

# ACCUMULATIVE POLLUTION, ENVIRONMENTAL REGULATION AND ENVIRONMENTAL COSTS: DYNAMIC APPROACH

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## ABSTRACT

This article analyses how social preference towards environmental costs for addressing environmental concerns can have an impact on the steady state solution concerning the stock of accumulative pollutants and the optimal environmental regulatory stringency as well as on the initial value of the optimal environmental regulatory stringency that leads to the steady state solution. The results found that the steady state value of the optimal environmental regulatory stringency was higher, and the steady-state value of the stock of accumulative pollutants was lower in the case where sufficient liability costs for environmental damages were estimated, compared with the case where the estimation of liability costs was insufficient. In addition, the results showed that the steady-state value of optimal environmental regulatory stringency was higher and the value of the stock of accumulative pollutants was lower in the case where the discount rate provided sufficient consideration for future generations, than when the discount rate provided insufficient consideration. Moreover, the study also indicated that, where the initial value of the stock of accumulative pollutants was within a given range, in the cases where sufficient liability costs for environmental damages were estimated and where the discount rate provided sufficient consideration for future generations, the initial value of the optimal environmental regulatory stringency was found to be at a higher level, compared to the initial value of the optimal environmental regulatory stringency for the cases where there was insufficient estimation of liability costs and when the discount rate provided insufficient consideration.

## KEYWORDS

accumulative pollution, environmental regulatory stringency, liability cost for environmental damage, discount rate

## CLASSIFICATION

JEL: Q52, Q53, Q54

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## **INTRODUCTION**

The pollution from various toxic substances which is often a by-product of production and industrialization can be a threat to the environment. Heavy metals such as mercury and cadmium; radioactive contamination caused by accidents or wastes from nuclear power stations; dioxins which is considered to impact endocrine disruptors; and global warming by greenhouse gases, are all growing social concerns. These pollutants which accumulate over time are referred to be the main causes of environmental degradation [1, 2]. In recent years, providing for both intergenerational and intra-generational equity which are vital principles of sustainability and taking into consideration the trend of environmentalism, the effects of such accumulative pollutants on the environment cannot be ignored. Accordingly, it would be necessary to appropriately understand how these accumulative pollutants and environmental policies which could be related to these pollutants change over time. Environmental costs which could affect accumulative pollutants would be important to examine along with how social planners consider the social preference towards environmental costs. The reason is that under a competitive market, it is considered that the social benefit is larger when social costs related to environmental issues are internalized compared to when the costs are ignored [3]. It is also considered that firms could receive benefits such as maintaining a strong brand image, enhance productivity and profitability and establish stronger social/stakeholders relationships when they have appropriate consideration for environmental costs and there are previous researches supporting this [4-6].

The environmental costs, here, are composed of liability costs for environmental damages and abatement costs. As for the abatement costs, it can be regarded as ex-ante costs that prevent pollution. These are costs necessary to reduce the volume of residuals emitted into the environment or to reduce the ambient concentrations [7]. They also include costs for pollution prevention, environmental preservation and the investment, administration, and transaction to support this. For example, activities such as changes in production technologies, input switching, recycling and treatment are covered in these costs [7]. With regards to the liability costs for environmental damages, they include environmental loss costs, environmental loss mitigation costs, environmental restoration costs, related administration costs and transaction costs. Concerning the estimation of liability costs, this will be regarded as “sufficient liability costs for environmental damages” when the private interests, which contain both rights to live and property rights; and the public interests from living organisms and their surrounding; and the intrinsic values of the environment are included. However, only a portion of the “sufficient liability costs for environmental damages” are usually covered. Holl and Howarth [8] identify a number of cases where environmental restoration costs were not adequately covered. Yoshimura [9] identified the lack of a system for administrators and citizens to play a role in the restoration of the environment, which implies that environmental restoration costs have not been adequately considered [9]. Moreover, there are cases where the compensation costs for environmental losses were not adequately paid [10, 11]. According to Otsuka [12], the effectiveness of the conciliation is limited to the settlement agreement and it does not impact the proceedings of the trial, which could be considered that the compensation fees were insufficient. Furthermore, with regards to laws related to environmental compensation, it is recognised that there are challenges for the compensation to cover the public interests which include living organisms and their surroundings and the intrinsic value of the environment [9]. When such liability costs for environmental damages are not sufficiently covered, we will consider this as “insufficient liability costs for environmental damages.” In this research we will examine both the cases of sufficient and insufficient coverage of liability costs for environmental damages.

It is necessary to estimate environmental costs which are composed of abatement costs and liability costs for environmental damages from the viewpoint of not only the present generation but also with consideration for future generations. This leads to the important consideration of the discount rates adopted when estimating environmental costs. Therefore, in this article, we will analyse the discount rates adopted for both insufficient and sufficient consideration of future generations. In the former case where there is insufficient consideration, there is the tendency to estimate lower environmental costs compared to the latter where the discount rate takes sufficient consideration for future generations and estimates higher environmental costs. Accordingly, the discount rate of the former case will be higher than the discount rate for the latter case [13-15].

Concerning the above, it is considered to be effective to examine the accumulative pollution where time proceeds are taken into account, that is, through a dynamic analysis. Numerous literature that examine pollutants including accumulative pollutants using the dynamic approach exists. For instance, there is literature on the examination of the relationship between pollutants and the economy such as economic growth, capital accumulation and consumption [16, 17]. There are also past research which study the relationship between pollution and the assimilative capacity and the degradation of this capacity under pollution excess [18-20]. Moreover, there are large amounts of past literature on the relationship between pollution and environmental policies in a dynamic setting. For example, there is a study that examine the relationship between tax and subsidy; and pollution flow and stock [17]. There are also studies which analyse the relationship between emissions quotas or permit and pollution flow and stock [21-23]. Furthermore, Saltari and Travaglini [24] question whether there is an intermediate phase where consumers, firms and society anticipate the effects of environmental constraints in planning their current economic-ecological decisions and so they examine if even an unconstrained regime can have an impact on pollution, by anticipating the latent constraint.

As seen previously, we know that dynamic analysis concerning pollution have been conducted from various aspects. The main purpose of this article is to examine the differences in the steady state solution and initial values for both the stock of accumulative pollutants and the stringency of environmental policies, affected by social preference towards environmental costs if a steady state solution exists. Therefore, our analysis will extend the study of Chukwuemeka [25] which focuses on both the stock of accumulative pollutants and emission tax from a dynamic approach. The study analyses the existence of a steady state solution for both the stock of accumulative pollutants and emission tax, their paths in the phase diagram, the set of initial values of emission tax, under the framework to minimize environmental costs which include liability costs for environmental damages and abatement costs. As emphasized before, this article will introduce the aspects of social preference towards environmental costs. First, it will examine the differences of the steady state solution in the case of sufficient estimation of liability costs for environmental damages and then the case of insufficient estimation of liability costs for both the stock of accumulative pollutants and the optimal environmental regulatory stringency. Second, similar to the first analysis, this article will observe the differences in the steady state solution in the case where the discount rate takes sufficient consideration for future generations and in the case where the consideration is insufficient for both the stock of accumulative pollutants and the optimal environmental regulatory stringency. Third, this article also examines, under a given range of initial stock of accumulative pollutants, how the initial value of the optimal environmental regulatory stringency is decided, depending on the social preference towards environmental costs as studied in the first and second analysis.

