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ROBUST SLIDING MODE CONTROL FOR FLEXIBLE JOINT ROBOTIC MANIPULATOR VIA DISTURBANCE OBSERVER

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ABSTRACT

In a flexible joint robotic manipulator, parametric variations and external disturbances result in mismatch uncertainties thus posing a great challenge in terms of manipulator's control. This article investigates non-linear control algorithms for desired trajectory tracking of a flexible manipulator subjected to mismatch perturbations. The manipulator's dynamics is derived based on Euler-Lagrange approach followed by the design of nonlinear control laws. The traditional Sliding Mode Control and Integral Sliding Mode Control failed to demonstrate adequate performance due to complex system dynamics. Disturbance Observer-based Sliding Mode Control has been thoroughly examined by defining a novel sliding manifold. The aforementioned control laws are designed and simulated in MATLAB/Simulink environment to characterize the control performance. Results demonstrated that the proposed Disturbance Observer-based Sliding Mode Control scheme over-performed on Sliding Mode Control variants and had three prominent features: robustness against mismatch uncertainty, improved chattering behaviour and ability to sustain nominal control performance of the system.

KEYWORDS

robotics, automation, modern control, flexible joint manipulator

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INTRODUCTION

In the recent decade, the desired trajectory tracking problem of flexible joint robotic manipulator got considerable attention in the scientific community. Numerous control strategies have been investigated to address the aforementioned problem, most of which assume torque as an input to the system, thus excluding actuator dynamics. However, the actuator dynamics is an essential component of an electromechanical system which must be considered in system modelling [1, 2].

Spong in [3], presented a dynamic model of a flexible joint manipulator, which instigated many researchers to carry out studies on its desired trajectory tracking problem. Various reported control methods to address this tracking problem are based on both linear and nonlinear control laws. In [4], the effects of joint flexibility on the dynamic response of flexible joint manipulator are studied. In [5], the trajectory tracking control of the manipulator is presented using Proportional Integral Derivative (PID) Controller with state feedback control law. Sliding Mode Control (SMC) based adaptive law for a flexible joint manipulator with parametric uncertainty is presented in [6]. Nguyen et al. [7] designed a robust position and vibration control approach for an elastic manipulator with actuator perturbation. Some prominent laws explored in literature to control the manipulator include; Integral control approach [8], adaptive feedback linearization methodology [9], the singular perturbation scheme [10], fuzzy error governing approach for counteracting the actuator saturation [11] and Proportional Derivative (PD) based control method [12]. In [13, 14], adaptive backstepping based control algorithms are designed for a flexible joint manipulator with varying parameters to control the desired trajectory. In [15], adaptation based controller is designed for the trajectory tracking problem of a flexible joint manipulator with varying parameters. Iterative regulation of an electrically driven manipulator with unknown payload and parametric model variation is developed in [16].

To the best of authors' knowledge, little attention has been paid in the literature towards the inclusion of actuator dynamics and designing of observer-based control scheme for a flexible joint manipulator with mismatch perturbations. Thus instigated by the literature, the SMC based non-linear control approaches with Disturbance Observer (DO) are presented in this article for desired trajectory tracking of the manipulator with the inclusion of actuator dynamics as well as mismatch perturbations. The mismatch perturbations must be non-vanishing and are not necessarily H2 norm bounded.

SMC is gaining popularity in various scientific applications owing to its computational simplicity and excellent robust nature [17-20]. However, the sliding manifold in SMC is only focused on the alleviation of match uncertainties. Thus, the matching condition may not be satisfied by certain uncertainties present in the practical systems and hence the traditional SMC would not work anymore [21]. In SMC, the chattering problem is still a concerning issue which needs to be handled. For this reason, an Integral SMC is preferred. It is verified that Integral SMC is more practical and robust as compared to SMC [22]. However, in Integral SMC, integral action in the sliding surface brings some adverse effects such as high overshoot and degradation of nominal control performance. It means that Integral SMC scheme counteracts mismatch perturbations by compromising on system control performance.

To tackle the situation, a state of the art non-linear disturbance observer based SMC algorithm is developed for the trajectory tracking problem of the manipulator. By investigating a novel sliding manifold based on estimated disturbance, the system trajectory can be asymptotically driven to the desired equilibrium point, while counteracting the mismatch perturbations. The novelty of the proposed control scheme lies in its excellent robust nature to handle the mismatch uncertainty while retaining good nominal control performance and the capability to substantially mitigate the undesirable chattering problem.

This article is organized into following sections: In section II, mathematical modelling of a flexible joint manipulator is presented. Control strategies with stability analysis are developed in section III. Section IV covers simulation results and discussion. Finally, section V comments on the conclusion.

MATHEMATICAL MODELLING

A flexible joint manipulator is an electromechanical system in which the manipulator's link is connected to the actuator's shaft. The system is actuated with a DC motor with voltage as the input. The output is the position of the manipulator's end effector which can freely move around its x-axis. At the joint, the actuator's shaft is connected to the manipulator's link through a chain of gears which possess flexibility. Due to this flexibility at the joint, undesirable oscillations are produced which prevent the end-effector from precisely tracking the desirable position. The flexibility is portrayed as a linear torsion spring which is depicted as the combined effect of damping factor, spring constant and opposing force [3]. The graphical view of a flexible joint manipulator is shown in Figure 1.

