentheses, e.g., “the  $y$  dependence on  $x$  shows (Fig. 1) that...”, or “... shows (Figs. 1-3) that ...”.

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