## ACCUMULATIVE POLLUTION AND OPTIMAL GROWTH MODEL

### EXPLANATION OF THE BASIC MODEL

In this section, based on Chukwuemeka [25], we will introduce the framework applied in this study. At first, we will confirm the existence of the steady state solution for the stock of accumulative pollutants and the optimal environmental regulatory stringency and the possible region to be able to move into a steady state solution. To more be specific, given the initial value of the stock of accumulative pollutants, and under the condition that abatement activities are being conducted, in order to manage environmental costs efficiently, that is, when minimizing the total environmental costs which include the liability costs for environmental damages and abatement costs, we will identify the existence of the steady state solution for the stock of accumulative pollutants and the optimal environmental regulatory stringency and the path to the steady state solution. Hence, the objective function and the constraint equations are as follows.

$$\min_A \int_0^{\infty} [D_p(S) + A(M)] e^{-r_q t} dt,$$

subject to

$$\begin{aligned} \dot{S} &= W - M - dS, \\ S(0) &= S_0. \end{aligned} \quad (1)$$

Here,  $S$  represents the stock of accumulative pollutants,  $W$  is the amount of the pollutants under the uncontrolled regime,  $M$  denote the amount of abatement activities. Quantity  $d$  ( $0 \leq d \leq 1$ ) is the rate of decay for the accumulative pollutants. The functional relationship between the stocks of pollutants and the rate of decay is little known [20]. Hence, this article adopts a constant proportionate rate of decay, which is often used in previous literatures concerning pollution stock [20, 25].  $D_p(S)$  is the liability costs for environmental damages. As identified by many economists and environmentalists, it assumes that  $D'_p > 0, D''_p > 0$ . It also assumes that  $D'''_p = 0, D_p(0) = 0$ . The  $p$  denotes the sufficiency of the consideration for the liability costs of environmental damages, which will be explained in the next section.  $A(M)$  represents abatement costs. According to Field and Field [7], it is assumed that  $A' > 0, A'' > 0$  since they increase and the marginal costs also increase as the reduction of emissions increase.  $S_0$  is the initial value of the stock of accumulative pollutants.  $r_q$  denotes the discount rate. The  $q$  represents the sufficiency concerning future generation consideration. It will be explained in a later section.  $t$  means time and the dot on top of the letter (e.g.  $\dot{S}$ ) represents the differential with respect to  $t$ .

From the previously stated, the present value Hamiltonian is as follows.

$$H = [D_p(S) + A(M)] e^{-r_q t} + \delta(W - M - dS). \quad (2)$$

Here, using the definition  $\varepsilon = \delta \cdot e^{r_q t}$ , the current value Hamiltonian is as follows:

$$H^C = D_p(S) + A(M) + \varepsilon(W - M - dS). \quad (3)$$

Since  $D_p$  and  $A$  are strictly convex, the current value Hamiltonian is also strictly convex in  $S$  and  $M$ . Accordingly, the following necessary conditions are sufficient.

$$\partial H^C / \partial M = 0, \quad \therefore \varepsilon = A'(M), \quad (4)$$

$$-(\partial H^C / \partial S) e^{-r_q t} = \partial(\varepsilon e^{-r_q t}) / \partial t, \quad \therefore \dot{\varepsilon} = -D'_p(S) + \varepsilon(r_q + d), \quad (5)$$

$$\partial H^C / \partial \varepsilon = \dot{S}, \quad \therefore \dot{S} = W - M - dS, \quad (6)$$

Here,  $\varepsilon$  indicates the shadow value of abating one unit of emissions. Since it can be interpreted that society will pay for the amount corresponding to the value of abating one unit

of emissions, it could be regarded as the optimal environmental regulatory stringency, assuming that it could be estimated. Hence, (4) suggests that the increase of marginal cost of abating the accumulative pollution represents the increase of the optimal environmental regulatory stringency.

Here, in order to find the steady state, making use of (4), (5), (6),  $\dot{S} = \dot{\varepsilon} = 0$ . At first, substitute  $\dot{\varepsilon} = 0$  into (5). As a result, we obtain the following:

$$\varepsilon = \frac{D_p'}{r_q + d}. \quad (7)$$

Here, differentiate (7) with respect to  $S$ , the following equation can be introduced:

$$\frac{\partial \varepsilon}{\partial S} = \frac{D_p''}{r_q + d} > 0. \quad (8)$$

From  $D_p''(S) > 0, r_q > 0, d > 0$ , we can find the sign of (8). Next, substitute  $\dot{S} = 0$  into (6),

$$W - M - dS = 0. \quad (9)$$

Rewrite (9),

$$M = W - dS. \quad (10)$$

Here, substitute (10) into (4),

$$\varepsilon = A'(W - dS). \quad (11)$$

Differentiate (11) with respect to  $S$ , we can obtain:

$$\frac{\partial \varepsilon}{\partial S} = -dA''(W - dS) < 0. \quad (12)$$

From  $A''(M) > 0$ , i.e.  $A''(W - dS) > 0, d > 0$ , we can confirm the sign of (12). From (4), (7) and (9), we can get  $(S^S, M^S, \varepsilon^S)$  as the steady state solution of  $(S, M, \varepsilon)$ .

Since this article aims to focus on the stock of accumulative pollutants and the optimal environmental regulatory stringency, and to illustrate them in two dimensions, we will focus on  $(S, \varepsilon)$ . Therefore, with regards to the path to a steady state solution, we will proceed as follows. In order to do so, first, we will set  $(\bar{S}, \bar{\varepsilon})$  such as  $\dot{\varepsilon} = 0$ . That is, from (5),  $\dot{\varepsilon} = \dot{\varepsilon}(\bar{S}, \bar{\varepsilon}) = -D_p'(\bar{S}) + \bar{\varepsilon}(r_q + d) = 0$ .

Fixing  $\varepsilon = \bar{\varepsilon}$ , if it assumes that  $S_a < \bar{S}$ , from (5), we obtain  $\partial \varepsilon / \partial S < 0$ . With  $\dot{\varepsilon}(\bar{S}, \bar{\varepsilon}) = 0$ , we can introduce  $\dot{\varepsilon}(S_a, \bar{\varepsilon}) > 0$ . On the other hand, keeping  $\varepsilon = \bar{\varepsilon}$ , assuming that  $S_b > \bar{S}$ , we attain  $\partial \varepsilon / \partial S < 0$ . With  $\dot{\varepsilon}(\bar{S}, \bar{\varepsilon}) = 0$ , we can make sure  $\dot{\varepsilon}(\bar{S}, \bar{\varepsilon}) = 0$ . From these results, in the region above  $\dot{\varepsilon} = 0$ , we can confirm  $\dot{\varepsilon}(S, \varepsilon) > 0$ , while in the region below  $\dot{\varepsilon} = 0$ , we can find  $\dot{\varepsilon}(S, \varepsilon) < 0$ .

Next, we set  $(\hat{S}, \hat{\varepsilon})$  such as  $\dot{S} = 0$ . That is, from (6),  $\dot{S} = \dot{S}(\hat{S}, \hat{\varepsilon}) = W - M - dS = 0$ . Here, as  $A'(M)$  is strictly increasing, its inverse is also strictly increasing. Let  $(A')^{-1} = v$ . By making use of (4) and (6),  $\dot{S} = \dot{S}(\hat{S}, \hat{\varepsilon}) = W - v(\varepsilon) - dS$ . Hence,  $\partial \dot{S} / \partial \varepsilon = v'(\varepsilon) < 0$ .

Keeping  $S = \hat{S}$  and supposing that  $\varepsilon_a < \hat{\varepsilon}$ , from  $\partial \dot{S} / \partial \varepsilon < 0$  and  $\dot{S}(\hat{S}, \hat{\varepsilon}) = 0$ ,  $\dot{S}(\hat{S}, \varepsilon_a) > 0$ . On the other hand, leaving  $S = \hat{S}$  and supposing that  $\varepsilon_b > \hat{\varepsilon}$ , from  $\partial \dot{S} / \partial \varepsilon < 0$  and  $\dot{S}(\hat{S}, \hat{\varepsilon}) = 0$ ,  $\dot{S}(\hat{S}, \varepsilon_b) < 0$ . From these results, we can identify that, in the region above  $\dot{S} = 0$ ,  $\dot{S}(S, \varepsilon) < 0$ . On the other hand we can confirm that, in the region below  $\dot{S} = 0$ ,  $\dot{S}(S, \varepsilon) > 0$ .