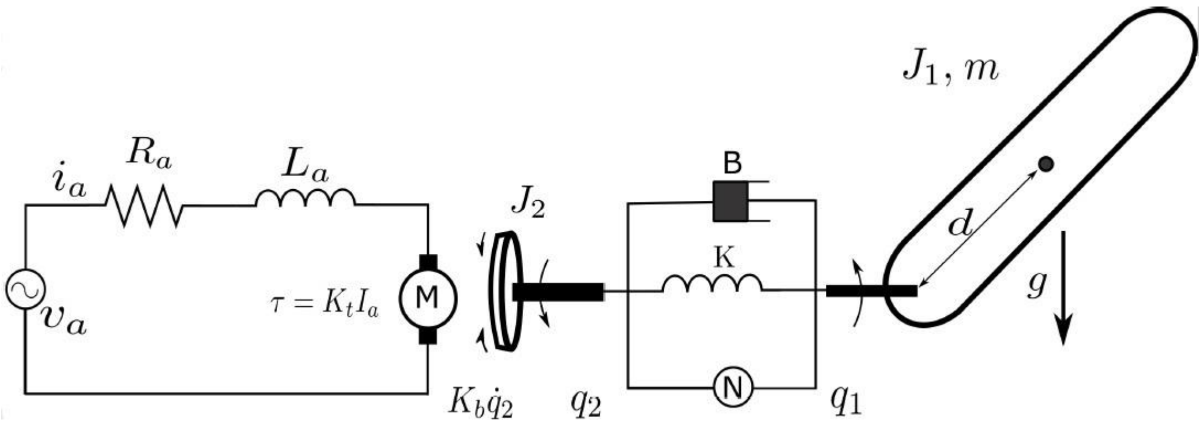

Figure 1. Block diagram of the flexible joint manipulator.

Dynamics of the electromechanical system is modelled using Euler-Lagrange equation in [23, 24], which are as follows:

$$J_l\ddot{q}_1 + mghsin(q_1) + k(q_1 - q_2) = 0 \quad (1)$$

$$J_{2\_d}\ddot{q}_2 - k(q_1 - q_2) + B_{\_d}\dot{q}_2 = k_{t\_d}I_a \quad (2)$$

$$u = R_aI_a + L_a\frac{dI_a}{dt} + K_b\dot{q}_2 \quad (3)$$

The description of the system parameters is listed in Table 1.

Table 1. System parameters and values.

Parameter	Symbol	Value	Unit	Parameter	Symbol	Value	Unit
Mass of link	m	1	kg	Length of link	h	0,5	m
Gears ratio	N	1	-	Gravitational acceleration	g	10	m/s^2
Armature resistance	R_a	1.6	Ω	Link moment of inertia	J_1	1	$kg{\cdot}m^2$
Motor torque constant	k_t	0,2	N·m/A	Motor shaft moment of inertia	J_2	0,3	$kg{\cdot}m^2$
Back EMF constant	B	0,001	N·m·s/rad	Armature inductance	L_a	0,001	H
Stiffness of joint	K	14	N·m/rad	Control input	u	-	V

ASSUMPTION 1. Certain parameters in (1), (2) and (3) are assumed to be time variant and can be written as $J_{2\_d}(.) = J_2 + \Delta J_2(t)$, $B_{\_d}(.) = B + \Delta B(t)$ and $k_{t\_d}(.) = k_t + \Delta k_t(t)$.

REMARK 1. The system parameters, described in assumption 1, are the combination of nominal and uncertain parts. The assumption made is pretty reasonable in case of practical applications. The parameters show uncertain behaviour around their nominal values due to the external environmental impacts. This is the reason that dynamics of the system (1), (2) and (3) is incorporated with such assumption.

The non-linear dynamical equations of the flexible joint manipulator can be represented in state space form as:

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = -\frac{mgh}{J_1}\sin(x_1) - \frac{k}{J_1}(x_1 - x_3) \tag{5}$$

$$\dot{x}_3 = x_4 \tag{6}$$

$$\dot{x}_4 = \frac{k}{J_2}(x_1 - x_3) - \frac{B}{J_2}x_4 + \frac{k_t}{J_2}x_5 + \varsigma(x,t) \tag{7}$$

$$\dot{x}_5 = -\frac{R_a}{L_a}x_5 - \frac{k_b}{L_a}x_4 + \frac{1}{L_a}u \tag{8}$$

$$y = x_1 \tag{9}$$

where $[x_1, x_2, x_3, x_4, x_5,] = [q_1, \dot{q}_1, q_2, \dot{q}_2, I_a]$, which represent angular position and angular velocity of manipulator's link, angular position and angular velocity of motor's shaft and the motor armature current respectively. y is the system output, while $\varsigma(x,t)$ represents the mismatch perturbation caused due to parametric variation, un-modelled dynamics and external disturbances.

ASSUMPTION 2. The mismatch perturbation in the system (4) – (9) is norm bounded and must satisfy $\varsigma^* = \lim_{t\to\infty}|\varsigma(x,t)|$, where ς^* is the upper bound of the mismatch uncertainty.

SMC BASED NONLINEAR CONTROL APPROACHES

The SMC based nonlinear control approaches are discussed as follows:

CONVENTIONAL SLIDING MODE CONTROL

SMC plays a vital role in the theory of variable structure system. It is a non-linear robust control algorithm, which has significant advantages in the field of control engineering [25]. The most prominent feature of SMC is the enforcement of the system's trajectories onto a defined switching manifold which is called a sliding or switching surface. Once the system trajectories reach the defined manifold, the configuration of the controller is then altered continuously to keep the states on the switching surface. SMC scheme exhibits insensitivity to parametric variations, un-modelled system dynamics and external disturbances. Besides offering the salient features, the high frequency switching along a sliding manifold results in the so-called chattering phenomena, which is the inherent property and is thus considered as the main limitation of SMC [26].

Sliding manifold for the system (1)-(3) is given as;

$$s = (\frac{d}{dt} + c)^{n-1}z \tag{10}$$

where c is a constant switching parameter i.e., c > 0. n is the system relative degree and z is the error variable, which can be expressed as,

$$z = y - x_d = x_1 - x_d \tag{11}$$

As the system has full relative degree i.e. $= 5$, so the sliding manifold (10) can be expressed as,

$$s = \frac{d\dddot{z}}{dt} + 4c\dddot{z} + 6c^2\ddot{z} + 4c^3\dot{z} + zc^4 \tag{12}$$

Taking derivatives of (12) and substituting (4)-(9) in (12), the surface manifold can be expressed as,

$$\dot{s} = \psi + \frac{kk_t}{J_1 J_2 L} u + \left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\varsigma(x,t) \tag{13}$$

where the notation ψ is used for the sake of brevity and can be expressed as,

$$\psi = c^4 x_2 + 4c^3 \dot{x_2} + 6c^2 \ddot{x_2} + 4c - c^4 \dot{x_d} - 4c^3 \ddot{x_d} + 6c^2 \dddot{x_d} - 4c\frac{d\dddot{x}_d}{dt} - \frac{d\dddot{x}_d}{dt}\Big(-\frac{mgd}{J_1}\cos(x_1)\dot{x_2} + \frac{mgd}{J_1}\sin(x_1)x_2{}^2 + \frac{2mghk}{J_1{}^2}\sin(x_1) + \frac{k^2}{J_1{}^2}(x_1 - x_3) + \frac{k^2}{J_1 J_2}(x_1 - x_3) - \frac{Bk}{J_1 J_2}x_4 + \frac{k_t k}{J_1 J_2}x_5\Big) - \frac{mgh}{J_1}\ddot{x_2}\cos(x_1) + \frac{mgh}{J_1}\sin(x_1)x_2\dot{x_2} + \frac{2mgh}{J_1}\sin(x_1)x_2\dot{x_2} + \frac{mgh}{J_1}x_2{}^3\cos(x_1) + \frac{mghk}{J_1{}^2}\cos(x_1)x_2 + \frac{k^2}{J_1{}^2}(x_2 - x_4) + \frac{k^2}{J_1 J_2}(x_2 - x_4) - \frac{Bk}{J_1 J_2}\Big(\frac{k}{J_2}(x_1 - x_3) - \frac{B}{J_2}x_4 + \frac{k_t}{J_2}x_5\Big) + \frac{k_t k}{J_1 J_2}\Big(\frac{-R}{L}x_5 - \frac{k_b}{L}x_4\Big). \tag{14}$$

The designed control input i.e. u, obtained using exponential reaching law is given as

$$u = \frac{LJ_1 J_2}{kk_t}(-\psi - k_1 sign(s) - k_2 s) \tag{15}$$

where k_1 and k_2 are switching gain parameters. Substituting (15) in (14) results in new surface dynamics, which can be expressed as

$$\dot{s} = -k_1 sign(s) - k_2 s + \left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\varsigma(x,t) \tag{16}$$

Stability Analysis

To examine the stability of the proposed control scheme, Lyapunov candidate function is considered, which can be expressed as;

$$V = \frac{1}{2}s^2 \tag{17}$$

To ensure asymptotic stability, the derivative of Lyapunov function must be negative definite for $s \neq 0$ i.e. $\dot{V} < 0$. The derivative of the Lyapunov candidate function (17) can be expressed as;

$$\dot{V} \leq -k_2|s|^2 - |s|\left\{k_1 - \left|\left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\right| |\varsigma(x,t)|\right\} \tag{18}$$

This proves the asymptotic stability of the system trajectory to the desired equilibrium point. The asymptotic stability $\dot{V} < 0$ holds if the gains in the control input hold the condition

$$k_2 > 0,\ k_1 > \left|\left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\right| \varsigma^*(x,t) \tag{19}$$

where $\varsigma^*(x,t)$ represents the maximum value of the mismatch perturbations. At s = 0 the solution of differential equation (12) can be given by,

$$z(t) = \left(e^{-ct} + \frac{c^2 t^2 e^{-ct}}{2} + \frac{c^3 t^3 e^{-ct}}{6} + cte^{-ct}\right) z(0) + \varsigma(x,t)\left\{\frac{1}{c^4} - e^{-ct}\left(\frac{1}{c^4} + \frac{t}{c^3} + \frac{t^2}{2c^2} + \frac{t^3}{6c}\right)\right\} \tag{20}$$

where $z(t)$ is the tracking error with convergence rate given in (20). It is evident from the equation that despite of using the control law, tracking error does not converge to zero i.e. $\lim_{t\to\infty} z(t) \neq 0$. This is because of the fact, that traditional SMC scheme is highly sensitive to mismatched perturbations.

INTEGRAL SLIDING MODE CONTROL

Integral SMC is proposed to overcome the limitations found in traditional SMC strategy. It is well established that Integral SMC is an effective control approach for compensation of mismatched uncertainty [27]. It concentrates on the system to be insensitive in the entire state space to any perturbation. Furthermore, it significantly alleviates the chattering problem.

The integral sliding manifold for the system (1)-(3) is defined as

$$s = \int (c + \frac{d}{dt})^n z, \quad (21)$$

where c is a constant parameter, n represents system relative degree and z is the difference between the desired and actual trajectory, which can be expressed as

$$z = y - x_d = x_1 - x_d. \quad (22)$$

Taking time derivative and substituting (22) in (21), the integral sliding manifold is formulated as

$$\dot{s} = \vartheta + \frac{kk_t}{J_1 J_2 L} u + (\frac{5ck}{J_1} - \frac{Bk}{J_1 J_2}) \varsigma(x,t), \quad (23)$$

where

$$\vartheta = c^5 z + 5c^4 \dot{z} + 10c^3(\dot{x_2} - \ddot{x_d}) + 10c^2\left(-\frac{mgh}{J_1}\cos(x_1)x_2 - \frac{k}{J_1}(x_2 + x_4) - \dddot{x_d}\right) + 5c\left(-\frac{mgh}{J_1}\cos(x_1)\dot{x_2} + \frac{mgh}{J_1}\sin(x_1){x_2}^2 - \frac{k}{J_1}\dot{x_2} + \frac{k}{J_1}\dot{x_4}\frac{d\dddot{x_d}}{dt}\right) - \frac{mgh}{J_1}\cos(x_1)\ddot{x_2} + \frac{mgh}{J_1}\sin(x_1)x_2\dot{x_2} + \frac{2mgh}{J_1}\sin(x_1)\,x_2\dot{x_2} + \frac{mgh}{J_1}\cos(x_1)\,{x_2}^3 - \frac{k}{J_1}\ddot{x_2} + \frac{k^2}{J_1 J_2}(x_2 - x_4) - \frac{Bk}{J_1 J_2}\left(\frac{k}{J_2}(x_1 - x_3) - \frac{B}{J_2}x^4 + \frac{k_t}{J_2}x_5\right) + \frac{kk_t}{J_1 J_2}\left(-\frac{R}{L}x_5 - \frac{k_b}{L}x_4\right) - \frac{d\dddot{x_d}}{dt} \quad (24)$$