By summarizing these results, we can obtain the direction of the paths in each region as follows:

1) in the region which is below  $\dot{S} = 0$  and above  $\dot{\varepsilon} = 0$ , the stock of accumulative pollutants

- increases and the optimal environmental regulatory stringency also increases as time proceeds. Hence, the paths composed of the stock of accumulative pollutants and the optimal environmental regulatory stringency move towards the upper right direction in the region;
- 2) in the region which is below  $\dot{S} = 0$  and below  $\dot{\varepsilon} = 0$ , the stock of accumulative pollutants increases and the optimal environmental regulatory stringency also increases as time proceeds. Accordingly, the paths composed of the stock of accumulative pollutants and the optimal environmental regulatory stringency move towards the lower right direction in the region;
  - 3) in the region which is above  $\dot{S} = 0$  and below  $\dot{\varepsilon} = 0$ , the stock of accumulative pollutants increases and the optimal environmental regulatory stringency also increases as time proceeds. Hence, the paths composed of the stock of accumulative pollutants and the optimal environmental regulatory stringency move towards the lower left direction in the region.
  - 4) in the region which is above  $\dot{S} = 0$  and above  $\dot{\varepsilon} = 0$ , the stock of accumulative pollutants increases and the optimal environmental regulatory stringency also increases as time proceeds. Therefore, the stock of accumulative pollutants and the optimal environmental regulatory stringency move towards the upper left direction in the region.

### **STEADY STATE COMPARION BETWEEN SUFFICIENT AND INSUFFICIENT LIABILITY COSTS ESTIMATIONS**

Based on the previous results, this section will examine the difference that can occur in terms of the steady state solution of the stock of accumulative pollutants and the optimal environmental regulatory stringency for the case where the liability costs for environmental damages estimated are sufficient and the case where those estimates are insufficient. As mentioned before, in the case of liability costs for environmental damages through sufficient consideration, the estimation includes costs corresponding to not only the private interests such as rights to live and property rights but also covers the public interests from living organisms and their surrounding and the intrinsic values of the environment. In this case, we will define that  $D_p = D_h$  (when the liability costs for environmental damages are sufficiently estimated). On the other hand, in the case of the liability costs for environmental damages estimated through insufficient consideration, it can be considered that the costs representing both the private and public interests may not be sufficiently covered. This will be defined as  $D_p = D_l$  (when the liability costs for environmental damages are insufficiently estimated). Taking into consideration the above and past literature that find that only a portion of the sufficient liability costs for environmental damages are usually covered and compensation costs for environmental losses are not adequately paid [8-12], the estimation of the liability costs for the case of sufficient estimation will be larger than those in the case of insufficient estimation. Hence, it assumes that the estimation of the liability costs for environmental damages in the former case are larger than those in the latter case with any amount of stock of accumulative pollutants. i.e.  $D_h(S) > D_l(S)$  for  $0 < S$ . It also assumes that  $D_h(0) = D_l(0) = 0$ .

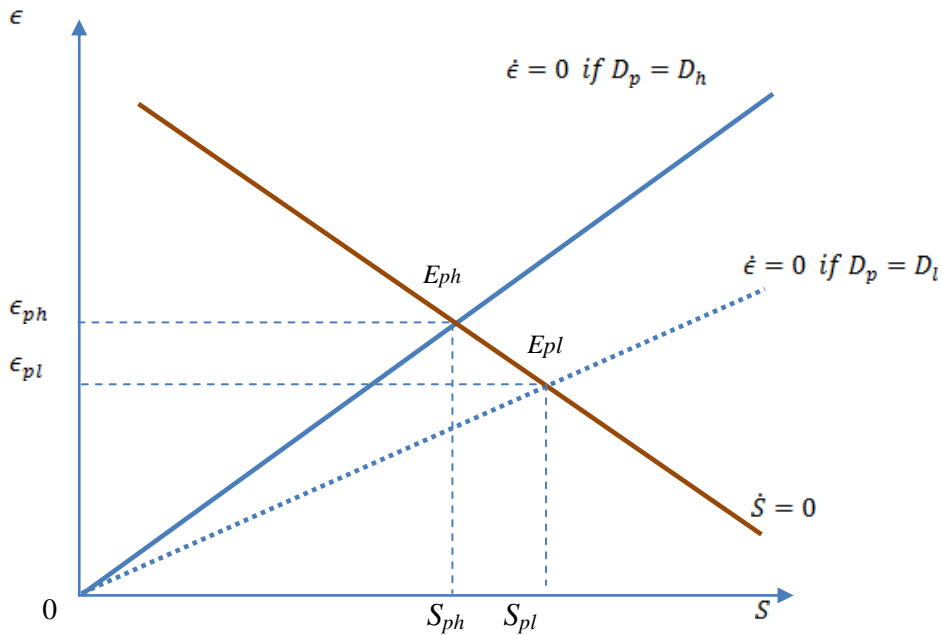
Accordingly,

$$D_h''(S) > D_l''(S). \quad (13)$$

Substitute (13) into (8), we can obtain:

$$\frac{D_h''(S)}{r_q + d} > \frac{D_l''(S)}{r_q + d} > 0. \quad (14)$$

Since this section and the next section only focuses on the differences between the case of sufficient estimations of liability costs for environmental damages and the case of insufficient estimations, for convenience, it assumes that the discount rates are identical in both cases. Hence, from (12) and (14), we can draw Figure 1.  $\dot{\varepsilon} = 0$  refers to the stock of accumulative



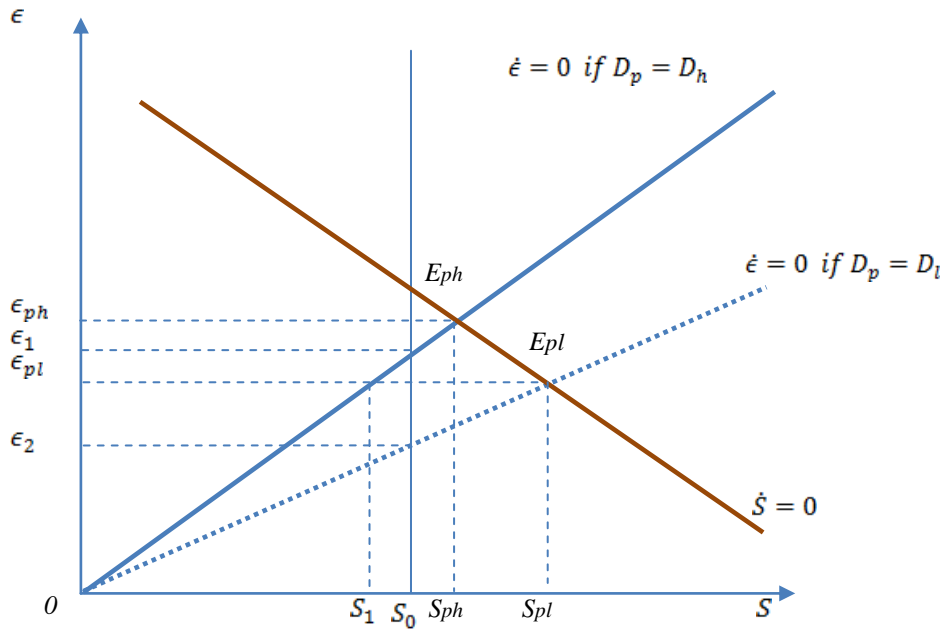
**Figure 1.** Steady state solutions in the case of sufficient estimation of liability costs for environmental damages and in the case of insufficient estimation of liability costs for environmental damages.

pollutants under the steady state;  $S_{ph}$  indicates the stock of accumulative pollutants at the steady state when the estimations of liability costs are sufficient; and  $S_{pl}$  indicates the stock of accumulative pollutants at the steady state when the estimations of liability costs are insufficient.

As seen in Figure 1, we can find that in the case where liability costs for environmental damages are sufficiently estimated, which include both costs covering the public and private interests, the steady state value of the optimal environmental regulatory stringency are higher and the stock of accumulative pollutants are lower, than compared with the case of insufficient estimation of liability costs for environmental damages. That is, this steady state could be regarded as the steady state solution of high environmental consideration. On the contrary, in the case of insufficient estimation of liability costs for environmental damages, the steady state value of the optimal environmental regulatory stringency are lower, and the stock of accumulative pollutants are higher, than compared with the case of sufficient estimation of liability costs. Hence, this state could be interpreted as the steady state solution of low environmental consideration.

### **DIFFERENCE IN THE INITIAL VALUE OF THE ENVIRONMENTAL REGULATIONS – COMPARING SUFFICIENT AND INSUFFICIENT LIABILITY COSTS ESTIMATIONS**

Next, in this section, we will examine, under the condition that the initial value of the stock of accumulative pollutants is within a given range, how the initial value of the optimal environmental regulatory stringency in the case of sufficient estimation of liability costs for environmental damages differs from the case of insufficient estimation of liability costs for environmental damages. To describe more precisely, under the condition that the initial value of the stock of accumulative pollutants is within a given range, this section will analyse how the initial value of the optimal environmental regulatory stringency which leads to the steady state solution of high environmental consideration differs from one which leads to the steady state solution of low environmental consideration. The reason for analysing this is that we can observe how the initial value of the optimal environmental regulatory stringency is set in order to converge into a steady state solution of higher environmental consideration compared to the



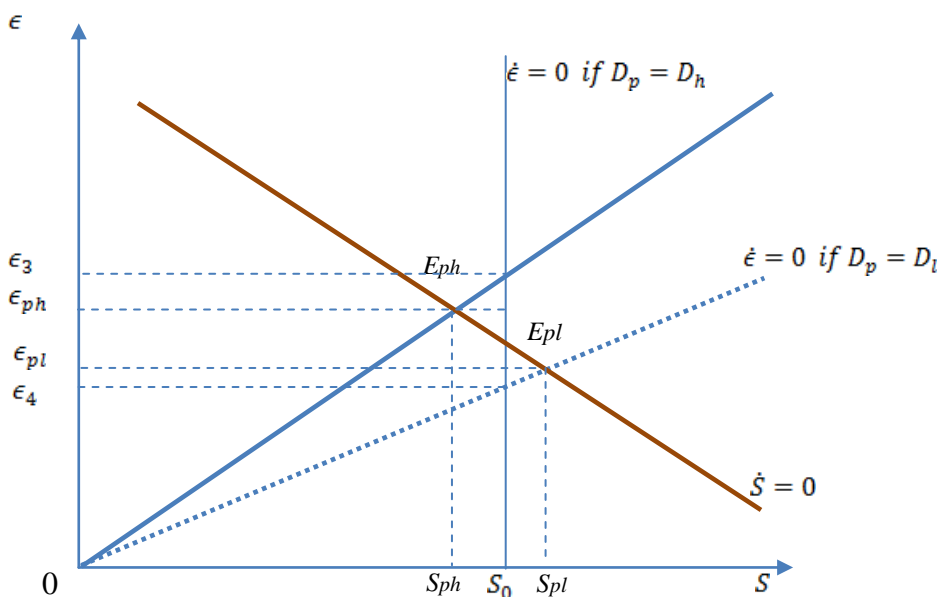
**Figure 2.** Initial values of the optimal environmental regulatory stringency under  $S_1 < S_0 < S_{ph}$  for both sufficient and insufficient estimations of liability costs for environmental damages.

compared to the steady state solution of lower environmental consideration, given that the initial value of the stock of accumulative pollutants is within a given range and under the condition that the social preference towards the consideration of liability costs for environmental damages does not change over time in each of the cases.

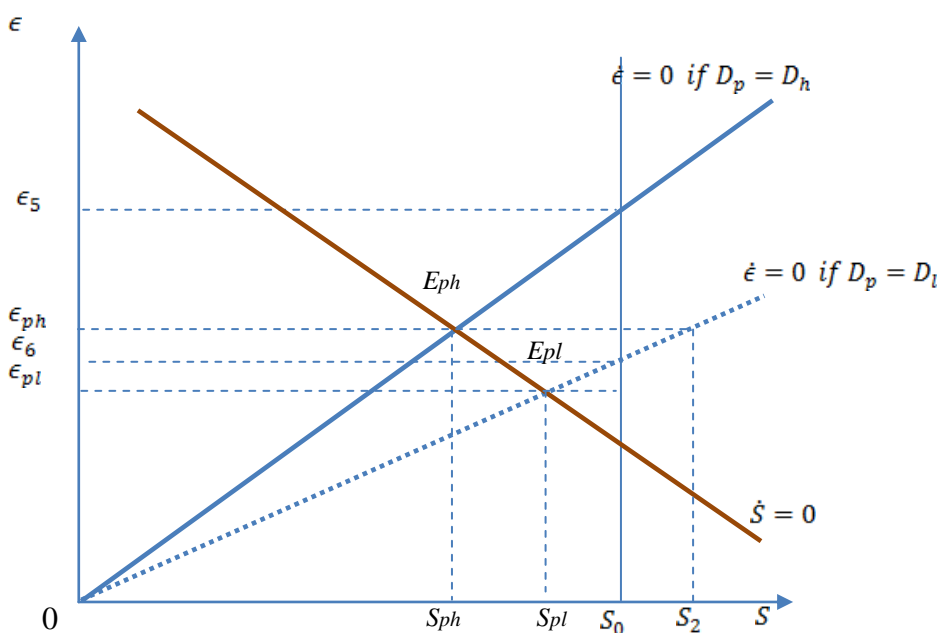
In Figure 2,  $S_1$  represents the stock of accumulative pollutants at the steady state with sufficient estimation of liability cost ( $D_p = D_h$ ) and at the level of environmental regulations stringency ( $\varepsilon_{pl}$ ) obtained under the steady state of the stock of accumulative pollutants ( $\dot{S} = 0$ ) and the steady state with insufficient estimation of liability costs ( $D_p = D_l$ ). As seen in Figure 2, given  $S_1 < S_0 < S_{ph}$ , concerning the case of sufficient estimation of liability costs for environmental damages, from the result of 1), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_1 < \tilde{\varepsilon}_{0h} < \varepsilon_{ph}$ . On the other hand, given  $S_1 < S_0 < S_{ph}$ , concerning the case of insufficient estimation of liability costs, from the result of 1), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_2 < \tilde{\varepsilon}_{0l} < \varepsilon_{pl}$ . From Figure 2, we can identify  $\varepsilon_2 < \tilde{\varepsilon}_{0l} < \varepsilon_{pl} < \varepsilon_1 < \tilde{\varepsilon}_{0h} < \varepsilon_{ph}$ . From transitivity,  $\tilde{\varepsilon}_{0l} < \tilde{\varepsilon}_{0h}$ . That is, when sufficient liability costs for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the estimation of the liability costs are insufficient and leads to a steady state of low environmental consideration. Accordingly, when insufficient liability costs for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of low environmental consideration, is at a lower level than the initial optimal environmental regulatory stringency when the liability costs are sufficiently estimated and leads to a steady state of high environmental consideration.

As indicated in Figure 3, given  $S_{ph} < S_0 < S_{pl}$ , with regard to the case of sufficient estimation of liability costs for environmental damages, from the result of 3), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be





**Figure 3.** Initial values of the optimal environmental regulatory stringency under  $S_{ph} < S_0 < S_{pl}$  for both sufficient and insufficient estimations of liability costs for environmental damages.