To stabilize the manipulator's link position under the influence of mismatch uncertainty, the control input can be designed as

$$u = \frac{LJ_1J_2}{kk_t}(-\vartheta - k_1 \mathrm{sign}(s) - k_2 s) \quad (25)$$

where k_1 and k_2 are the switching gain parameters. Substituting the designed control law (25) in (24), we get

$$\dot{s} = -k_1 sign(s) - k_2 s + (\frac{5ck}{J_1} - \frac{Bk}{J_1 J_2})\varsigma(x,t) \quad (26)$$

Stability Analysis

To analyze the stability of the close loop system using integral SMC approach, Lyapunov candidate function is considered, which can be expressed as

$$V = \frac{1}{2}s^2 \quad (27)$$

Taking derivative of Lyapunov function and putting (26) in (27), we get,

$$\dot{V} \le -k_2|s|^2 - |s|\left\{k_1 - \left|\left(\frac{5ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\right| |\varsigma(x,t)|\right\}. \quad (28)$$

Hence, the negative definiteness of the Lyapunov candidate function is proved i.e. $\dot{V} < 0$ which guarantees the asymptotic stability of the system trajectories in the presence of mismatch perturbations. In (28), the condition for asymptotic stability i.e. $\dot{V} < 0$ holds, if the switching gains in the designed control input satisfy the condition.

$$k_2 > 0, \ k_1 > \left|\left(\frac{5ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\right| \varsigma^*(x,t). \quad (29)$$

To work out the convergence rate of the steady state error, the integral sliding manifold with incorporated mismatch uncertainty is given as

$$s = \int (c^5 z + 5c^4 z + 10c^3 \dot{z} + 10c^2 \ddot{z} + 5c\dddot{z} + \frac{d}{dt}\dddot{z} + \varsigma(x,t))\,dt. \quad (30)$$

At sliding mode i.e. s = 0, the solution of differential equation (30) becomes;

$$z(t) = e^{-ct}(1 + \frac{c^2t^2}{2} + \frac{c^3t^3}{6} - \frac{c^4t^4}{6} + ct)z(0) + \varsigma(x,t)(\frac{e^{-ct}t^4}{24}), \quad (31)$$

where $z(t)$ is the steady-state error, whose convergence rate is given by (31). It is an exponentially decaying function i.e. $\lim_{t\to\infty} z(t) = 0$.

DISTURBANCE OBSERVER-BASED SMC APPROACH

The numerical analysis reported in the literature validates that traditional SMC has the most sensitive nature to mismatched uncertainty and results in a severe steady-state error, thus enforcing the system trajectories to deviate from the desired one. Contrary to this, Integral SMC counteract mismatched uncertainty in a robust way, but on the price of sacrificing nominal control performance of a system. DO-based SMC is a non-linear control technique, which acts in a robust fashion to counteract any perturbations while sustaining the nominal control performance of a system. In DO-based SMC, the internal state observer estimates the instantaneous value of mismatched uncertainty and updates the parameters in the control law.

The system formulated in (4)-(9) can be generally expressed as;

$$\dot{x} = f(x) + g_1(x)u + g_2\varsigma(x,t), \tag{32}$$

$$y = h(x), \tag{33}$$

where $x \in \Re^n$ is the states matrix, $\varsigma(x,t)$ is an unknown bounded uncertainty vector, u and y are input and output variables respectively, while $f(x)$, $g(x)$ and $h(x)$ are nonlinear smooth functions. To nullify the effect of the disturbance, the estimated version of disturbance is required, which is done with disturbance observer expressed as [28] and [29],

$$\dot{p} = -lg_2p - l(g_2lx + f(x) + g_1(x)u), \tag{34}$$

$$\hat{\varsigma} = p + lx, \tag{35}$$

where p is the internal state of the observer, l is the observer gain and $\hat{\varsigma}$ is the estimated disturbance. A novel sliding manifold for the system (20) can be expressed as,

$$s = 4\dddot{z}c + 6\ddot{z}c^2 + 4\dot{z}c^3 + zc^4 - \frac{mgh}{J_1}\cos(x_1)\dot{x_2} + \frac{mgh}{J_1}\sin(x_1)\,{x_2}^2 + \frac{mghk}{{J_1}^2}\sin(x_1) + \\ + \frac{k^2}{{J_1}^2}(x_1 - x_3) + \frac{k^2}{J_1J_2}(x_1 - x_3) - \frac{Bk}{J_1J_2}x^4 + \frac{kk_t}{J_1J_2}x_5 + \frac{k}{J_1}\hat{\varsigma}(x,t). \tag{36}$$

Taking derivative of (36) and substituting error dynamics, we obtain,

$$\dot{s} = \upsilon + \frac{kk_t}{J_1J_2L}u + \left(\frac{4ck}{J_1} - \frac{Bk}{J_1J_2}\right)\varsigma(x,t) + \frac{k}{J_1}\dot{\hat{\varsigma}}(x,t), \tag{37}$$

where υ is introduced for the sake of simplicity and can be expressed as,

$$\upsilon = c^4x_2 + 4c^3\dot{x_2} + 6c^2\ddot{x_2} + \\ 4c\left(\frac{mgh}{J_1}\sin(x_1)\,{x_2}^2 - \frac{mgh}{J_1}\cos(x_1)\dot{x_2} + \frac{mghk}{{J_1}^2}\sin(x_1) + \frac{k^2}{{J_1}^2}(x_1 - x_3) + \\ \frac{k^2}{J_1J_2}(x_1 - x_3) - \frac{Bk}{J_1J_2}x^4 + \frac{kk_t}{J_1J_2}x_5\right) - \frac{mgh}{J_1}\cos(x_1)\ddot{x_2} + \frac{mgh}{J_1}\sin(x_1)x_2\dot{x_2} + \\ \frac{2mgh}{J_1}\sin(x_1)\,x_2\dot{x_2} + \frac{mgh}{J_1}\cos(x_1)\,{x_2}^3 + \frac{mghk}{{J_1}^2}\cos(x_1)x_2 + \frac{k^2}{{J_1}^2}(x_2 - x_4) + \\ \frac{k^2}{J_1J_2}(x_2 - x_4) - \frac{Bk}{J_1J_2}\left(\frac{k}{J_2}(x_1 - x_3) - \frac{B}{J_2}x_4 + \frac{k_t}{J_2}x_5\right) + \frac{kk_t}{J_1J_2}\left(-\frac{R}{L}x_5 - \frac{k_b}{L}x_4\right) - c^4\dot{x_d} - \\ 4c^3\ddot{x_d} + 6c^2\dddot{x_d} - 4c\frac{d}{dt}\dddot{x_d} - \frac{d}{dt}\dddot{x_d} \tag{38}$$

The control law required to track the system's desired trajectory under the influence of mismatch uncertainty can be formulated as,

$$u = \frac{LJ_1J_2}{kk_t}\left(-\upsilon - k_1\text{sign}(s) - k_2s + \left(\frac{4ck}{J_1} - \frac{Bk}{J_1J_2}\right)\hat{\varsigma}(x,t)\right), \tag{39}$$

where k_1 and k_2 are the switching gain parameters. Back substitution of (39) in (37) gives,

$$\dot{s} = -k_1 \text{sign}(s) - k_2 s + \left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2}\right)\tilde{\varsigma}(x,t) + \frac{k}{J_1}\dot{\hat{\varsigma}}(x,t), \quad (40)$$

where $\tilde{\varsigma}(x,t)$ is the difference between the estimated and original disturbances i.e. $\tilde{\varsigma}(x,t) = \hat{\varsigma}(x,t) - \varsigma(x,t)$.

Assumption 3. The derivative of disturbance is bounded and must satisfy the condition $\lim_{t\to\infty} \dot{\varsigma}\,(z,t) = 0$.

Assumption 4. The estimated disturbance error $\tilde{\varsigma}(x,t)$ is norm bounded and is given by $\tilde{\varsigma}^*(x,t) = \lim_{t\to\infty} \sup\left|\tilde{\varsigma}(x,t)\right|$.

Lemma 1. Suppose the system (32) and (33) satisfies both the assumptions 3 and 4. In that case the disturbance estimation error $\tilde{\varsigma}(x,t)$ will converge to zero asymptotically i.e.

$$\dot{\tilde{\varsigma}}(x,t) + lg_2\tilde{\varsigma}(x,t) = 0 \quad (41)$$

This condition holds if the observer gain l is selected such that $lg_2 > 0$.

To design a disturbance observer, (34) and (35) can be expressed as

$$\hat{\varsigma} = p + lx \quad (42)$$

Taking time derivative of (42) and substituting (32) - (35), we get

$$\dot{\hat{\varsigma}} = -lg_2 p - l(g_2(x)lx + f(x) + g_1(x)u) + l(f(x) + g_1(x)u + g_2(x)\varsigma(x,t)). \quad (43)$$

Since $\dot{\tilde{\varsigma}} = \dot{\hat{\varsigma}}$, the solution of (43) can be expressed as,

$$\tilde{\varsigma}(t) = \tilde{\varsigma}(0)e^{-lg_2 t}. \quad (44)$$

It is well established in (44) that the observer gain l, if chosen such that $lg_2 > 0$, the disturbance estimation error would converge to zero asymptotically. To analyse the stability of a close-loop control system, Lyapunov candidate function is considered, which is as follows:

$$V = \frac{1}{2}s^2 \quad (45)$$

Taking time derivative of Lyapunov candidate function and substituting (40) and (41), we get

$$\dot{V} \le -k_2|s|^2 - |s|\left(k_1 - \left|\left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2} - \frac{lg_2 k}{J_1}\right)\right|\left|\tilde{\varsigma}(x,t)\right|\right), \quad (46)$$

where k_1 and k_2 are tuning parameters and can be expressed as,

$$k_2 > 0,\ k_1 > \left|\left(\frac{4ck}{J_1} - \frac{Bk}{J_1 J_2} - \frac{lg_2 k}{J_1}\right)\right|\ \left|\tilde{\varsigma}^*(t)\right| \quad (47)$$

It is clearly validated that the designed control law ensures the asymptotic stability of the system's trajectory as long as the gains in the control law and internal observer l satisfy their respective conditions.

SIMULATION RESULTS AND DISCUSSION

The aforementioned non-linear control approaches have been simulated in the MATLAB/Simulink environment. The key objective of the designed control algorithms is the tracking of the desired trajectory along with the stabilization of closed-loop system under the influence of a non-vanishing mismatch perturbation. The desired trajectory to be tracked is of constant amplitude.

Figure 2. depicts the performance of traditional SMC for desired trajectory tracking of the flexible joint manipulator. It can be seen that initially the system is relaxed and is perfectly

tracking the desired trajectory. However, at t = 10 s when the disturbance is injected into the system, the system trajectory deviates from the desired one and causes a constant steady state error. The steady state error is non-vanishing and exists till infinity.