**Figure 4.** Initial values of the optimal environmental regulatory stringency under  $S_{pl} < S_0 < S_2$  for both sufficient and insufficient estimations of liability costs for environmental damages.

set within the range of  $\varepsilon_{ph} < \tilde{\varepsilon}_{0h} < \varepsilon_3$ . On the other hand, given  $S_{ph} < S_0 < S_{pl}$ , concerning the case of insufficient estimation of liability costs, from the result of (i), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_4 < \tilde{\varepsilon}_{0l} < \varepsilon_{pl}$ . As seen in Figure 3, we can identify  $\varepsilon_4 < \tilde{\varepsilon}_{0l} < \varepsilon_{pl} < \varepsilon_{ph} < \tilde{\varepsilon}_{0h} < \varepsilon_3$ . From transitivity,  $\tilde{\varepsilon}_{0l} < \varepsilon_{0h}$ . That is to say, when sufficient liability costs for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the liability costs are insufficiently estimated which leads to a steady state of low environmental consideration.

In Figure 4,  $S_2$  represents the stock of accumulative pollutants at the steady state with insufficient estimation of liability cost ( $D_p = D_l$ ) and at the level of environmental regulation stringency ( $\varepsilon_{ph}$ ) obtained under the steady state of the stock of accumulative pollutants ( $\dot{S} = 0$ ) and the steady state with sufficient estimation of liability costs ( $D_p = D_h$ ).

As shown in Figure 4, given  $S_{pl} < S_0 < S_2$ , as for the case of sufficient estimation of liability costs for environmental damages, from the result of (iii), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{ph} < \tilde{\varepsilon}_{0h} < \varepsilon_5$ . On the other hand, given  $S_{pl} < S_0 < S_2$ , concerning the case of insufficient estimation of liability costs, from the result of (iii), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{pl} < \tilde{\varepsilon}_{0l} < \varepsilon_6$ . As indicated in Figure 4, we are able to identify  $\varepsilon_{pl} < \tilde{\varepsilon}_{0l} < \varepsilon_6 < \varepsilon_{ph} < \tilde{\varepsilon}_{0h} < \varepsilon_5$ . From transitivity,  $\tilde{\varepsilon}_{0l} < \tilde{\varepsilon}_{0h}$ . In other words, when sufficient liability costs for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the liability cost is insufficiently estimated which leads to a steady state of low environmental consideration.

To summarize these results, given  $S_1 < S_0 < S_2$ , when sufficient liability costs for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the liability cost is insufficiently estimated which leads to a steady state of low environmental consideration. Accordingly, when insufficient liability cost for environmental damages are covered, the initial optimal environmental regulatory stringency that leads to the steady state of low environmental consideration, is at a lower level than the initial optimal environmental regulatory stringency when the liability cost is sufficiently estimated which leads to a steady state of high environmental consideration.

### **STEADY STATE COMPARISON BETWEEN SUFFICIENT AND INSUFFICIENT DISCOUNT RATES FOR FUTURE GENERATIONS**

Next, in this section, we will compare the steady state in the case of a discount rate with sufficient consideration for future generations with the steady state in the case of a discount rate with insufficient consideration. Here, with respect to the discount rate with insufficient consideration for future generations, we define it as  $r_q = r_h$ . On the other hand, with regards to the discount rate with sufficient consideration for future generations, we define it as  $r_q = r_l$ . When future generations are not adequately considered, there is the tendency that environmental costs along with human activities including economic activities are not sufficiently estimated and the discount rate is relatively high. On the other hand, when future generations are adequately considered, there is the tendency that environmental costs along with human activities including economic activities are sufficiently estimated and the discount rate is low. Hence,

$$r_h > r_l. \quad (15)$$

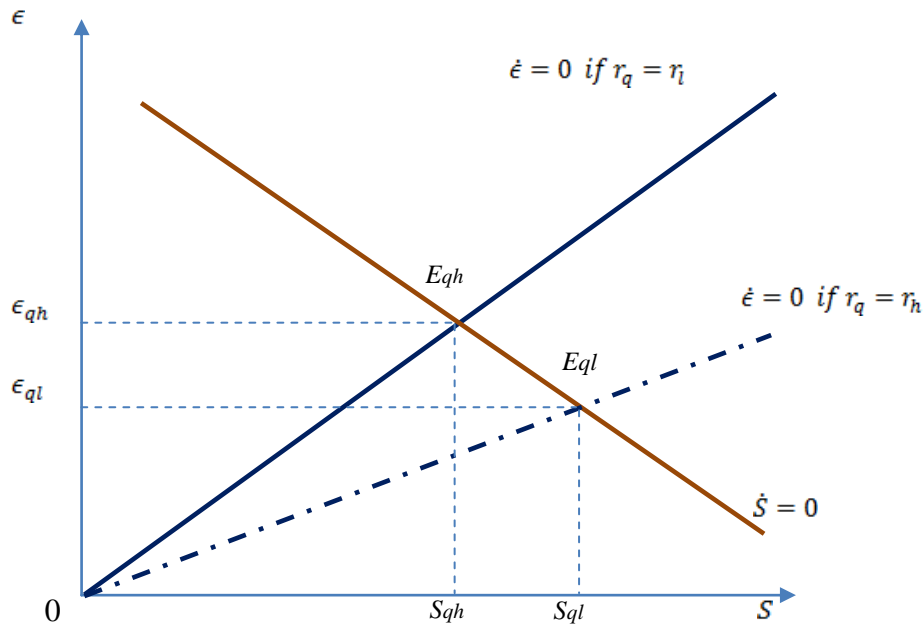
Substitute (15) into (8), we can attain:

$$\frac{D_p''(S)}{r_l + d} > \frac{D_p''(S)}{r_h + d} > 0. \quad (16)$$

Since this section and the next section only focuses on the differences between the cases with discount rates with sufficient consideration for future generation and the discount rate with insufficient consideration, for convenience, it assumes that the function concerning the

estimation of liability costs for environmental damages are identical for both cases. Hence, from (12) and (16), we can draw Figure 5.

In Figure 5,  $S_{qh}$  represents the stock of accumulative pollutants at the steady state when the discount rate for future generations is sufficient and  $S_{ql}$  indicates the stock of accumulative pollutants at the steady state when the discount rate for future generations is insufficient. As seen in Figure 5, we can find that in the case of environmental costs estimated adopting a discount rate with sufficient consideration for future generations, the steady state value of the optimal environmental regulatory stringency are higher, and the stock of accumulative pollutants are lower, than compared with the case of environmental costs estimated adopting a discount rate with insufficient consideration for future generations. That is, this could be regarded as the steady state solution of high environmental consideration. Accordingly, in the case of a discount rate with insufficient consideration for future generations, the steady state value of the optimal environmental regulatory stringency are lower, and the stock of accumulative pollutants are higher, than compared with the case of a discount rate with sufficient consideration for future generations. Hence, they could be interpreted as the steady state solution of low environmental consideration.



**Figure 5.** Steady state solutions in the case of a discount rate with sufficient consideration for future generations and in the case of a discount rate with insufficient consideration.

From the previous results, we can find that the results of the case of sufficient estimation of liability costs for environmental damages is similar to the results of the case of a discount rate with sufficient consideration of future generations. We can also find that the results of the case of insufficient estimation of liability costs for environmental damages is similar to the results of the case of a discount rate with insufficient consideration for future generations.