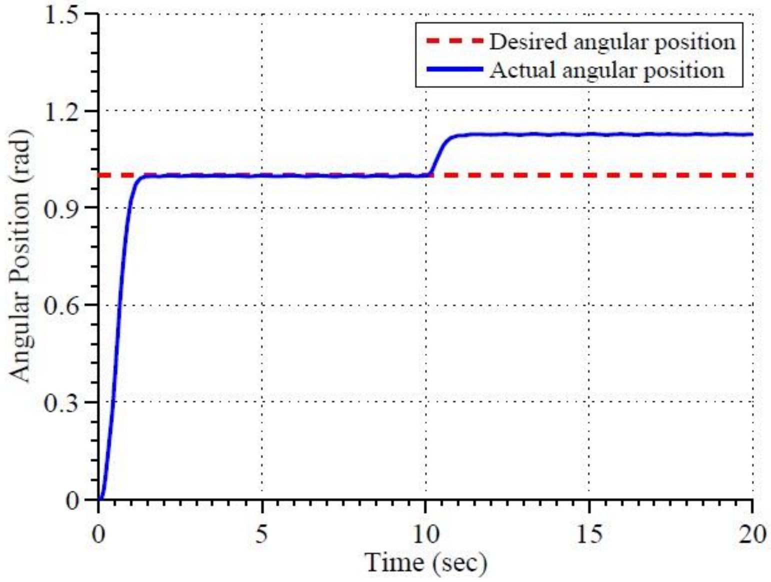

Figure 2. Desired trajectory tracking of the flexible joint manipulator by SMC law.

Figure 3. illustrates the effectiveness of integral SMC for the desired trajectory tracking by the flexible joint manipulator. The system is exposed to mismatched disturbances at t = 10 s. It is well established from the results that integral SMC shows excellent robustness property and eradicates the mismatch disturbance effectively but on the price of sacrificing nominal control performance, such as introducing an overshoot of high amplitude and suffering from long settling time.

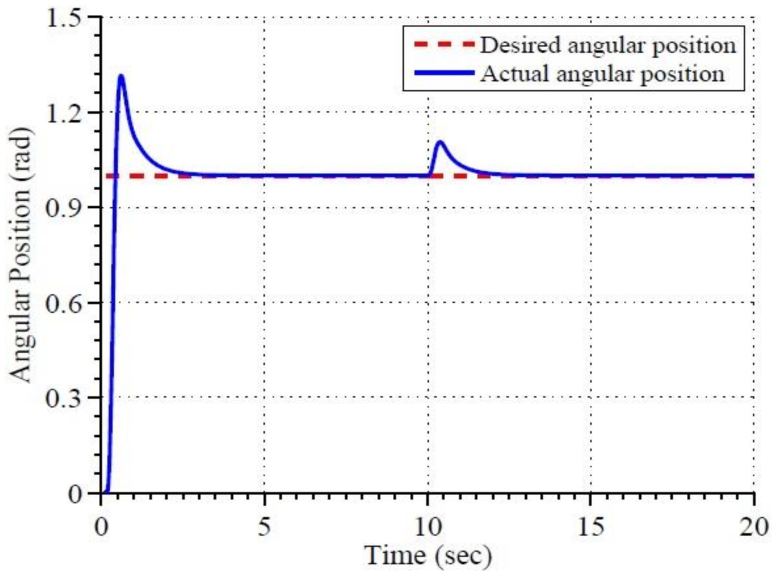

Figure 3. Desired trajectory tracking for flexible joint manipulator by integral SMC law.

Figure 4. demonstrates the control performance of nonlinear DO-based SMC. The proposed control method acts as traditional SMC in the absence of any perturbations. However, when a disturbance is introduced in the system at t = 10 s it counteracts the effect of disturbance. This is primarily possible due to the excellent sharp update law of the internal observer, which estimates the disturbance at each instant and thus nullifies its effects. It is pretty unambiguous

that the DO based SMC algorithm accurately compensates the mismatch disturbances, thus enabling the system to track the desired trajectory accurately with a negligible amount of steady-state error.

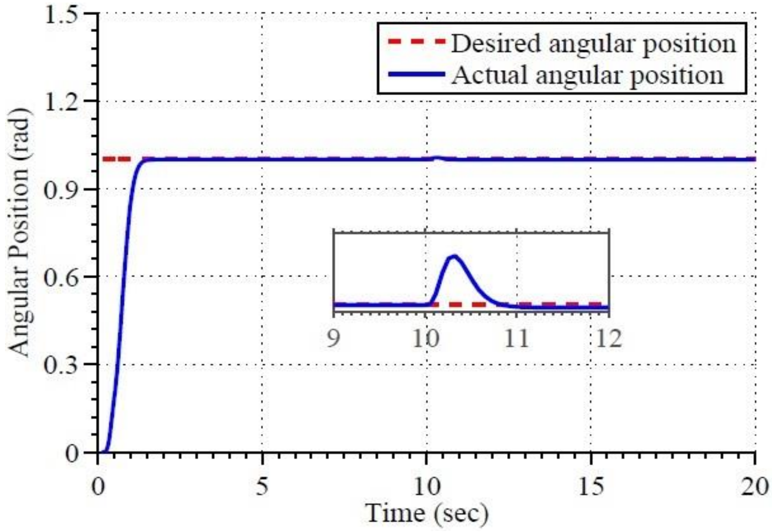

Figure 4. Desired trajectory tracking for flexible joint manipulator by DO based SMC law.

In Figure 5., the profile of estimation error between actual and estimated disturbance is presented. It is clearly evident that the estimation error approaches to zero because of the excellent update law of the proposed control scheme.

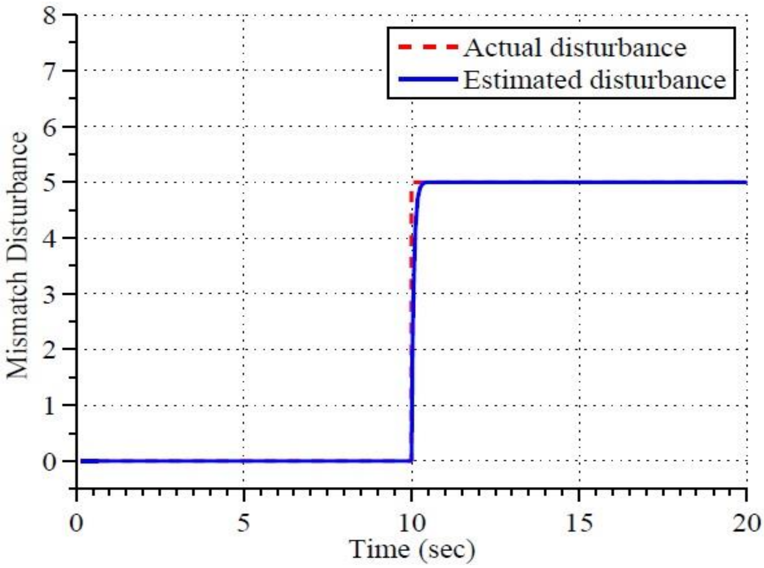

Figure 5. Profile of actual and estimated disturbance.

Figure 6. presents the comparative results of the three control strategies under investigation. The comparison is based on transient parameters like settling time and overshoot as well as steady state error. The comparative results justify the effectiveness of nonlinear DO-based SMC scheme.

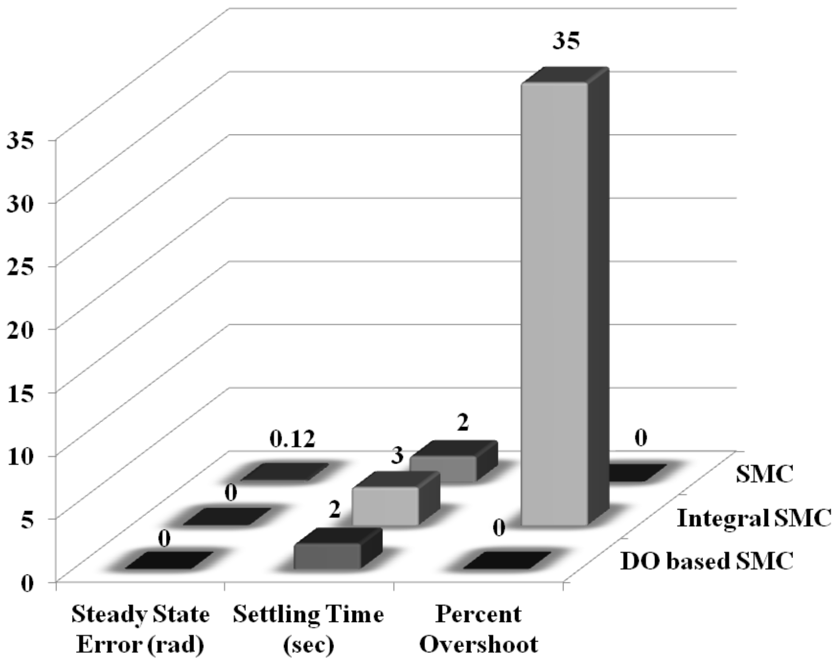

Figure 6. Performance comparison of proposed control strategies.

CONCLUSION

In this article, a robust sliding mode control approach, via a nonlinear DO, is designed for the desired trajectory tracking of a flexible joint manipulator with mismatch perturbations. The novelty exists in the sliding manifold, which is based on the estimated version of disturbance. Once the disturbance is estimated, the system trajectory converges onto the desired equilibrium point asymptotically. Furthermore, the comparison between traditional and integral SMC is carried out. It is verified from both the numerical and simulation results that traditional SMC and integral SMC schemes face degradation of nominal control performance in the presence of mismatched perturbations, while the DO-based SMC technique showed remarkable advantages including sustainability of control performance and alleviation of chattering problem. The aforementioned control schemes have been simulated in MATLAB environment to validate the effectiveness and analysis of the proposed control law. The results dictate that DO-based control algorithm accurately counteracts the mismatched disturbances while retaining the system nominal control performance.

REFERENCES

[1] Li, S.; Shao, Z.; Xu, P. and Yin, H.: *Robust adaptive control for coordinated constrained multiple flexible joint manipulators with hysteresis loop*.
Mathematical Problems in Engineering **2018**, Article ID 2507637,

[2] Alam, W.; Mehmood, A.; Ali, K.; Javaid, U.; Alharbi, S. and Iqbal, J.: *Nonlinear control of a flexible joint robotic manipulator with experimental validation*.
Strojniški vestnik-Journal of Mechanical Engineering **64**(1), 47-55, 2018,

[3] Spong, M.W.: *Modeling and control of elastic joint robots*.
Journal of Dynamic Systems, Measurement and Control **109**(4), 310-318, 1987,
http://dx.doi.org/10.1115/1.3143860,

[4] Zaher, M.H. and Megahed, S.M.: *Joints flexibility effect on the dynamic performance of robots*.
Robotica **33**(7), 1424-1445, 2015,
http://dx.doi.org/10.1017/S0263574714000848,

[5] Akyuz, I.H.; Yolacan, E.; Ertunc, H.M. and Bingul, Z.: *PID and state feedback control of a single-link flexible joint robot manipulator*.
In: *IEEE International Conference on Mechatronics*. pp.409-414, 2011,
http://dx.doi.org/10.1109/ICMECH.2011.5971320,

[6] Barjuei, E.S.; Boscariol, P.; Gasparetto, A.; Giovagnoni, M. and Vidoni, E.: *Control design for 3D flexible link mechanisms using linearized models*.
In: Advances on Theory and Practice of Robots and Manipulators. Springer, pp.181-188, 2014,

[7] Nguyen, M.N.; Tran, D.T. and Ahn, K.K.: *Robust position and vibration control of an electrohydraulic series elastic manipulator against disturbance generated by a variable stiffness actuator*.
Mechatronics **52**, 22-35, 2018,
http://dx.doi.org/10.1016/j.mechatronics.2018.04.004,

[8] Ferrara, A. and Incremona, G.P.: *Design of an integral suboptimal second-order sliding mode controller for the robust motion control of robot manipulators*.
IEEE Transactions on Control Systems Technology **23**(6), 2316-2325, 2015,
http://dx.doi.org/10.1109/TCST.2015.2420624,

[9] Li, Y.; Tong, S. and Li, T.: *Adaptive fuzzy output feedback control for a single-link flexible robot manipulator driven dc motor via backstepping*.
Nonlinear Analysis: Real World Applications **14**(1), 483-494, 2013,
http://dx.doi.org/10.1016/j.nonrwa.2012.07.010,