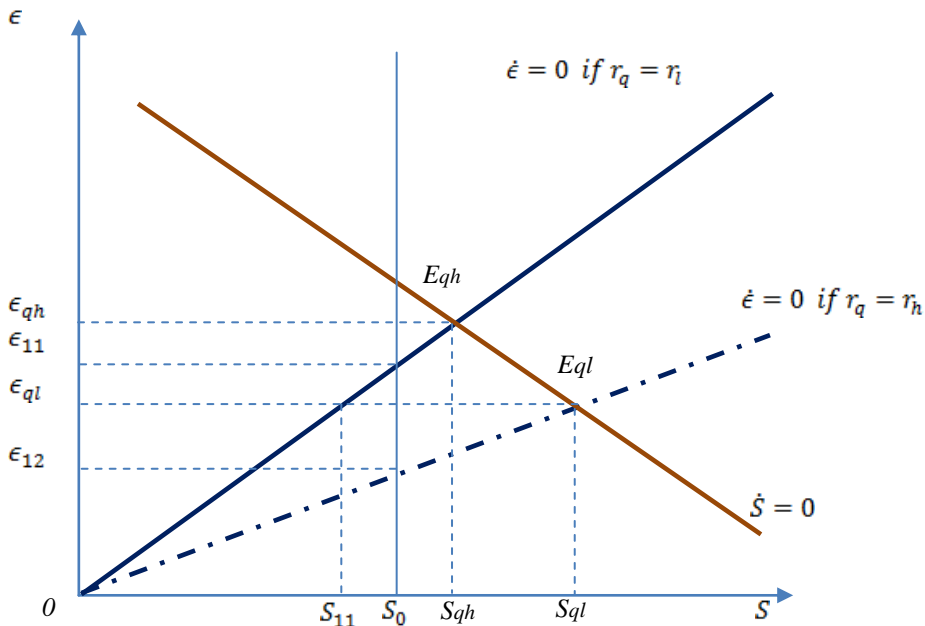
### **DIFFERENCE IN THE INITIAL VALUE OF THE ENVIRONMENTAL REGULATIONS – COMPARING SUFFICIENT AND INSUFFICIENT DISCOUNT RATES FOR FUTURE GENERATIONS**

Next, in this section, similar to analysis on the consideration of liability costs for environmental damages, under the condition that the initial value of the stock of accumulative pollutants is within a given range, we will examine, how the initial value of the optimal

environmental regulatory stringency in the case of a discount rate with sufficient consideration for future generations differs from the initial value of optimal environmental regulatory stringency in the case of a discount rate with insufficient consideration.

In Figure 6,  $S_{11}$  represents the stock of accumulative pollutants at the steady state with sufficient consideration for future generations ( $r_q = r_l$ ) and at the level of environmental regulations stringency ( $\varepsilon_{ql}$ ) obtained under the steady state of the stock of accumulative pollutants ( $\dot{S} = 0$ ) and the steady state with insufficient consideration for future generations ( $r_q = r_h$ ). As seen in Figure 6, given  $S_{11} < S_0 < S_{qh}$ , concerning the case of a discount rate with sufficient consideration for future generations, from the result of 1), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{11} < \hat{\varepsilon}_{0h} < \varepsilon_{qh}$ . On the other hand, given  $S_{11} < S_0 < S_{qh}$ , concerning the case of a discount rate with insufficient consideration for future generations, from the result of 1), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{12} < \hat{\varepsilon}_{0l} < \varepsilon_{ql}$ . From Figure 6, we can confirm  $\varepsilon_{12} < \hat{\varepsilon}_{0l} < \varepsilon_{ql} < \varepsilon_{11} < \hat{\varepsilon}_{0h} < \varepsilon_{qh}$ . From transitivity,  $\hat{\varepsilon}_{0l} < \hat{\varepsilon}_{0h}$ . That is, when the discount rate is adopted, sufficiently considering future generations, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the discount rate is adopted with insufficient consideration of future generations and leads to a steady state of low environmental consideration.

From these results, the initial value of the optimal environmental regulatory stringency in the case of a discount rate with sufficient consideration of future generations and in the case of a discount rate with insufficient consideration under  $S_{11} < S_0 < S_{qh}$ , corresponds with the trend of the initial value of the optimal environmental regulatory stringency in the case of sufficient liability for environmental damages and in the case of insufficient liability costs for environmental damages under  $S_1 < S_0 < S_{ph}$ .



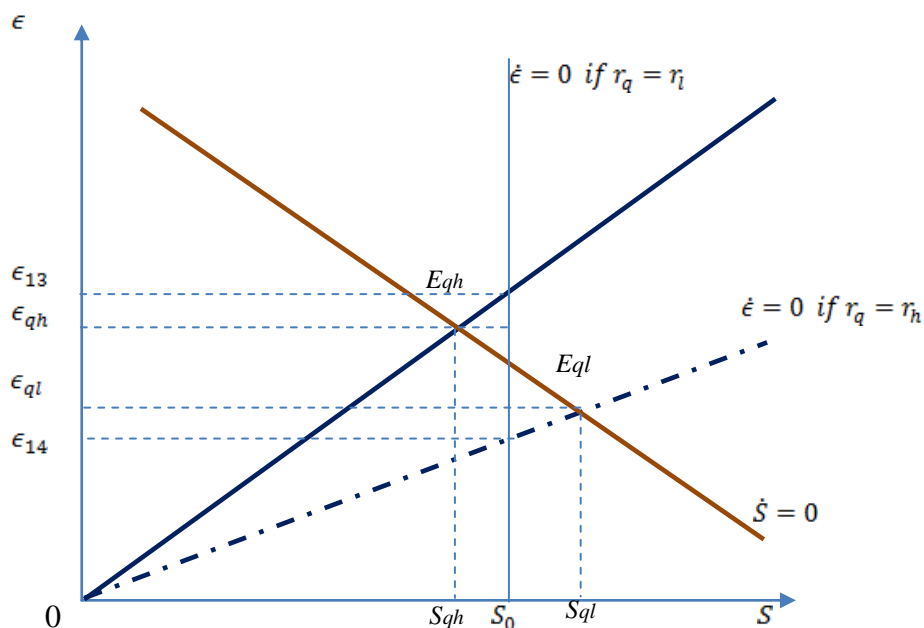
**Figure 6.** Initial values of the optimal environmental regulatory stringency under  $S_{11} < S_0 < S_{qh}$  for both discount rates with sufficient and insufficient consideration for future generations.

As indicated in Figure 7, given  $S_{qh} < S_0 < S_{ql}$ , with respect to the case of a discount rate with sufficient consideration of future generations, from the result of 3), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{qh} < \hat{\varepsilon}_{0h} < \varepsilon_{13}$ . On the other hand, given  $S_{qh} < S_0 < S_{ql}$ , concerning the case of a discount rate with insufficient consideration of future generations, from the result of 1), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{14} < \hat{\varepsilon}_{0l} < \varepsilon_{ql}$ . As found in Figure 7, we are able to see  $\varepsilon_{14} < \hat{\varepsilon}_{0l} < \varepsilon_{ql} < \varepsilon_{qh} < \hat{\varepsilon}_{0h} < \varepsilon_{13}$ . From transitivity,  $\hat{\varepsilon}_{0l} < \hat{\varepsilon}_{0h}$ . That is to say, when discount rate is adopted, sufficiently considering future generations, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the discount rate is adopted with insufficient consideration for future generations which leads to a steady state of low environmental consideration.

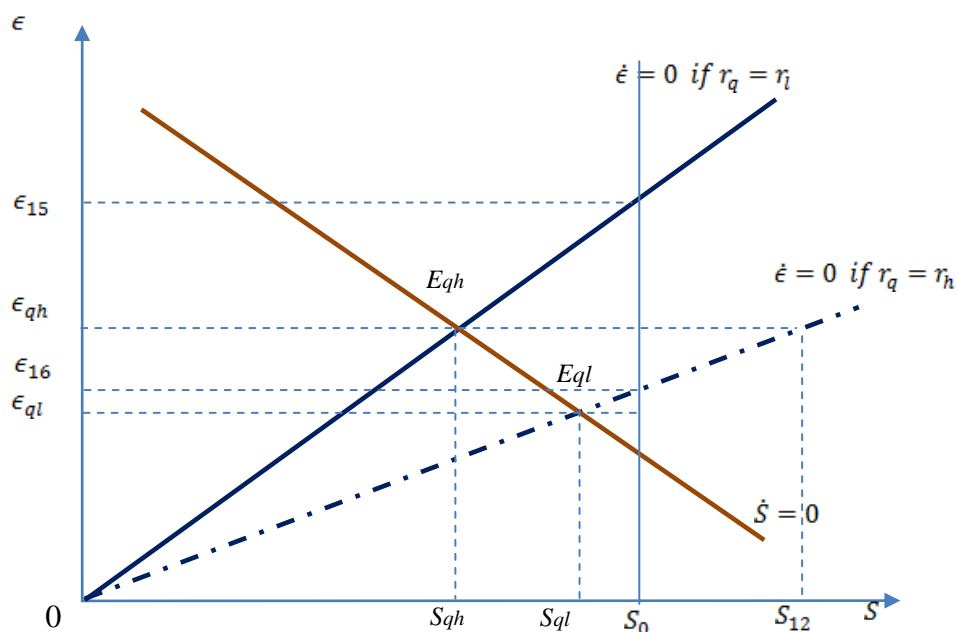
From these results, the initial values of the optimal environmental regulatory stringency in the case of a discount rate with sufficient consideration of future generations and in the case of a discount rate with insufficient consideration under  $S_{qh} < S_0 < S_{ql}$ , corresponds to the initial values of the optimal environmental regulatory stringency in the case of a sufficient estimation of liability costs for environmental damages and in the case of an insufficient estimation under  $S_{ph} < S_0 < S_{pl}$ .

In Figure 8,  $S_{12}$  represents the stock of accumulative pollutants at the steady state with insufficient consideration for future generations ( $r_q = r_h$ ) and at the level of environmental regulations stringency ( $\varepsilon_{qh}$ ) obtained under the steady state of the stock of accumulative pollutants ( $\dot{S} = 0$ ) and the steady state with sufficient consideration for future generations ( $r_q = r_l$ ). As shown in Figure 8, given  $S_{ql} < S_0 < S_{12}$ , as for the case of a discount rate with sufficient consideration for future generations, from the result of 3), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{qh} < \hat{\varepsilon}_{0h} < \varepsilon_{15}$ . On the other hand, given  $S_{ql} < S_0 < S_{12}$ , concerning the case of a discount rate with insufficient consideration of future generations, from the result of 3), the initial value of the optimal environmental regulatory stringency which leads to the steady state solution can be set within the range of  $\varepsilon_{ql} < \hat{\varepsilon}_{0l} < \varepsilon_{16}$ . Through Figure 8, we can find out  $\varepsilon_{ql} < \hat{\varepsilon}_{0l} < \varepsilon_{16} < \varepsilon_{qh} < \hat{\varepsilon}_{0h} < \varepsilon_{15}$ . From transitivity,  $\hat{\varepsilon}_{0l} < \hat{\varepsilon}_{0h}$ . In other words, when the discount rate is adopted, sufficiently considering future generations, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the discount rate is adopted with insufficient consideration of future generations which leads to a steady state of low environmental consideration.

From these results, the initial values of the optimal environmental regulatory stringency in the case of a discount rate with sufficient consideration of future generations and in the case of a discount rate with insufficient consideration under  $S_{ql} < S_0 < S_{12}$ , corresponds to the initial values of the optimal environmental regulatory stringency in the case of a sufficient estimation of liability costs for environmental damages and in the case of an insufficient estimation under  $S_{pl} < S_0 < S_2$ . To summarize the results, given  $S_{11} < S_0 < S_{12}$  when the discount rate with sufficient consideration of future generations is adopted, the initial optimal environmental regulatory stringency that leads to the steady state of high environmental consideration, is at a higher level than the initial optimal environmental regulatory stringency when the adopted discount rate is high and leads to a steady state of low environmental consideration.



**Figure 7.** Initial values of the optimal environmental regulatory stringency under  $S_{qh} < S_0 < S_{ql}$  for both discount rates with sufficient and insufficient consideration for future generations.



**Figure 8.** Initial values of the optimal environmental regulatory stringency under  $S_{ql} < S_0 < S_{12}$  for both discount rates with sufficient and insufficient consideration for future generations.

From the results described, the case of a discount rate with sufficient consideration of future generations is similar to the case of sufficient estimation of liability costs for environmental damages. Accordingly, the case of a discount rate with insufficient consideration of future generations corresponds to the case of insufficient estimation of liability costs for environmental damages.

## CONCLUSIONS

Along with the expansion of economic activities, accumulative pollution such as global warming, radioactive contamination, and dioxins is a growing concern. In order to tackle this

concern, environmental costs which include the liability costs of environmental damages and abatement costs will be required and it will be important to understand the social preference towards such environmental costs.

Therefore, this article examines the impacts the difference in social preference towards environmental costs have on the steady state solution of the stock of accumulative pollutants and the optimal environmental regulatory stringency. Furthermore, this article analyses the difference in terms of the initial value of the optimal environmental regulatory stringency, depending on the social preference towards environmental costs.

The results find that the steady state value of the optimal environmental regulatory stringency was higher, and the steady state value of the stock of the accumulative pollutants was lower in the case when the estimation of the liability costs for environmental damages was sufficient, compared to when the estimation was insufficient. This article also finds that the steady state value of the optimal environmental regulatory stringency was higher, and the steady state value of the stock of accumulative pollutants was lower in the case when the discount rate adopted took sufficient consideration for future generations, than compared to the adoption of a discount rate with insufficient consideration. These results indicate that in the case of high social preference towards addressing environmental issues accepting environmental costs, we can find the convergence of the steady state solution with high environmental consideration where the optimal environmental regulatory stringency is higher and the stock of accumulative pollutants is lower, than compared with the case of low social preference towards costs concerning environmental issues. Accordingly, as in the case of insufficient estimation of the liability for environmental damages and the case with the adoption of a discount rate with insufficient consideration for future generations where there is low social preference towards costs concerning environmental issues, we find the convergence of the steady state solution with low environmental consideration where the optimal environmental regulatory stringency is lower and the stock of accumulative pollutants is higher, than compared with the case of high social preference towards costs to address environmental issues. Therefore, it is desirable to conduct policies to encourage a shift towards the social preference to address environmental issues and accepting the environmental costs. For example, it would be necessary to promote educational activities to penetrate the concept of intergenerational equity and the intrinsic values of the environment, and promote the concept and measures of environmental accounting to companies and public organizations.

The results indicate that the steady state solution converges at a higher state of environmental consideration if society sufficiently estimates the liability costs for environmental damages, and/or adopts discount rates with sufficient consideration of future generations. When aiming to achieve such a steady state of high environmental consideration and the initial value of the stock of accumulative pollutants are within a given range, the initial value of the stringency of environmental regulations are set much higher than when society is not aiming for such a high environmental consideration and converges to a steady state of low consideration. Hence, in order to support the convergence it would be necessary to establish appropriate systems and/or institutions. For instance, the gathering and accessibility of information and an objective monitoring system would be required to support society to make well informed decisions. Here, we will refer to the shortcoming of this study. Given that  $S_0 < S_1$  and  $S_2 < S_0$ , the possible range of the initial value of the optimal environmental regulatory stringency that converges to a steady state solution for the case of a sufficient estimate of liability costs for environmental damages, partially overlaps with the possible range of the initial value of the optimal environmental regulatory stringency for the case of an insufficient estimation of liability costs. Hence, it would be uncertain if the initial value of the optimal environmental regulatory stringency in the case of a sufficient estimation of liability costs is higher than that

in the case of an insufficient estimation. It would depend on the function of abatement costs and/or of liability costs for environmental damages in the case of those sufficiently estimated and in the case of those insufficiently estimated. Given that  $S_0 < S_{11}$  and  $S_{12} < S_0$  the possible range of the initial value of the optimal environmental regulatory stringency that converges to a steady state solution in the case of the adoption of a discount rate with sufficient consideration of future generations partially overlaps with the possible range of the initial value of the optimal environmental regulatory stringency when a discount rate with insufficient consideration is adopted. Hence, it would be uncertain if the initial value of the optimal environmental regulatory stringency in the case of a discount rate with sufficient consideration for future generations is higher than that of the case of a discount rate with insufficient consideration. It would depend on the difference between the discount rate of sufficient and insufficient consideration of future generations, or the function of the abatement costs and/or of liability costs for environmental damage. Hence, this would be a possible area for future studies.