[10] Chen, Z.; Zhang, T. and Li, Z.: *Hybrid control scheme consisting of adaptive and optimal controllers for flexible-base flexible-joint space manipulator with uncertain parameters*.
In: 9th International Conference on Intelligent Human-Machine Systems and Cybernetics. Vol. 1. pp.341-345, 2017,
http://dx.doi.org/10.1109/IHMSC.2017.84,

[11] Ozgoli, S. and Taghirad, H.: *Fuzzy error governor: A practical approach to counter actuator saturation on flexible joint robots*.
Mechatronics **19**(6), 993-1002, 2009,
http://dx.doi.org/10.1016/j.mechatronics.2009.05.004,

[12] Zollo, L.; Siciliano, B.; De Luca, A. and Guglielmelli, E.: *PD control with on-line gravity compensation for robots with flexible links*.
In: IEEE European Control Conference. pp.4365-4370, 2007,
http://dx.doi.org/10.23919/ECC.2007.7068344,

[13] Bang, J.S.; Shim, H.; Park, S.K. and Seo, J.H.: *Robust tracking and vibration suppression for a two-inertia system by combining backstepping approach with disturbance observer*.
IEEE Transactions on Industrial Electronics **57**(9), 3197-3206, 2010,
http://dx.doi.org/10.1109/TIE.2009.2038398,

[14] Liu, C.; Cheah, C.C. and Slotine, J.-J.E.: *Adaptive task-space regulation of rigid-link flexible-joint robots with uncertain kinematics*.
Automatica **44**(7), 1806-1814, 2008,
http://dx.doi.org/10.1016/j.automatica.2007.10.039,

[15] Chien, M.-C. and Huang, A.-C.: *Adaptive control for flexible-joint electrically driven robot with time-varying uncertainties*.
IEEE Transactions on Industrial Electronics **54**(2), 1032-1038, 2007,
http://dx.doi.org/10.1109/TIE.2007.893054,

[16] Ailon, A., et al.: *Iterative regulation of an electrically driven flexible-joint robot with model uncertainty*.
IEEE Transactions on Robotics and Automation **16**(6), 863-870, 2000,
http://dx.doi.org/10.1109/70.897798,

[17] Ajwad, S.A.; Iqbal, J.; Islam,R.U.; Alsheikhy, A.; Almeshal, A. and Mehmood, A.: *Optimal and robust control of multi DOF robotic manipulator: Design and hardware realization*.
Cybernetics and Systems **49**(1), 77-93, 2018,
http://dx.doi.org/10.1080/01969722.2017.1412905,

[18] Ilyas, M.; Khaqan, A.; Iqbal, J. and Riaz, R.A.: *Regulation of hypnosis in propofol anesthesia administration based on non-linear control strategy*.
Brazilian Journal of Anesthesiology (English Edition) **67**(2), 122-130, 2017,
http://dx.doi.org/10.1016/j.bjane.2015.08.011,

[19] Ullah, M.I.; Ajwad, S.A.; Irfan, M. and Iqbal, J.: *Non-linear control law for articulated serial manipulators: Simulation augmented with hardware implementation*.
Elektronika ir Elektrotechnika **22**(1), 3-7, 2016,
http://dx.doi.org/10.5755/j01.eee.22.1.14094,

[20] Iqbal, J.; Ullah, M.I.; Khan, A.A. and Irfan, M.: *Towards sophisticated control of robotic manipulators: An experimental study on a pseudoindustrial arm*.
Strojniški vestnik-Journal of Mechanical Engineering **61**(7-8), 465-470, 2015,

[21] Ajwad, S.; Ullah, M.; Khelifa, B. and Iqbal, J.: *A comprehensive state-of-the-art on control of industrial articulated robots*.
Journal of Balkan Tribological Association **20**(4), 499-521, 2014,

[22] Hu, Q.; Xie, L.; Wang, Y.and Du,C.: *Robust track-following control of hard disk drives using improved integral sliding mode combined with phase lead peak filter*.
International Journal of Adaptive Control and Signal Processing **22**(4), 413-430, 2008,
http://dx.doi.org/10.1002/acs.995,

[23] Chang, Y.-C. and Yen, H.-M.: *Robust tracking control for a class of electrically driven flexible-joint robots without velocity measurements*.
International Journal of Control **85**(2), 194-212, 2012,
http://dx.doi.org/10.1080/00207179.2011.643241,

[24] Ajwad, S.; Adeel, M.; Ullah, M.I. and Iqbal, J.: *Optimal v/s robust control: A study and comparison for articulated manipulator*.
Journal of the Balkan Tribological Association **22**(3), 2460-2466, 2016,

[25] Khan, O.; Pervaiz, M.; Ahmad, E. and Iqbal, J.: *On the derivation of novel model and sophisticated control of flexible joint manipulator*.
Revue Roumaine des Sciences Techniques-Serie Electrotechnique et Energetique **62**(1), 103-108, 2017,

[26] Alam, W.; Ali, N.; Ahmad, S. and Iqbal, J.: *Super twisting control algorithm for blood glucose regulation in type 1 diabetes patients*.
In: *15th IEEE Conference on Applied Sciences and Technology*. pp.298-303, 2018,
http://dx.doi.org/10.1109/IBCAST.2018.8312239,

[27] Ajwad, S.A.; Iqbal, J.; Ullah, M.I. and Mehmood, A.: *A systematic review of current and emergent manipulator control approaches*.
Frontiers of Mechanical Engineering **10**(2), 198-210, 2015,
http://dx.doi.org/10.1007/s11465-015-0335-0,

[28] Chen, W.-H.: *Nonlinear disturbance observer-enhanced dynamic inversion control of missiles*.
Journal of Guidance, Control, and Dynamics **26**(1), 161-166, 2003,
http://dx.doi.org/10.2514/2.5027,

[29] Chen, M. and Chen, W.-H.: *Sliding mode control for a class of uncertain nonlinear system based on disturbance observer*.
International Journal of Adaptive Control and Signal Processing **24**(1), 51-64, 2010.