From the previous results, the steady state solution of the stock of accumulative pollutants and the optimal environmental regulatory stringency, and the initial value of the optimal environmental regulatory stringency depend on social preference toward environmental costs to address environmental issues. Therefore, it could lead to the establishment of a more sustainable society, by conducting the appropriate policies based on these results.

## APPENDIX

Explanation concerning equation (4)

$$\frac{\partial H^C}{\partial M} = 0.$$

Therefore,

$$A'(M) - \varepsilon = 0.$$

$$\therefore \varepsilon = A'(M).$$

With respect to equation (5),

$$-\frac{\partial H^C}{\partial S} e^{-r_q t} = \frac{\partial(\varepsilon e^{-r_q t})}{\partial t}.$$

Therefore,

$$-\left[D'_p(S) - \varepsilon d\right] e^{-r_q t} = \dot{\varepsilon} e^{-r_q t} - r_q \varepsilon e^{-r_q t}.$$

Divide both sides of the above equation by  $e^{-r_q t}$ , then we can obtain:

$$\dot{\varepsilon} - r_q \varepsilon = -D'_p(S) + \varepsilon d,$$

$$\therefore \dot{\varepsilon} = -D'_p(S) + \varepsilon(r_q + d).$$

## REFERENCES

- [1] Solow, R.M.: *An almost practical step toward sustainability*. Resources Policy **19**(3), 162-172, 1993, [http://dx.doi.org/10.1016/0301-4207\(93\)90001-4](http://dx.doi.org/10.1016/0301-4207(93)90001-4),
- [2] Toman, M.A. and Withagen, C.: *Accumulative pollution, "clean technology," and policy design*. Resource and Energy Economics **22**(4), 367-384, 2000, [http://dx.doi.org/10.1016/S0928-7655\(00\)00022-1](http://dx.doi.org/10.1016/S0928-7655(00)00022-1),
- [3] Yokoyama, A.: *Kankyo-mondai to gaibu-fukeizai [Environmental issues and negative externalities]*. In: Hosoda, E. and Yokoyama, A., eds.: *Kankyo-keizaigaku [Environmental Economics]*. Yuhikaku-arma, Tokyo, 2007,



- [4] Ministry of the Environment: *Kankyo-kaikei guidebook [Environmental accounting guidebook]*.  
<http://www.env.go.jp/policy/kaikei/guide05.pdf>,
- [5] U.S. Environmental Protection Agency: *U.S. Environmental Protection Agency Design for the Environment Program Environmental Accounting Project*.  
<https://www.epa.gov/sites/production/files/2014-01/documents/busmgt.pdf>,
- [6] Shimamoto, K.: *Empirical Study on the Relationship between Environmental Management, Productivity and Profitability at the Firm Level: The Case of Japan*.  
*Hirao School of Management Review* **3**, 66-90, 2013,
- [7] Field, B.C. and Field, M.K.: *Environmental Economics: An Introduction*.  
McGraw-Hill Education, Boston, 2009,
- [8] Holl, K.D. and Howarth, R.B.: *Paying for restoration*.  
*Restoration Ecology* **8**(3), 260-267, 2000,  
<http://dx.doi.org/10.1046/j.1526-100x.2000.80037.x>,
- [9] Yoshimura, R.: *Kankyo-higai no baisho-kankyo-hogo niokeru koushikyodo no ichisokumen [Compensation for Environmental Damages – An examination of the cooperation between the public and private sector for environmental protection]*.  
*Ritsumeikan Law Review* **5-6**, 1769-1802, 2010,
- [10] Martin-Ortega, J.; Brouwer, R. and Aiking, H.: *Application of a value-based equivalency method to assess environmental damage compensation under the European Environmental Liability Directive*.  
*Journal of Environmental Management* **92**(6), 1461-1470, 2011,  
<http://dx.doi.org/10.1016/j.jenvman.2010.12.001>,
- [11] Hosei University Ohara Institute for Social Research: *The Labour Year Book of Japan 1983*.  
No. 53, 1983,
- [12] Otsuka, N.: *Kankyo-hou [Environmental law]*.  
Youhikaku, Tokyo, 2007,
- [13] Farber, D.A. and Hemmersbaugh, P.A.: *The shadow of the future: discount rates. Later generations, and the environment*.  
*Vanderbilt Law Review* **46**(2), 267-304, 1993,
- [14] Ueda, K.: *Kankyo-keizai [Environmental Economics]*.  
Iwanami-shoten, Tokyo, 2009,
- [15] Hasegawa, H.: *Kankyo ni hairyo shita waribiki-ritsu ya waribiki- keisan [Environmentally considerate discount rate and estimation]*.  
*Proceedings of 1997 Annual Meeting of Society for Environmental Economics and Policy Studies*, 406-411, 1997,
- [16] Withagen, C.: *Pollution, abatement and balanced growth*.  
*Environmental and Resource Economics* **5**(6), 1-8, 1995,  
<http://dx.doi.org/10.1007/BF00691906>,
- [17] van der Ploeg, F. and Withagen, C.: *Pollution control and the Ramsey problem*.  
*Environmental and Resource Economics* **1**(2), 215-236, 1991,  
<http://dx.doi.org/10.1007/BF00310019>,
- [18] Leandri, M.: *The shadow price of assimilative capacity in optimal flow pollution control*.  
*Ecological Economics* **68**(4), 1020-1031, 2009,  
<http://dx.doi.org/10.1016/j.ecolecon.2008.11.019>,
- [19] Chev e, M.: *Irreversibility of Pollution Accumulation: New Implications for Sustainable Endogenous Growth*.  
*Environmental and Resource Economics* **16**(1), 93-104, 2000,  
<http://dx.doi.org/10.1023/A:1008367226371>,
- [20] Moslener, U. and Requate, T.: *The Dynamics of Optimal Abatement Strategies for Multiple Pollutants – An illustration in the Green House*.  
*Ecological Economics* **68**(5), 1521-1534, 2009,  
<http://dx.doi.org/10.1016/j.ecolecon.2008.10.010>,

- [21] Hole, M. and Karp, L.: *Taxes versus quotas for a stock pollutant*.  
Resource and Energy Economics **24**(4), 367-384, 2002,  
[http://dx.doi.org/10.1016/S0928-7655\(02\)00014-3](http://dx.doi.org/10.1016/S0928-7655(02)00014-3),
- [22] Kling, C. and Rubin, J.: *Bankable permits for the control of environmental pollution*.  
Journal of Public Economics **64**(1), 101-115, 1997,  
[http://dx.doi.org/10.1016/S0047-2727\(96\)01600-3](http://dx.doi.org/10.1016/S0047-2727(96)01600-3),
- [23] Rubin, J.D.: *A model of intertemporal emission trading, banking, and borrowing*.  
Journal of Environmental Economics and Management **31**(3), 269-286, 1996,  
<http://dx.doi.org/10.1006/jeem.1996.0044>,
- [24] Saltari, E. and Travaglini, G.: *Pollution control under emission constraints: Switching between regimes*.  
Energy Economics **53**, 212-219, 2016,  
<http://dx.doi.org/10.1016/j.eneco.2014.07.010>,
- [25] Chukwuemeka, O.: *Managing an Accumulative Inorganic Pollutant: An Optimal Tax Prescription for the Social Planner*.  
International Journal of Economics, Commerce and Management **3**(8), 1-14, 2015.